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# THE NECESSITY OF THE MATERIAL CONDITIONAL

## I

That the notion of the material conditional occupies a central place in contemporary logic is no longer in doubt, and one would have thought that the resistance to its employment had died down long back. This apparently is not so. Even now many students of logic when first exposed to that notion feel intrigued and frustrated by it, and the attempts to carry conviction to them quite often fail. Recently a young and intelligent teacher narrated to me how some of his M. A. students violently reacted to the notion and called it preposterous and worthless, and how his repeated attempts to defog their minds failed miserably ! (See Tarski 1946 : 27). And I suspect that his is not a lone experience. It is this situation which has prompted me to flog the dead horse once again, and to try to show that the exercise is after all not wholly worthless.

To begin with, a word or two about the paradoxical character of the title. Since a material conditional need not be necessary (i.e. need not be a logical truth) the reader might wonder in what sense one could talk about its necessity. The sense is simply this that the material conditional is necessarily involved, directly or indirectly in the conceptual framework of our thought and language; that it is a consequence of some of our most basic and unchallenged intuitions and that, therefore, we can not do without it. The burden of this paper is to emphasise and substantiate the claim underlying this sense.

## II

The material conditional is truth functional in character and is defined solely in terms of its truth conditions. It is false just in case the antecedent is true and the consequent is false, and true otherwise. This means that the material conditional is true both when its antecedent is false (whether or not its consequent also is false), and when its consequent is true (whether or not its antecedent also is true). It also means, of course, that the material conditional is true when its antecedent and consequent are both

true. The following, then, are all examples of a true material conditional :

1. If Charan Singh survives the confidence vote (on August 20, 1979) then Janata (S) will attract more defectors.
2. If grass is green then Mrs. Gandhi is a politician.
3. If grass is white then Mrs. Gandhi is a professor.
4. If grass is white then Mrs. Gandhi is a politician.

While example (1) will be accepted as perfectly natural, it is (?)—(4) that are likely to be felt as counterintuitive and to engender resistance. This is mainly because in (1) there is some sort of a connection between the sentential components, whereas there is no connection whatsoever between the components of (2)—(4). Examples (2)—(4) in fact illustrate the so-called paradoxes of material implication, namely, that a false statement (materially) implies any statement whatever, and that a true statement is implied by any statement whatever. These paradoxes amount to saying that a material conditional can be true without there being any connection between its antecedent and consequent, and this is what worries some people and generates resistance to the material conditioal. In ordinary language and in traditional logic, the conditionals considered are generally the ones which have such a connection, and thus an unconscious conditioning (or 'habit' as Hume would call it) is formed in our minds which makes us expect it in every case. Another consideration responsible for the resistance against the material conditional is that a sentence containing 'if....then....' is normally used when the truth-values of its sentential components are not known, as is the case with example (1)[1]. If the truth-values of the components are known, there no longer is any need for the compound sentence. If for example, we already know that the Janata (S) will attract more defectors, there is no point in saying "If Charan Singh survives the confidence vote, Janata (S) will attract more defectors".

But our resistance to the material conditional is no more justified than our belief in the causal connection (as, again, Hume would say). Our intuition which demands that there be *always* a connection between the antecedent and the consequent of a material conditional is believed by many to be in conflict with some of our other and apparently more fundamental intuitions.

In a situation like this it is necessary to take a closer look at the seemingly conflicting intuitions to determine whether there really is a conflict, and if there indeed is one, to isolate the errant intuition and to deal with it so as to eliminate the conflict. Among the intuitions involved in the case of the material conditional are those concerning negation and conjunction. Our intuition regarding negation— that the negation of a true statement is false, and that the negation of a false statement is true—is clear enough and is accepted on all hands. Our intuition regarding conjunction is not as clear, but the truth functional interpretation is by and large accepted and it is agreed that a conjunction is true if and only if both of its conjuncts are true and false otherwise. The truth-functional interpretation of a conjunction also includes commutativity, associativity and idempotency. It is recognised of course that in ordinary language there are uses of 'and' which are not commutative ( hence not truth functional ) ( see Strawson 1964: 80), but these have no immediate bearing on the material conditional and can be ignored in the present context.

