Indian Philosophical Quarterly, Vol. IX, No. 3, April (1982)

REDEFINING UNIVERSALS

Our aim here is to *redefine universals* by means of a *symbolic representation*, built upon a mathematical model. In the words of *Arthur Eddington*, "It is not at all necessary that every individual symbol that is used should represent some thing in common experience or even something explicable in terms of of common experience."[1] *F. Ramsey* in "*Foundations of Mathematics*" shows that for *Russell*, the class of universals is 'the sum of the class of predicates and the class of relations".

I have tried to modify this view-point by considering the *class of universals as the class of* relations alone.

I

BASIC CONCEPTS AND ASSUMPTIONS INVOLVED

I shall explain below the concept of an '*equivalence relation*' (relevant to our discussion).

Defn. : Consider a set X such that X is non-empty. Consider the cartesian product—

$X \times X = \} (x_1, x_2) / x_1 \in X_1\ x_2 \in X \{$

Take a non-empty subset R of X × X i.e $R \subset X \times X$
Then if—

(i) $x_1, Rx_1, \forall\ x_1 \in R$ (property of reflexivity).

(ii) $x_1\ Rx_2 \Leftrightarrow x_2\ Rx_1\ \forall\ x_1, x_2\ \in R$ (property of *Symmetry*).

(iii) $x_1\ Rx_2, \quad x_2\ R\ x_3 \Rightarrow x_1\ R\ x_3, \quad \forall\ x_1, x_2\ x_3 \in R$ (property of *transitivity*).

R is an equivalence relation defined on set X

Now we define "*multi-equivalence*[2] *relations*".

Defn. : Let X be a non-empty set.

Let $x,\ y, \in X$.

Then x, Ry iff x R_i y for all i ....

Where R = (R_1, R_2, ... Rn) and each R_i is an equivalence relation on X.

In words this means that—

"x is R — related to y" iff

"x is Ri — related to y".

It is easy to *check*[3] that R is an equivalence relation.

Defn. : If R is an equivalence relation, and $a \in X$ then the '*equivalence class*' containing a denoted by (a) is the set $\} x \in X / x Ra \{$.

Note[4] : (a) = (b) iff aRb.

To avoid misconceptions associated with words like " predicates " or " properties " or " qualities" we shall consider an object as having ' *aspects* '.

(i) When we consider a single aspect of objects we shall arrive at the concept of *Simple Universals*[5] e. g. whiteness

We shall see later that a single aspect can be represented by a simple equivalence relation R on set of objects X.

(ii) When we consider more than one aspect of objects we shall arrive at the concept of *Complex Universals.*[6] We shall see later that it can be represented by a multi-relational equivalence relation R = (R_1, R_2...) e. g. humanness.

Analogous to *Plato's remark*[7] in *Book X* of the *Republic* we shall make the following assumption.

Assumption I : The admittance of any universal U implies atleast one instance of it (which we shall) call '*primary instance*' denoted by I.

II

UNIVERSALS REDEFINED AS EQUIVALENCE RELATIONS

Let X be the universe of discourse.

Let U be any universal (to be redefined).

We *define a relation* R on X as follows :—

For all $x, y \in X$.

x Ry iff both x and y are instances of **U**.

Then it can be easily *checked*[8] that R is an equivalence relation.

Note : R may be a simple or multi-equivalence relation.

We consider R as the logical representation of the Universal U.

RECONCILIATION WITH LANGUAGE

Most Linguistic analysts like *Ryle* and *Pears* feel that the problem of universals is contained in the *problem of naming;* viz. the problem of how abstract and common names function in Language.

Mccloskey[9] comes *closer* to my definition when he says that " the problem of universals is not merely one of naming but of *resemblance* ".

