MODERN ANALYSIS OF GENERAL PROPOSITION :
# A CRITICAL EVALUATION

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This paper is an attempt to question the veracity of the modern analysis of universal and existential propositions in two different logical forms by maintaining that their analysis is not logically well grounded. To establish it, let me begin with their analysis of universal and existential categorical propositions.

Modern logicians analyse the inner logical structure of a universal and and existential categorical propositions and their relationships in terms of the following square array :

$(x) (\phi x \supset \psi x)$ A   E $(x) (\phi x \supset \sim \psi x)$

$(\exists x) (\phi x . \psi x)$ I   O $(\exists x) (\phi x . \sim \psi x)$
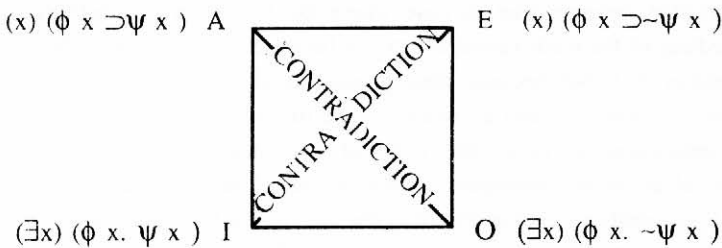
From this square array two points are quite clear : (i) The universal and the existential propositions differ not only in terms of their quantifications but also in terms of their propositional functions quantified ($\phi x \supset \psi x$ on the one hand, and $\phi x . \psi x$ on the other). (ii) The universal and the existential propositions are assertions of classes/functions. They are not assertions of individuals. Individuals, in fact, do not form as quantitative elements of a general proposition. When general propositions, whether universal or existential, are used, they always assert certain things about their propositional functions quantified by the quantifiers and assertions of propositional functions are not assertions of individuals. They are ascertains of classes / functions. On both the

points, modern logic differs from the Aristotlean logic. Because according to the Aristotlean logic, the universal and the existential propositions differ only in terms of their quantifications and not in terms of their propositional functions quantified; and the subject and the predicate terms of a general proposition always involve a reference to some existent entity and property or relation being non-empty terms which the modern logic transcendents in its analysis.

By associating a universal proposition with a material conditional and an existential proposition with a conjunction the modern logicians invalidates Aristotle's doctrine of square of opposition. On their analysis except contradictories all relationships are invalid even where we assume that the universe of discourse is non-empty, that is, there exists at least one individual in it. This they say because on their analysis A and E propositions can both be true and I and O propositions can both be false; and as a result of it neither the truth of A implies the truth of I nor does the truth of E imply the truth of O, as Aristotle claimed. $(x) (\phi x \supset \psi x)$ and $(x) (\phi x \supset \neg \psi x)$ forms of proposition become true in case where ϕx has no true substitution instances regardless of the truth-values of ψx not because they contain in them a universal quantifier, (x), but because their propositional functions are in the form of a material conditional and according to the rule of material conditional, whenever the antecedent is false, the material conditional is true regardless of the truth-values of its consequent. What is true of in the cases of the forms of A and E propositions is also true of in the case of the forms of I and O propositions. $(\exists x) (\phi x . \psi x)$ and $(\exists x) (\phi x . \neg \psi x)$ forms of propositions become false in cases where ϕx has no true substitution instances regardless of the truth-values of ψx not because they contain in them an existential quantifier $(\exists x)$, but because their propositional functions are in the form of a conjunction and according to the rule of conjunction if one of the conjunct is false, the conjunction is false regardless of the truth-values of the other conjunct. Thus by associating universal with a conditional and an existential with a conjunction the modern logicians reject the validity of Aristotle's doctrine of square of opposition.

