# A NOTE ON EXISTENTIAL INSTANTIATION

R. N. Karani

## I

In his "natural deduction" (axiomless) system for the Predicate Calculus, Irving M Copi (*Symbolic Logic*)[1] sets out four formal structures, each constructed as "an elementary valid argument form accepted as a Rule of inference".[2]

Two of these are Rules of *Instantiation (or Quantifier Elimination)* which permit the valid *elimination dropping* of a quantifier (universal quantifier, 'all' or existential quantifier, 'some') in passing from premises to conclusion - in effect, Rules that permit the *freeing* of *bound* variables. These are the Rules of *Universal Instantiation (UI)* and *Existential Instantiation (EI)*

Two others are the Rules of *Generalization* (or *Quantifier Introduction*) which permit the valid *introduction* or *adding* of a quantifier (universal or existential) in the transition from premises to conclusion in effect, Rules that permit the *binding* of *free* variable. These are the rules of *Universal Generalization (UG)* and *Existential Generalization (EG)*.

This set of four Rules governing Quantificational Deductions, the "Quantification Rules", is offered by Copi "To construct formal proofs of validity for arguments symbolized by means of quantifiers and propositional functions"[3]

Actually Copi presents two graded versions of his Quantification Rules:

1) An earlier "Preliminary Version", confined in its scope to proofs of

arguments "involving the simplest kind of quantification"[4] where "we have limited our attention to general propositions involving only a single quantifier. A general proposition that contains exactly one quantifier is said to be singly-general".[5]

2) Copi, then, advances to a final, re-formulated version of his set of Quantification Rules, comprehensively governing the construction of formal proofs of validity of more "complex kinds" of arguments involving "multiply-general propositions which contain two or more quantifiers"[6] "and finally relational arguments"[7] containing two and more place predicates. Not excluding, of course, the earlier analyzed simpler, singly quantified argument forms.

The aim of this note is twofold: a) to seek to demonstrate that Copi's "Preliminary Version" of the Rule of Existential Instantiation (EI) is flawed, and, indeed, furthermore, that the entire approach to what Copi classifies as "Preliminary Quantification Rules" rests on a certain ill-conceived stipulation and assumption. B) to seek to pinpoint where, precisely, Copi's formulation has gone awry.

## II

It can be shown that the "Preliminary version" of EI is logically flawed (even considered within its limited domain of arguments involving only singly-quantified Propositions): it is not *truth-preserving* (truth-transmitting): it does not succeed in blocking *all* transitions from true premiss(es) to false conclusion.

Copi's symbolic formulation of the EI Rule is as follows: [8]

$$\frac{(\exists x)(\phi x)}{\therefore \phi u}$$ (where u is a constant other than 'y' that has no prior occurrence in the context)

Earlier, Copi had introduced the special symbol, 'y' [in symbolically representing his Rule of Universal Generalization (UG)] as a "notation analogous to that of the geometer"[9] "to denote any arbitrarily *selected individual*".[10]

Thus, UG is symbolically represented as follows: [11]

$$\frac{\phi y}{\therefore (x)(\phi)x}$$ (Where 'y' denotes any arbitrarily selected individual...)

(More on this score later)

However, coming back to our critique of EI, it is to be stressed that (granting certain underlying assumptions to be critically tested later) Copi is right in imposing, and building into the Rule, the given restriction on the application of EI, namely, that the substitution-instanceu (nu) in the conclusion must be an individual "constant" "*that has no prior occurrence in the context*", (emphasis added). This restriction Copi correctly regards as *necessary* to "prevent the construction of an erroneous 'formal proof of validity'[12] for some obviously invalid arguments; in other words, necessary to block transition from true premiss(es) to false conclusion in such invalid arguments like(to use one of Copi's own example): "Some men are handsome. Socrates is a man. Therefore, Socrates is handsome". Call it Argument A.

Let us formalize Copi's argument symbolically as follows :

(∃x) (Mx & Hx), Ms ∴ Hs

formal "Proof" A

| | | |
|---|---|---|
| 1) | (∃x) (Mx & Hx) | Premiss |
| 2) | Ms | Premiss |
| 3) | Ms & Hs | From 1) EI (wrong) |
| 4) | Hs & Ms | From 3) by commutation |
| 5) | Hs | From 4) by simplification |

Now, clearly the above displayed "Proof" is erroneous.[12]

The application of the EI rule in line 3) is plainly erroneous since the substitution instance, 's' does indeed have a prior occurrence in line 2) violating the restriction built into the Rule.