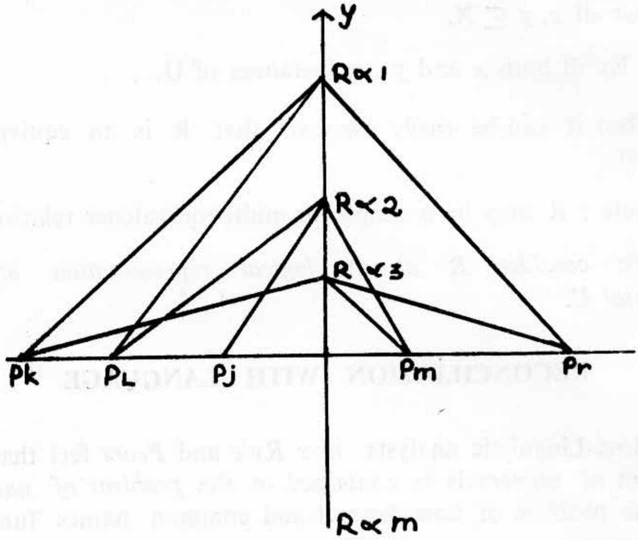
Moore[10] has also at one point said that " is a universal is simply logically equivalent to is either predicable of something or is a relation. ".

Wittgenstein appears to have also thought along similar lines as the view represented here. For he says, " games, what is common to them all ? "

" There is nothing at all that is common but one can only see *similarities or relationships* and a whole series at that."

We can pictorially depict this view as follows. Let $F = (R_\alpha)_{\alpha \in A}$ where A is an index set, be a family of resemblances. (Viz. equivalence relations reperesenting universals). Conasider the cartesian co-ordinate system. We shall represent universals along the y-axis, whereas their instantiations will be mapped on the x-axis.

PICTORIAL REPRESENTATION



P_k, P_v, P_r .... being instantiations of $R\alpha_1$,

P_l, P_j, P_m ..... being instantiations of $R\alpha_2$

P_k, P_m, P_r .... being instantiations of $R\alpha_3$

and so on.

Note :— The use of suffixes α_1, α_2, α_3, need not be interpreted that the use of the family of equivalence classes } Rα { α ∈ A is a countable set.

J. R. Bambrough has stated that[11] " *all propositions involve universals* " i. e. according to our interpretation " all propositions involve the use of some relation R, where R is the logical representation of universal U."

In particular, consider the equivalence relation R ∝ *o* and the class C ∝ *o* where

C ∝ *o* = } / P/P is a proposition involving R ∝ *o* {

To be more explicit we take the following example :—

Let P be the propositional function

" *x* is bald "

We consider the class of all such propositional functions where ' bald ' is the *logical predicate.*

(What Bambrough calls logical representation of universal U in a propositon)

Hence C = { P/P is a propositional function such that 'bald' is the logical predicate 'bald' in 'P' }

Now[12] let P be an *atomic proposition.* Therefore, P corresponds to a fact in the world.

Hence for each occurrence of the logical predicate 'bald' there corresponds atleast one instance in the world. Then the class [I].

[I] = } I/where I is an instance in the world corresponding to each occurrence of logical predicate 'bald' in P. {

Then we *define C as the naming class of (I).*

The 'logical predicate' occurring in C enables us to name the relation which relates the members of (I).

Perhaps this may be satisfying to the '*nominalist*' to the extent that the "problem of naming" is the distinguishing factor between relations.

Clearly the equivalence relation R which is the representative of some universal U is *not different* from the equivalence relation R which is the representative of some other universal U'-in *so far as R and R' are considered as mere "logical constructs" representing universals.*

It is only the 'naming class' which allow R to be named differently from R'.

But then the question arises as to how *does one actually determines the "naming class"*. Or rather when and why and how does one use a logical predicate corresponding to an instance.

This problem will be tackled in the next section "Reconciliation with Experience".

It will be shown that one empirically recognizes an instance. This empirical recognition is determined by "*measurement methods*", e. g. one ascertains that the colour seen by one is an instance of 'redness' by its wavelenth, viz. given a spectral line, there corresponds a unique wave length.

One then expresses the aspect of an instance as a 'logical predicate' viz. red, in the proposition. Hence one arrives at the 'naming class' that enables us to name the relation existing between members of its corresponding equivalence class as 'redness'.