So to judge whether the modern logician's rejection of Aristotle's doctrine of square of opposition is valid or not we have to first examine that analysis of universal and existential propositions since their rejection of Aristotle's doctrine

of square of opposition is based upon their analysis of universal and existential propositions themselves. If we take for granted that their analysis of a categorical proposition is correct, then there is no doubt that A and E propositions can both be true and I and O propositions can both be false; and as a result of it neither the truth of A can be said to imply the truth of I nor does the truth of E can be said to imply the truth of O. But when we question their analysis without assuming of its validity and ask the question : Why do they associate a universal proposition with a conditional and an existential proposition with a conjunction, not the other way round? Or why didn't they associate both the universal and existential propositions either with a conditional or with a conjunction when the subject and the predicate terms and their order in the proposition remain the same, we do not find a satisfactory answer to it on the ground of which their analysis could be said to be logically sound. We cannot say that they did it because universal proposition implicitly or explicitly always contain in them a universal quantifier while existential propositions contain in them an existential quantifier and the notions of universal and existential quantifier are essentially connected with the notions of conditionality and conjunctivity respectively. Because to say so would amount to mean that universal and existential quantifiers cannot legitimately be used in case where the elements of conditionality and conjunctivity are not found to be present which on the modern account is not true. Because according to the modern account they do have legitimate use in cases where the elements of conditionality and conjunctivity are found to be absent. For them universal and existential quantifiers are monadic operators. And being monadic operators they require for their operation only a single element, this is, a propositional function. To say this does not mean that on the modern account quantifiers cannot operate on a complex propositional function. It only means that the compoundness or the complexity of a propositional function is not a logical requirement of quantifiers for them to have meaningful uses. What is logically required for them to have a meaningful use is a propositional function and not of its simplicity of complexity. Nonetheless whenever the quantifiers operate on a propositional function, they always produce a general question and quantify the value of their individual variable falling under their range. Quantifiers are the functors of functors that form propositions out of names and numbers are implicitly components of such

functors (this is why they are called quantifiers). If at all universal and existential quantifiers were essentially associated with the notions of conditionality and conjunctivity respectively, their meaningful uses would have not been possible in cases where these elements are found to be absent because of their essentiality. But this is not true. Take, for example, the proposition, 'Everything is mortal' and 'Something is mortal'. In both the propositions since quantifiers are used meaningfully, they can be said to be either true or false. But in spite of this none of the quantifiers can be said to be the logical subject of the propositions in which they occur nor can they be said to be the expressions of clauses. That is why the propositions in which they appear cannot be said to state a conditional (in the case of a universal quantifier) and a conjunctive (in the case of an existential quantifier) relationship between the two closes / functions. 'Everything is mortal' and 'something is mortal' propositions are not examples of a categorical proposition. They are examples of a simple general proposition because their quantifier operates on a simple propositional function and any proposition whose quantifier operates on a simple propositional function is a simple general proposition. The former is a universal and the latter is an existential simple general proposition. They are of the forms of $(x) \phi x$ and $(\exists x) \phi x$ respectively. And these forms clearly indicate that neither a universal quantifier is essentially associated with a conditional nor an existential quanitifier is essentially associated with conjunction; and yet they do have meanings in their respective proposition. Quantifier words are syncategormatic words. They acquire their meanings in use and in use they always go with categorematic words. They do not refer to any class different from the classes of categorematic words. They are not in fact class expression. They are expression of quantification and to quantify a class is not to refer to that class because the act of quantifying is different from that of referring. The universal quantifier words indicate that the subject terms of a proposition with which they are associated should be taken in the entire or the whole range of its extension while the existential quantifier words, on the other hand, merely indicate that the subject term of a proposition with which they are associated should be taken in the part, not in the whole range of its extension. But in either case the quantifier words do not indicate anything about any particular individual thing that exists in the subject - class of the proposition. We can say from this point of view that

quantifier notions involve in them an indication of generality opposed to specificity. But since they always require for their operation a propositional function, they can also be said to be the indicator of the presence of a propositional function. And to say all this does not imply that universal and existential quantifiers essentially involve in them a reference to conditionality and conjunctivity; nor does it mean that whenever they are used, they always refer to what they quantify. All that means is that the quantifier words are used to indicate about quantity or number of the values of an individual variable occurring in the propositional function quantified by them without referring to any particular value of that variable.

In fact, according to the analysis of modern logic all the relationships between universal and existential propositions are valid in the cases of simple general propositions which is quite obvious from the following theorems of logical truth.