Thus, Copi's restriction on EI may be recognized as a *necessary* condition to block transition from true to false propositions.

The question is" is the given restriction, by itself, *sufficient* for the purpose?

I submit that it is not. And it is significant (as we shall observe later) that Copi himself, at the end of his analysis of Quantification is, inevitably, led to the same conclusion.

Consider the following example of an obviously invalid argument from a true premises to a false conclusion. (For another example refer Notes)[14]. "Some men are Buddhists. Therefore, Socrates is Buddhist." Symbolically:

(∃x) (Mx & Bx).∴ Bs

**Formal Proof - B**

| | | |
|---|---|---|
| 1) | (∃x) (Mx & Bx) | Premiss |
| 2) | Ms & Bs | From 1) by EI (correct) |
| 3) | Bs & Hs | From 2) by commutation |
| 4) | Bs | From 3) by simplification |

The point to note, and, to stress, is that in the above displayed Formal Proof (which satisfies the syntactic definition of proof to be introduced later unlike the so called Formal 'Proof' -A discussed earlier) Copi's Rule of EI applied in line 2) is *formally* impeccably correct; that it does not violate the given restriction demanded by Copi as *necessary* to block transition from true to false propositions: the substitution-instance, the "constant", 's', used in line 2) has no prior occurrence in the context - no prior occurrence in the premiss in line 1).

From this we may justifiably conclude that: a) the restriction on EI stipulated by Copi, though *necessary*, is *not sufficient* for the stated purpose, and, hence b) that Copi's "Preliminary Version" of EI is fundamentally flawed (even in the limited domain of arguments with singly-quantified propositions) inasmuch as it fails to be truth-preserving (truth-transmitting) It permits an invalid argument to come out as valid.

## III

Now to assert of a proof that it makes an invalid argument come out as valid presupposes that we are invoking and applying two distinct criteria to test and evaluate validity. In the present context the two criteria may specifically be identified as the *syntactic* as distinguished from the *semantic* criterion.

In simplest terms, the notion of *syntactic* validity (as David Hilbert conceived it) is to be understood only within the framework of "the *complete formalization* of a deductive system"[15]

This procedure "entails the conversion of the ... system into a calculus of uninterpreted signs"[16] a calculus or system of "empty signs" formed by "draining the expressions occurring within the system of all meaning"[17] along with a "set of precisely stated rules" prescribing "[H]ow these signs are to be combined and manipulated" [18] combined by means of the *Formation Rules* [19] recursively defining the structure of a *well-formed formula* in a *specific* formal symbolic language, L; and *manipulated* in accordance with the accepted Transformation Rules (Rules of Inference) of a *specific* system, S, governing the law-like derivation of one abstract formula from others.

(The above characterization brings out an important point: the Rules of *syntax* are to be understood as holding contextually only in relation to a *specific* formal, symbolic language and a *specific* formal deductive system-they are, in other words, "system-relative").

Accordingly we get the following definition of a purely formal or *syntactic proof of validity* in Copi's Deductive system: CS (which, it must be kept in mind, is an *axiomless* system, deploying a "natural deduction" technique).

***Definition***: A completely formal or *syntactic* proof of validity in CS is a finite (but not empty) sequence (or string) of (abstract, uninterpreted) formulas in the symbolic language of CS, and, such that each formula in the sequence is either a premise, or, derived from some preceding formula(s) in the sequence by the proper application of one or other of the accepted Rule of Inference (Transformation Rule) explicitly stated in CS. A proof in CS is a proof of the last formula in the string.

Tested against this criterial definition, it emerges plainly, that, in the earlier discussed examples, *Proof B*(but not, Proof-A), is *syntactically valid* in CS: it is a sequence of four abstract formulas (in the formal, symbolic language of CS), where, the first formula is *premiss*, and each of the succeeding three formulas has been legitimately derived from a preceding formula by the proper and correct application of the following accepted Inference Rules state in CS: EI, commutation, Simplification[20]

*Semantically* considered, however i.e. when the formulas in the Proof-sequence are *interpreted* as true/false sentences (propositions), and not merely as "meaningless marks" in an abstract, formal symbolic calculus- the argument formalized by Proof-B turns out to be *invalid*: *semantically* invalid. A *semantically valid* symbolic argument (standardly defined) being such that *every interpretation of the symbolic language of a deductive system, S, which makes the premiss(es) true, must make the conclusion also true.*

In this respect both symbolic arguments *A* and *B* (discussed above) comė out as *semantically* invalid, *21*, though, interestingly, as shown in our previous analysis, Proof-*B*, (though not Proof -*A*) tests as *syntactically* valid. Leading us to conclude that *at least one* of the Rules of Inference (Transformation Rules) though correctly and properly applied (viz. EI, Commutation and Simplification) fails to be *truth preserving*: that it permits us to infer false from all true propositions.