III

RECONCILIATION WITH EXPERIENCE THROUGH PHYSICS (REALIST VIEW-POINT)

Mccloskey[13] also thinks that " a behaviouristic account of the problem of resemblance and of referring rules could be offered i. e. an account of names in terms of rules governing their use, must explain why a name applies to the particular group of things and not to other things. This would involve reference to resemblance."

The *realist* tries to explain properties and qualities in terms of other properties and qualities until he is forced to admit the resemblances between the objects that are characterized by such a property or quality.

Let X be the universe of discourse.

Let $\} R_\alpha \{_{\alpha \in A}$ be a family of equivalence relations on X. Corresponding to each R_α we have a family of equivalence classes $\} [I_\alpha \beta] \{_{\alpha \in A}$ i. e. $[I_\alpha \beta] = \} I / I_\alpha \beta \, R_\alpha \, I \{$

Then $X = \bigcup_{\alpha \beta A} [I_\alpha \beta]$. Suppose R_w is an equivalence relation characterizing the universal of whiteness. Then we get the equivalence class $[I_w \beta] = \} I / I_w \beta \, R_w \, I \{$. Now consider set of all subsets of $[I_w \beta]$, viz $P[I_w \beta]$. Then[14] $P[I_w \beta]$ is a σ – algebra.

Hence $([I_w \beta], \; P[I_w \beta])$ is a *measurable space.*

According to the[15] nature of $[I_w \beta]$ viz. Physical or mental space we may be able to *define a measure* μ on $P(I_w \beta)$.

Now in our case 'given a spectral line (for the white colour) there exists a unique wave length'. Hence for *the instance* I_w *we have the corresponding measure* $\mu(I_w)$.

Therefore, corresponding to the set $(I_w \beta)$ we have the set of elements of the form $\mu(I_w \beta)$. Denote this *set by* S.

Define a relation R_w on S as follows :—

$\mu\,(I_{wi})\ Rw'\ \mu\,(I_{wj}) \longleftrightarrow I_{wi}\ Rw\ I_{wj}$.

For all $I_{wi} \in [\,I_w\beta\,]$ and all $\mu\,(I_{wi}) \in S$. It can be easily checked[16] that Rw' is an equivalence relation on $\mu\,(I_w\beta)$.

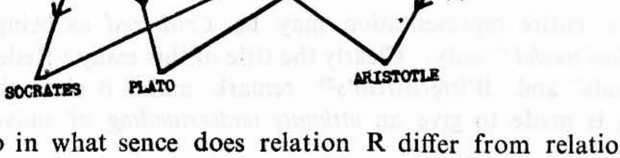
Therefore corresponding to the equivalence relation Rw characterizing universal of whiteness we have the equivalence relation Rw′ i. e. we have shown correspondence between the abstract "*logical construct Rw*" and the calculable "*Physical construct Rw′*".

I think this *merges* to a certain extent the nominalist and the realist view-points.

IV

ANTICIPATED CRITICISMS

It may be argued against my view that how does one *distinguish* one universal from the other if each is an equivalence relation. Hence this would imply a nominalist view-point in another guise for merely defining universals by naming differently the same equivalence relation. This criticism can be represented pictorially[17] as follows :—



So in what sence does relation R differ from relation R′ ?

In answer to the criticism mentioned above we shall review our definition.

I think this criticism is not justified as I merely assert that the universal is of a *relational character.* (but a universal is incompletely defined that way). Since we cannot talk of an equivalence relation without the set of objects on which it is defined.

The fact that each object is an instance of the universal viz. a particular is a matter of sense-perception **18.** Hence each individual's " private space of perçeptions " gives us the required " set of objects or instances ".

With respect to this set we have " our universe of discourse " say X.

This set x is split into equivalence classes **19.**

$$\} [I_{\propto} \beta] \{ \propto \in A$$

by defining a family of equivalence relations $\} R_{\propto} \{ \propto \in A$

i. e. $X = \bigcup_{\propto \in A} [I_{\propto} \beta]$ such that $[I_{\propto} \beta_1] \cap [I_{\propto} \beta_2] = \phi$

In the section " Reconciliation with experience " we saw that corresponding to a logical construct R*w*, we have a physical construct R*w*′. Clearly the physical constructs enable us to distinguish between the logical constructs. Hence we can distinguish one equivalence relation from another by behaviouristic methods.