(i) $(x)\ Fx \supset (\exists\ x)\ Fx$

(ii) $(x) \sim Fx \supset (\exists\ x) \sim Fx$

(iii) $(\exists\ x)\ Fx\ v\ (\exists\ x) \sim Fx$

But in the cases of categorical propositions none of these laws holds good on the modern interpretation. Neither the truth of universal proposition implies the truth of the corresponding existential proposition nor the disjunction of a positive and its negative existentials is a tautology. None of the following expressions is treated as an expression of the logical truth.

(a) $(x)\ (Fx \supset Gx) \supset (\exists\ x)\ (Fx\ .\ Gx)$

(b) $(x)\ (Fx \supset Gx) \supset (\exists\ x)\ (Fx\ .\ \sim Gx)$

(c) $(\exists\ x)\ (Fx\ .\ Gx)\ v\ (\exists\ x)\ (Fx\ .\ \sim Gx)$

The reason why the expressions of (a), (b) and (c) are not considered as the theorems of logical truth is that their antecedent with the negation of consequent does not imply contradiction while on the other hand, in the cases of (i), (ii) and (iii) the antecedent with the negation of consequent does not imply contradiction. This happens in both the cases because of the nature of their propositional functions, from which it is quite obvious that the validity of the

laws governing the relationships between universal and existential depend, in fact, not upon the nature of their quantifiers but upon the nature of their propositional functions. Quantifiers as such do not involve in them any indication about the relationships of their propositional functions. But since the modern logicians interpret universal and existential propositions in two different logical structures in spite of the fact that the subject and the predicate terms and their orders in vernacular language remains the same, the question arises : Why is a universal associated with a conditional and an existential with a conjunction and not the other way round? Why shouldn't they associate both universal and existential either with a conditional or with a conjunction when the subject and the predicate terms and their orders in them remains the same? We cannot say that they did it because of the involvement of different quantifiers. This we have already seen, So there must be some other ground behind their interpretation.

Generally, it is said that the reason why a universal categorical proposition is associated with a conditional and an existential categorical proposition is associated with a conjunction is that universal propositions, whether affirmative or negative, are assertions of condition; they are not assertions of existence (this is why they are called hypothetical in disguise) while existential propositions, whether affirmative or negative, are assertions of existence; they are not assertions of condition (this is why they are called existential). But this line of argument does not seem to hold much water. Because when we look into the logical structure of an existential categorical proposition and its symbols, we do not find in it any element which could be said to indicating that something exists in the subjcet class of the proposition when it is asserted and provides ground to say that the assertion of proposition does involve an assertion of existence. Symbolization merely indicates that existential proposition is an assertion of classes/functions and assertions of classes/functions are not assertions of individuals.

Classes are different from individuals and the existence of the latter is not semantically structured in the conception of the former. That is the reason why an assertion of existence cannot be said to be a part of the assertions of classes. The 'existential' in fact, does not have any connection with the notion of existence. When certain propositions are characterized as existential by the modern logicians, it is done not because they involve in them an assertion of existence when they are asserted but because they are used to assert certain

things about some part of the classes of subject and the notion of *some* does not contain in its semantic structure the notion of existence. In other words, the notion of existence does not form a constitutive element of the notion of *some*. The notion of *some* is a notion of quantifier and the notion of number is implicit in it (this is why it is called quantifier). And being a notion of quantifier it acquires its meaning only in the connection with some categorematic words, not in isolation. But when it goes with categorematic words and links them as subject and predicate, it does not refer to anything that can be said to existing in the subject-class of the proposition. What it indicates is that the propositional function quantified by it is true of some of the values of an individual variable appearing in it. And to say that the existential quantification of a propositional function is true of some of the values of an individual variable is not to assert that something of a certain kind does actually exist in the subject-class of the proposition which satisfies the propositional function in question. An assertion of existence is not a part of the assertion of the existential quantification of a propositional function. Saying this would amount to mean muddling of an assertion of existence with the assertion of classes.