Now, a simple *Truth-Tables* test for the *Truth-Functional* sentential (propositional) connective for *conjunction* would demonstrate that both, the Rule of Simplification, as also the rule of Commutation (for *Conjunction*), in no row of the table, permits us to pass from true premises to false conclusion.

Both these Rules of Sentential Logic are therefore unquestionably truth preserving, and hence, guarantee semantic validity when applied.

By elimination this leaves us with the conclusion for which the note has argued, namely that Cop's "Preliminary Version" of EI is flawed in this respect. That, even when legitimately applied in purely formal or syntactic terms, it does not ensure or guarantee *semantic validity*, which, surely, is one of the more important aims of logicians as system-constructors: the "intended interpretation" in mind which guides and disciplines their otherwise creative freedom to legislate abstract logical rules any way they please, by an appropriate well-defined fiat. If they intend the logical system to be *applied* in testing ordinary language arguments in science as well as in mundane contexts.

## IV

Where has Copi gone wrong?

The explanation, I submit, is to be located in Copi's attempt (misconceived, as it turns out) to simplify proof construction procedures of arguments involving exclusively singly-quantified propositions - arguments free from the complexities and complicated procedures of multiply- quantified and relational propositions.

Towards this end, Copi so frames his "Preliminary Quantification" Rules, as to dodge the need to introduce, or, rather, the need to "explicitly acknowledge" in any line of the proof, the occurrence and role of *propositional functions* containing free individual variables: though (as will be discussed later) he is led to admit, eventually, that such occurrence "was implied in our...usage"[22]

He seeks to achieve this sidestepping maneuver, by the simple device (a gambit, really) of masking, or passing off, a *free* individual variable as an individual *constant* - in effect a pseudo-constant.

And by this stratagem, in the operation of his "Preliminary Quantification Rules", Copi ensures, however, superficially that no line in the proof is seen to appear manifestly or explicitly as a prepositional-function containing a *free* occurrence of an individual variable. Every line is made to *appear* as a *proposition* where every individual symbol appears to occur either as a *bound* (or quantified) variable, or as an individual constant (a pretended, or pseudo constant as in EI, or genuine as in some applications of UI and EG).

The following two observations are stressed here (subsequently substantiated by citing the relevant texts):

1) that the special individual symbol, 'y', introduced by Copi in the premises, '$\phi$y' of UG (the preliminary version) is, in its true logical status, a variable, a *free* variable, (though not acknowledged as such, in the enunciation of the Rule).

2) Also that $\upsilon$ as it appears in the conclusion of the preliminary version of EI is *not in* its true logical status a genuine, full-blown individual *constant*, as made out to be and ostensibly characterized in Copi's formulation, but rather, a pseudo-constant, a masked or disguised free individual variable. And, hence that the conclusion of the preliminary version of EI is, in point of fact, under its guise of a singular proposition, a propositional function "rather than a (pretended) substitution instance"[23] of that function.

Both these contentions are eventually acknowledged by Copi at a later stage. To quote: "However in our previous use" (Copi is referring to the Preliminary Quantification Rules) "of the letter 'y' to denote any *arbitrarily selected* individual, we were, in effect, using it as a variable without acknowledging this fact. In introducing a letter by EI to denote some particular individual having a specific attribute without knowing which individual was denoted by it, we were, in effect, using that letter as a variable, also. We now proceed to acknowledge explicitly what was implicit in our former usage". [24]

A little later, Copi adds: "The instantiating rules of UI and EI ... must now permit the freeing of bound variables to permit the introduction of prepositional functions themselves rather than (pretended) substitution instances of them". [25]

Thus crucial admission helps to uncover the basic flaw in Copi's Preliminary Version of EI; and to pinpoint what has gone askew in the construction of Proof-*B* where syntactic and semantic validity do not match.