Nelson Goodman[20] in " The Question of Classes and Nominalism " points that it is often taken for granted that everything called logic including the calculus of classes is *purely neutral machinery* that can be used wihout ontological implication in any constructional system. However, in our case we have not used this " Mathematical Model " without reference to ontological considerations as made clear by the introduction of " Physical constructs ".

The entire representation may be *Critisized* as being " *a theoretical model* " only. Clearly the title of this essay "Redefining Universals" and *Wittgenstein's*[21] remark makes it clear that no attempt is made to give an *ultimate understanding* of universals.

However, the further criticism that the idea of being able to determine "the physical constructs" is very far-fetched. I agree that in certain fields (like psychology or parapsychology) it may at the moment be difficult to determine the 'physical constructs'.

However in the future this may be made possible. Whereas in the case of physics I think the determination of "Physical constructs" would not be far-fetched.

CONCLUSION

We have at *one and the same time* asserted the *realist's claim* that there exists an objective justification for the application of the word "Whiteness" to white objects (by our physical construct R*w*′) and to the *nominalist claim* that there does not exist any element common to all white objects. (by our logical construct R*w*)

The use of constructs may not appeal to all readers but Russell himself advocated *Logical constructions* for *inferred entities.*

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NOTES

1. "*The Nature of the Physical World*",
2. I have myself *extended* the previous concept of "equivalence relations" from set-theory and called it "multi-equivalence" relations in order to define complex universals.
3. (1) For all $x \in X$, $x R_i x$

   $\Rightarrow x R x$ (*reflexivity*)

   (2) If $x R y$ then $x R i y$

   $\Rightarrow y R i x$

   $\Rightarrow y R x$ (*symmetry*)

   (3) If $x R y$, $y R z$, then $x R i y$ & $y R i z$

   $\Rightarrow x R i z$

   $\Rightarrow x R z$ (*transitivity*)
4. Terminology 'Simple' and ' Complex' understood as in *Russell's An Enquiry Into Meaning and Truth.*
5. An equivalence relation partitions a set into mutually disjoint equivalence classes, i. e.

   $$X = \bigcup_{a \in x} [a] \text{ and } [a] \cap [b] = \phi \ \forall a, b \in X.$$
6. Refer to footnote (4).

7. 'Well then shall we proceed as usual and begin by assuming the existence of a single essential narure or form for every set of things which we call by the same name'.
8. (i) Clearly both X and X are instances of U.
   X R X (*reflexive*)
   (ii) Also if X and Y are both instances of U so are *y* and
   X R $y \rightarrow y$ R X (*symmetry*)
   (iii) If X and *y* are both instances of U, *y* and *z* are both instacnes of U then X and *z* are both instances of U.
9. "The Philosophy of Linguistic Analysis and Problem of Universals" *Philosophy* and *Phenomenological Research* Vol. XXIV, No. 3.
10. " Are the Characteristics of Particular Things Universal or Particular" *Proceedings of the Aristotelian Society supplementary* Vol. III, 1923.
11. " Universal and Family Resemblance " " *Proceedings of the Aristotelian Society*. Vol LXI, 1960-1961.
12. Proposition P is obtained from the propositional function P by substituting the value of the variable *x*, where the value has a *referrent*.
13. "The Philosophy of Linguistic Analysis and Problem of Universals " *Philosophy and Phenomenological Research* Vol. XXIV, No. 3.
14. Refer to any standard Book on " Measure Theory ".
15. This would require the specialized work of a physicist, chemist, biologist, psychiatrist or as the case may be.
16. Check (1) reflexivity, (2) Symmetry, (3) Transitivity obvious because R*w* is an equivalence relation.
17. This criticism was offered by a reader with hardly any knowledge of mathematics.
18. Acquaintance with the universal *x* - ness by repeated expeience of *x* (Russell).
19. Repetition to enable the reader to grasp clearly the context of my argument.
20. *The Structure of Appearence.*
21. "Even a concept which can be explained in necessary and sufficien terms cannot be *ultimately explained* in such terms".