Given these generally accepted notions of negation and conjunction, we may ask under what circumstances a conditional, irrespective of whether or not there is any connection between the antecedent and the consequent, is false. At least in the case of a large majority of conditionals there is only one answer to this question, namely, that a conditional is false if and only if its antecedent is true and its consequent is false. In other words, the following is a logical truth .

$$5. \quad -(p \rightarrow q) \leftrightarrow (p \,\&\, -q).$$

But once we have this result, we are logically committed to the material conditional. For, (5) is logically equivalent to

$$6. \quad (p \rightarrow q) \leftrightarrow (p \,\&\, -q)$$

which in turn is logically equivalent to

$$7. \quad (p \rightarrow q) \leftrightarrow (-p \vee q).$$

( 7 ) Specifies the circumstances under which a conditional is true. It says that a conditional is true either when the antecedent is false or when the consequent is true. Thus (5) and (7) together explicitly and completely describe the truth conditions, and hence in effect define, the material conditional.

We see then that the material condiotional is a direct conse-

quence of some of our intuitions to which we generally have no objection. As such, it also should evoke no objection and if there is any other intuition which conflicts with these, the fault must lie with the former. We have, therefore, to reject our intuition which stipulates that a connection between the component of a conditional is a *sine qua non* of its truth or falsity, and hence of its very meaningfulness, in so far as it is incompatible with those of negation and conjunction. Its rejection of course, does not mean that the presence of such a connection, interferes in any way with the truth-value of a conditional, but only that it is immaterial to the truth-value whether the connection is present or not. The cases of conditionals which altogether lack such a connection and are yet considered true or false are not entirely lacking in our ordinary language (see Tarski 1946 : 26-27). Such cases should make it clear that the class of material conditionals is wider than that of ( truth - functional ) conditionals involving connection. Every ( truth - functional ) conditional involving connection is a material conditional but not conversely. As a matter of fact, it is questionable whether we really have an intuition which demands the presence of connection in *every* conditional. The truth, as remarked earlier seems to be rather that we are attuned to expect such a connection because it is present in most cases, even though we occasionally do use conditionals that cannot be said to involve connection of any kind. The absence of a connection should alert us to the unusual case. If this is so, there really is no conflict of intuitions involved in the case of the material conditional.

What gives substance to this feeling that there should be a connection between the antecedent and the consequent of a conditional is, as noted before, the fact that ordinarily a conditional is used when the truth-values of its components are unknown. And often such a conditional is accepted as true as shown by (1). What enables us to assert a true conditional even in the absence of a knowledge of the truth-values of its components is the particular connection between its components (Tarski 1946 : 24-25; Quine 1972 : 21-22). But this is quite all right, and does not in the least mean that the truth-value of a conditional does not depend on the truth-values of its constituents even though the latter truth-values may not be known.

It is quite consistent with using conditionals when the truth-values of its components are known. Truth-functionality is an objective relation between the truth-value of a ( truth - functional) sentence and the truth-values of its components, and is not affected by whether or when these truth-values are known.

The foregoing considerations should make it clear that it is misleading to describe the material conditional ( as is done by some) as constituting some strange or peculiar or special sense of the words ' if...then.... ,' far removed from ordinary usage. For, as seen above, it is but a natural and inevitable consequence of intuitions which we do not regard as strange or peculiar. The correct way to describe the material conditional is perhaps to say that it constitutes the weakest sense of ' if. . . . then . . .'

III

I remarked above on the general agreement regarding when a conditional is false. [ See (5) ]. Another general agreement which is inextricably connected with (5), and therefore with our in-uitions regarding negation and conjunction is that a conditional together with the negation of its consequent entails (or logically implies) the negation of its antecedent. In other words,