The source of this defect plainly lies in the now acknowledged fact that the formula in the last line - the conclusion of Proof -B the formula, "Bs, is in point of fact, a *propositional function*, "rather than" (to use Copi's words) "(pretended) substitution instance" of this function. The now admitted fact that in using the letter, 's' for the EI in line 2), and, thus subsequently in the conclusion line, we are (again to use Copi's words)" using that letter as a variable" - a free variable, not as a full-blooded individual constant.

Now, immediately, this analysis can be shown to collide with the following syntactic demand stressed by Copi in expounding his proof-theory. To quote: "In constructing a formal proof of validity for a given argument, the premises with which we begin and the conclusion with which we end are propositions."[26]. Againt, later, referring to both the preliminary, as well as the final version of the Quantification Rules, Copi asserts: "here, as in earlier sections, we are concerned with constructing proofs of validity only for arguments whose premiss and conclusions are propositions... Hence, we never end a proof with a propositional function that contains a free variable".[27]

That, precisely, is what is wrong about Proof-*B*: the formula in the conclusion line is a disguised prepositional function, and the individual symbol is, hence, a disguised free variable.

This leads Copi on to the realization that Existential Instantiation is to be permitted "only under very stringent restrictions"[28] to prevent the construction of erroneous formal proofs of validity' certainly more stringent than the single restriction he had earlier deemed sufficient to impose in the enunciation of the preliminary version. (a point which we questioned in this note).[29]

Thus, we find, now, Copi tightening up EI with the addition of one further and important restriction. Writes Copi: "Not only must the instantiating variable not have any prior free occurrence (as discussed on pages 74-75)' where Copi had enunciated the preliminary version - but to prevent erroneous proofs a further restriction is necessary on the application of EI, so that (Copi states): "the formula or line *finally* inferred by its means contains no free variables that are introduced by it."[30]

A scan of proof-B shows that this restriction has been indeed violated: the formula 'Bs' finally inferred by means of EI contains the free variable, 's' introduced by EI in line 2). Again the same result: the proof of Argument B is syntactically invalid. Thus it is invalid both on the syntactic and semantic criterion.

**NOTES**

1) Irving M Copi, *Symbolic Logic*: 5th edition Chapter 4, Quantification Theory", pages 63 - 115.

Except where specified, all subsequent reference are to this text.

2 page 89

3 page 71

4 *Preface*: page vii

5 page 83

6 page 83

7 *Preface*: page vii

8 Page 74

9 Page 72

10 Page 72'

11 Page 72

12 Page 75

13 Indeed, strictly speaking it is not a *proof* - it does not satisfy the *syntactic* definition of *proof* to be enunciated later (see page 85-86 of this article)

14 Another example of an invalid argument from a true premisse to false conclusion, but in whose symbolic proof Cop's EI restriction is strictly observed: "All University teachers are graduates. Some University teachers are mathematicians. Therefore, Socrates is a mathematician and a graduate." Some University teachers are mathematicians. Therefore, Socrates is a mathematician and a graduate." Call this argument -*C*.

**FORMAL PROOF C**

| | | |
|---|---|---|
| 1) | (x) (Ux ⊃ Gx) | Premiss |
| 2) | (3x) (Ux & Mx) | Premiss |
| 3) | Us & Ms | From 2) By EI (correct) |
| 4) | Us ⊃ Gs | From 1) by UI |
| 5) | US | From 3) by Simplification |
| 6) | Gs | From 4 5 by MP |
| 7) | Ms & Us | From 3) by commutation |
| 8) | Ms | From 7) by Simplification |
| 9) | Ms & Gs | From 8 6) by conjunction |

15) Nagel & Newman : *Godel's Proof* (New York University Press, 1964) page 26

16) Nagel & Newman, page 45

17) Nagel & Newman, page 26

18) Nagel & Newman, page 26

19) Referring to the Formation Rules, Nagel & Newman observe: "The rules may be viewed as constituting the grammar of the system": pages 45-46.

20 Note also that Proof-*C* displayed above in Note 14, satisfied the definition of syntactic validity, though, it, too, formalizes an argument which proceeds from true premises to a false conclusion.

21 So also, Proof-C in Note 14 above.

22 Copi : page 85

23 page 90

24 page 90

25 page 90

26 page 96

27 page 96

28 page 96

29 Refer page 84 in this article.

30 Copi: page 96.