A NOTE ON LEWIS SYSTEM—I AND THE CONSISTENCY POSTULATE

It is well known that the difficulties which made C. I. Lewis to propound his system of strict implication were few 'awkward' proporties of material implication. All of them centre around the question : Can we interpret the horseshoe symbol of *Principia Mathematica* as the relation of 'deducibility' between two propositions? His answer was clearly negative for the following reason.

We have in PM the following thesis

$$(p \supset q) \vee (p \supset \sim q) \qquad (1)$$

That it is a wff of PM may be easily verified and hence by weak completeness of PM it is a thesis. We also have the following thesis.

Law of double negation : $p \equiv \sim \sim p$ (2)

De Morgan's law : $\sim (p \vee q) \equiv (\sim p \,.\, \sim q)$ (3)

(1), (2) x Eq : $\sim \sim ((p \supset q) \vee (p \supset \sim q))$ (4)

(4), (3) x Eq : $\sim (\sim (p \supset q) \,.\, \sim (p \supset \sim q))$ (5)

by 'Eq' we mean Rule for substitution of material equivalents, which is available in PM. Now if we interpret '$p \supset q$' as q is deducible from p, then $\sim (p \supset q)$ becomes, q is not deducible from p or in more familiar terms, q is independent of p. Similarly $\sim (p \supset \sim q)$ becomes q is consistent with p. So (5) becomes: "No two propositions can be at once consistent and independent"[1]. But we do claim consistency as well as independence for the set of axioms of any of our axiomatic systems. So Lewis develops "the calculus of propositions that it accords with the usual meaning of 'implies' and includes the relation of consistency with its ordinary properties"[2].

Such being his intention one would expect to find the consistency postulate at the outset. That is, the consistency postulate which appears in section five of chapter VI[3] could have appeared as one of 1st set of postulates. Following him one may read this postulate as : if the joint assertion of a proposition with another is self-consistent, then the former one is itself self-consistent.

$$M(pq) \prec Mp \qquad (19.01)^4$$

It at first seems to be intuitively obvious. For if a proposition itself is not self-cosistent then the joint assertion of it with another can't be self-consitent. In fact Lewis proves a theorem to the same effect almost immediately after introducing the consistency postulate. But the very fact that Lewis comes to it in Section 5 makes one suspicious. One asks : Can there be an assertion which contradicts it? As an answer what immediately occurs is the following proposition

$$\sim Mp. \sim Mq : \prec . M(pq) \qquad (6)$$

Reading 'M' as possible it amounts to saying that if p is impossible and q is impossible then, joint assertion of p and q is possible. One can verify easily that it is not a thesis of S1 by showing that the matrix of group V, appendix II[5] falsifies that. For it takes 3 when p is 4 and q is 4 and it takes 4 when p is 1 and q is 2 but it verifies all postulates of S1. It should be noted that, this fact only proves that our formula (6) is not a thesis of S1, but does not give us any information regarding whether (6) can be added consistently with the set of axioms of S1. The reason is very simple : None of his systems are strongly complete.

Now we shall look closely at the formula (6) and try to answer some questions. The first one is that : Does our intuition disagree with it? Well, the answer is — it does, even if we accept the "narrower meaning" of the term 'impossible'. For 'impossible' means 'logically inconsistent', and a joint assertion of two inconsistent propositions cannot be consistent. But Lewis also pointed out another meaning of 'impossible' and that is 'impossible in relation to given data'. 'Even then I am unable to find an instance of it.

Next question is : Can we modify our formula so that it becomes consistent with S1?

Here my answer is positive. A look at the interpretation of 'M'[7] appendix II gives us some operative idea. Semantically speaking, the formula (6) is intolerable because it takes the values 3 and 4 for some values of p and q. So we have to proceed in a way so that we can lower its value. Table of 'M' shows that this operator lowers the value of its argument. So let us prefix 'M' to our formula (6) to get

$$M (\sim M p. \sim M q : \prec . M (p q) \qquad (7)$$

But for p = 1 and q = 2, (7)
takes 3. So we prefix a 'M' once more to get

$$M M (\sim p. \sim M q : \prec . M (p q)) \qquad (8)$$

(8) never takes any value other than 1 and 2. But here did we get any new formula which is given in (8)? The answer is again negative. For (8) is simply a substitution instance of C 13 as ntoed by Lewis[8]

$$M M p \qquad (C 13)$$

to give a counter example of the formula

$$\sim M \sim p. \prec . \sim M \sim \sim M \sim p \qquad (9)$$

which is the characteristic axiom of S4. He was interested in the consequence $\sim (\sim M \sim \sim M \sim p)$ i. e., 'for every proposition p, p is necessarily necessary' is false[9]. So in a world where no proposition is necessarily necessary, we can have the formula C13 and consequently (8). In fact C13 is the characteristic axiom of the non-regular system S7.

Finally Lewis pointed out that C13 is independent of each of the systems S1, S2, S3 though consistent[10] with each of them.*

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For typographical convenience, we have replaced diamond symbol in the manuscript by M.

— *Editors*

NOTES

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All references are to : *Symbolic Logic*, C. I. Lewis and C. H. Langford, 2nd ed. Dover, 1959

1. p. 122
2. pp. 122-123
3. pp. 124-125
4. p. 166
5. p. 494
6. p. 161
7. *ibid*
8. p. 497
9. p. 499
10. p. 498