8. $[ ( p \rightarrow q ) \,\&\, -q ] \rightarrow -p$

is a tautology. Underlying ( 5 ) and ( 8 ) is, of course, the belief that a conditional ( at least the most common and familiar variety of it ) is truth functional; that when its elementary sentential components are assigned truth-values, it is also thereby assigned a specific truth-value. Taking (5) and (8) as our crucial test, we can see that of all the possible (truth-functional) interpretations of the expression ' if.. . .. then . . ..' (or its equivalents), the one embodied in the material conditional is the only acceptable interpretation. There are in all sixteen possible truth-functional interpretations as listed below :

| p, q : | If p, then q | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T T: | T | T | T | F | F | F | T | F | T | T | T | F | F | F | T | F |
| T F: | F | F | F | F | F | F | F | F | T | T | T | T | T | T | T | T |
| F T: | T | F | F | T | F | F | T | T | T | F | F | T | F | F | T | T |
| F F: | T | T | F | T | T | F | F | F | T | T | F | T | T | F | F | F |
| | i | ii | iii | iv | v | iv | vii | viii | ix | x | xi | xii | xiii | xiv | xv | xvi |

Interpretations (ix)—(xvi) assign T to a conditional even when its antecedent is true and its consequent is false. They thus go against (5), and have to be rejected. Similarly, interpretations (ii)—(viii) go against (8), and have to be ruled out. Under each one of them there is at least one assignment which assigns F to (8). But this is unacceptable since (8) is a logical truth and can never be false. Consider, for example, the third assignment in interpretation (ii). This makes "$p \rightarrow q$", and hence the antecedent of (8), false. It also makes the consequent of (8) true, and thereby (8) as a whole false. Take again the fourth assignment in interpretation (iii). It also makes the antecedent of (8) false and the consequent of (8) true and so as per the third assignment of the same interpretation (8) as a whole has to be false. The same result is obtained by trying out the remaining interpretations (iv)—(viii). Thus, the only interpretation that is faithful to (5) and (8) is the remaining interpretation (1), and that is the one represented by the material conditional.

That the material conditional is not a new-fangled idea of the modern logician, and that the fuss made about it is not justified can also be seen from the fact that it was fully recognised as early as 4th century B. C. by the Megarian logician, Philo. The Stoic logician Sextus Empiricus (3rd Century A. D.) states Philo's view with astonishing clarity :

" Since, then, there are four possible combinations of the parts of a conditional—true antecedent and true consequent, false antecedent and false consequent, false and true, or conversely true and false—they say that in the first three cases the conditional is true....; but in one case only is it false, namely, whenever the antecedent is true and the consequent is false" (cited in Mates 1965:204).

This is not to suggest that the Megarians and the Stoics accepted Philo's view without question. On the contrary, there was a lively controversy regarding the nature of a conditional (as also of other truth-functions) and alternative views were also advanced (Mates 1965 : 203; Tarski 1946 : 27 fn 3). But the point is that the notion of the material conditional is almost as old as the founding of logic (by Aristotle in 4th Century B. C.) even though by the vicissitudes of history it was lost sight of until revived in modern times by Frege and Peirce.

## IV

The case of the material conditional is similar to that of the logician's disjunction, though curiously enough the detractors of the former raise little, if any, objection against the latter (Tarski 1946:27). Ordinary discourse presents us with two senses of 'or', the so-called inclusive and exclusive senses. In the inclusive sense the disjuncts of a true disjunction are collectively exhaustive i. e. at least one of them is true though both may be. In the exclusive sense, the disjuncts in addition to being collectively exhaustive are also mutually exclusive; that is, only one of them is true and the other is false. The inclusive sense is said to be the weaker, and the exclusive sense the stronger, of the two senses, because while the former gives less information, the latter gives more information about the nature of the disjuncts. In logic the weaker sense is preferred. One reason of this preference is perhaps that the stronger sense is easily definable in terms of the weaker sense (along with the already available concepts of negation and conjunction). In other words, where ' $\bar{v}$ ' is the symbol for the exclusive ' or ',

9. $(p \bar{v} q) \leftrightarrow (p v q) \& -(p \& q)$

is a logical truth. So is its logical equivalent,

10. $(p \bar{v} q) \leftrightarrow (p v q) \& (-p v -q)$[2].

But this interdefinability of the stronger sense in terms of the weaker sense is not by itself a theoretically adequate reason for preferring the weaker sense. For, it can be said in favour of the stronger sense also that the weaker sense is definable in terms of the stronger sense ( together with conjunction). That is,

11. $(p v q) \leftrightarrow (p \bar{v} q) \bar{v} (p \& q)$

is also, like (9) and (10), a logical truth. In principle, therefore the weaker sense is no less dispensable than the stronger sense.

Presumably another reason that is responsible for the logician's preference for the weaker sense of ' or ' is that mutual exclusiveness or otherwise of the disjuncts of a disjunction is a matter of content which logic as a formal science cannot take into account (Mates 1965 : 75). Collective exhaustibility, on the other hand, is a matter of form, and is constitutive of the very

meaning of the *logical* word 'or'. This again, is not an altogether satisfactory reason, since to some extent it is arbitrary which words are classified as logical and which others as non-logical. Just as collective exhaustibility is built into the logical constant 'or', mutual exclusiveness also can be built into it. This is in fact what is done by devising the symbol ' $\bar{v}$ ' mentioned above, and defining it partly in terms of the symbol 'v'.

The decision, then, between the inclusive and the exclusive senses of 'or' is in principle arbitrary, and it seems that the logician's preference for the weaker sense can be justified, if at all, only on grounds of practical convenience (neatness, economy etc., of the system). Perhaps the inclusive sense ties in better with our ordinary notions about disjunction. It has also the advantage of being wider, the logical rules governing inferences involving it governing also those involving the stronger sense. The converse, of course, is not true. There have to be additional rules for inferences involving the stronger sense.

That the case of the material conditional is similar to that of the weak disjunction in important respects can be seen from the following considerations. The quarral about the material conditional is not so much about its truthfunctionality as about whether the truthfunctionality is to be limited to those conditionals where there is some connection between the antecedent and the consequent. This means that like 'either···or···', 'if···then···' also can be said to have a weak sense and a strong sense. The weak sense does not require any connection between the antecedent and the consequent while the strong sense does. As in the case of disjunction, in the case of conditional also, the weaker sense is preferred perhaps for similar reasons, namely, first, that the weaker sense covers the stronger sense and that the rules (such as those of affirming the antecedent and denying the consequent) governing the former govern the latter as well; and, second, that connection is a matter of content and can have no role in determining the logical status of a conditional. Further, in the case of both disjunction and conditional, the components usually have some connection, though not always, and both disjunctions and conditionals are used usually, though again not always, when the truth-values of the sentential components are unknown (Quine 1972: 21-22; Tarski 1946: 22). These similarities

also make it difficult to understand why there should be objections to accepting the weaker sense in the one case there are few, if any, in the other case. Besides, there is the further fact that the material conditional (i. e. the conditional in the weaker sense) is definable in terms of (weak) disjunction and negation. One would expect that any objection against the material conditional would carry over to the weak disjunction (since, as noted before, negation is beyond suspicion). That this is not so also shows that the resistance to the material conditional is not justified.

This, of course, is not to say that there is no difference between the case of the material conditional and that of the weak disjunction. In fact, the difference is so important as to render the case of the material conditional even stronger than that of the weak disjunction. It is that while the weaker and the stronger senses of 'or' are interdefinable (with the help of some other concepts), the weaker and the stronger senses of 'if' are not. The reason for this is that the stronger sense of 'if', unlike the stronger sense of 'or', is not clearly characterizable. There is no vagueness or ambiguity in the characterization that the disjuncts of a disjunction are mutually exclusive (i.e. that they cannot be true together). But there is considerable vagueness and ambiguity in the characterization that the antecedent and the consequent of a conditional are somehow connected. Such a connection can be of different kinds—definitional, logical, causal etc; and it is not much help to say that the antecedent and the consequent are connected unless we also specify the type of connection involved. But once we try to do this the stronger sense ceases to be one sense, but splits into several senses. A general, but precise, characterisation of all these senses is not possible.[3]

Even if a precise characterization of the stronger sense of 'if' were possible, it is doubtful if it could be defined in terms of the weaker sense (and some other known concepts). If it could not be so defined and yet we were to prefer it to the weaker sense, we would have to sacrifice the weaker sense altogether, and that would impoverish our conceptual apparatus. The impoverished apparatus would not be able to account, for example, for those conditionals of ordinary language which do not involve connection. In any case, lack of interdefinability of

the two senses of 'if' does not come in the way of its truth-functionality. The weaker sense—that represented by the material conditional—more than adequately takes care of it. In fact, lack of interdefinability makes the case of the material conditional for our acceptance stronger in that its rejection results in conceptual impoverishment.

## V

The material conditional, then far from being contrary to our intuitions is a necessary consequence of some of our important intuitions, and actually provides the least objectionable interpretation of 'if ... then ...'. All the fuss that is made about it is without justification. Truthfunctionality is an essential feature of a vast majority of conditionals, and the only interpretation of them which preserves that features and is yet faithful to our ideas of their nature is the one provided by the material conditional. But this is by no means to say that it is the only interpretation of all conditionals. Ordinary language also provides types of conditionals which are not truthfunctional. One such type, the most important and common, is made up of counterfactual conditionals. Another type, rather uncommon, is made up of what Austin (1961 : 159–60) calls conditionals of doubt or hesitation (see also Edwards 1967 : IV 127). Still another type consists of universal conditionals. Counterfactual (or contrafactual, or unfulfilled or contrary-to-fact) conditionals (or simply counterfactuals) are in the subjunctive mood unlike material conditionals which are in the indicative mood[4] and are illustrated by statements like the following :

12. If Sri Ram Narain were elected, he would have become a minister.

13. (Even) if Sri Ram Narain were elected, he would not have become a minister.

That counterfactuals are not truthfunctional is easily seen. On a truth-functional analysis both (12) and (13) would have to be true since their antecedent is false (Ram Narain being, as a matter of fact, not elected). But clearly one of them is intended to be true and the other false. Again,

14. (Even) if Sri Ram Narain had become a minister he (still) would not have been elected.

which is a contrapositive of (13), cannot be considered logically equivalent to it, as required by the truth-functional analysis. This also means that (13) and (14) do not conform to (8). The problem of counterfactuals is to define circumstances under which a given counterfactual [such as (12)] holds while the opposing conditional with the contradictory consequent [such as (13)] fails to hold (Goodman 1965 : 4). This problem has a vital bearing especially on Philosophy of Science, but is yet far from being solved.

Probably Austin (1961 : 159-60) is among the first, if not the first, to draw attention to conditions of doubt or hesitation. Examples of these are :

15. There are biscuits in the side board if you want them.

16. I paid you back yesterday if you remember.

The distinguishing feature of such conditionals is that their consequents can be legitimately inferred from them. They also share with counter-factuals the feature that they are not logically equivalent to their contrapositives. For example, (15) is not logically equivalent to its contrapositive.

17. If there are no biscuits in the side board, you do not want them.

An adequate analysis of such conditionals also is yet to be found.

Universal conditionals are such sentences as :

18. If any thing is a man it is mortal.

Or in symbols,

18. $(x)\,(Mx \rightarrow Tx)$.

Such conditionals also are not truthfunctional, since their components are not sentences, and cannot be assigned truth-values. But an adequate analysis of them is available in terms of the material conditional : they are regarded as bundles of singular material conditionals instantiating the relevant conditional propositional function (Quine 1972: 20; see Tarski 1946: 25-26).

But whatever the analyses of these types of non-truth-functional conditionals, they do not conflict with the material

conditional. The material conditional correctly captures one of the different uses of 'if', one which is very important and widespread, and makes no claim to capturing other uses (Strawson 1964: 35-40; Austin 1961 : 153-80). As such it is to be heartily welcomed, not demurred.

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## NOTES

1. This paper was completed before the confidence vote was taken.
2. Of course, the stronger sense of 'or' can also be defined in terms of negation and conjunction alone as "—(P & q) & — (—P & —q)," but that is beside the point here.
3. Attempts to determine, the nature of the connection between the antecedent and the consequent, initiated by C. I. Lewis, are, of course, continuing, but these are in the area of 'relevant' or non-classical lo gics (as against first-order or classical, including truthfunctional, logic). The concept of the material conditional is consistent with such attempts in non-classical logics.
4. Quine ( 1972:21 ) says that the material conditional is put forward 'at most as an analysis of the ordinary singular conditional in the indicative mood". But this seems to be a slip since the components of a material conditional need not be singular; they can as well be general, as in, for example, "If all mangoes are sweet then some marbles are green."