What I have been saying not only follows from symbolization of the modern logicians of an existential proposition but also from the extension of their predicate logic. Modern logicians extend the scope of predicate logic by allowing the subject and the predicate terms of a general proposition to refer both to empty and non-empty classes which Aristotle did not do. Aristotle restricted his analysis only to those categorical propositions whose terms all refer to non-empty classes, classes that have some members but excluded all other propositions whose terms refer to empty classes, classes that have no members. Here one might ask the question : Why did Aristotle do so? No doubt, there might have been good reasons behind his analysis of categorical propositions. But one of the reasons which comes to my mind is that he must have thought that truth conditions for general propositions always presuppose the existence of certain thing in the subject-class of the proposition which makes the admissionability of predication possible. Nonetheless, whatever reasons there might have been, the fact still remains that his analysis was restricted only to those categorical propositions whose terms all refer to non-empty classes which the modern logicians transcendent in their analysis by extending the scope of predicate logic to both empty and non empty classes of a general proposition.

If this be the case, and I think it is, then we cannot say on the modern account that it is the logical characteristic of an existential proposition that whenever it is used, it always involves in it a reference to the existence of certain thing which is not the logical characteristic of a universal proposition. The subject and the predicate terms of an existential proposition may not have any existential import. This is perfectly quite possible on the modern view. For example, 'some Pegasus has squared circle wings' is an existential proposition but its subject and predicate terms all refer to empty classes. There is no Pegasus in the subject-class which can be said does or does not satisfy the characteristic of squared circle wings. And yet the proposition is false on the modern account. This shows that existential commitment does not, in fact, figure in the formal analysis of the modern logicians. The modern logicians do not theoretically structure the notion of existence in their analysis of a categorical proposition no matter whether it is a universal or an existential. If it were so, they would have not extended the scope of predicate logic to empty classes of a general proposition; they would have accepted the Aristotelian assumption of existential import. But this they did not do. They rather reject it. And this is an evidence to say that an assertion of existence does not form a part of the assertion of an existential proposition on the modern account. If what I have said is true, then the argument based upon the dichotomy of existence and non-existence fails to provide an answer to the question as to why a universal proposition is interpreted by associating it with a conditional and an existential proposition is interpreted by associating with a conjunction.

An objection may be preferred against my view that although modern analysis of the existential quantification of a propositional function does not involve in it an assertion of existence as a part but it follows from their analysis. Because anyone who asserts existential proposition, on their account, commits himself thereby to the assertion that there exists at least one thing in the universe of the subject-class which fulfills or does not fulfill certain characteristic as described by the predicate term of the proposition. But this argument does not seem to have any logical strength. Because the modern logicians do not base their analysis upon this assumption that corresponding to each constituent of an existential proposition, there always exists certain object and property or relation, as Aristotle did. And if they do not base their analysis of an existential proposition upon the assumption of existential import, we cannot legitimately

say that an assertion of existence on the modern account is implied by the assertion of an existential proposition. An assertion of existence follows from the assertion of the existential proposition only when we assume that the subject and the predicate terms of the proposition refer to non-empty classes which modern logicians do not assume. On their account an existential proposition does not follow from its corresponding universal proposition not because the former implies an assertion of existence and the latter does not, but because the former is associated with a conjunction while the latter is associated with a conditional and the notions of conjunctivity and conditionality do not involve in their semantic structure the notion of existence. In analysing a general proposition, whether universal or existential, the modern logicians always keep their analysis free from the commitment about the existence of something. That is quite evident from the extension of their predicate logic itself. So it does not matter from the logic point of view whether the terms of a general proportion refer to existent entities or not, the modern analysis of a universal and an existential proposition always remains the same and the proposition does not fail to be either true or false. That is because the modern logicians do not base their truth-value account upon the assumption of existential import. Without presupposing the existence of anything, general propositions, whether universal or existential, on their account are possible to be either true or false. For example, 'Pegasus has wings', is a false proposition on their account in spite of the fact that its subject expression does not refer to the existence of Pegasus. Above all, when the modern logicians assume the universe of discourse non-empty while analysing universal and existential propositions from the semantic point of view, they do not assume the non-emptiness of the subject and the predicate classes of universe. Without assuming the latter, they make the assumption of the former. Hence, on the basis of the assumption of non-emptiness of the universe, we cannot legitimately argue that the modern analysis of an existential proposition always assumes or entails an assertion of existence. The modern analysis of an existential proposition neither involves in it an assertion of existence, nor does it presuppose or entail an assertion of existence. Moreover, the non-emptiness of the universe of discourse is not only assumed by the modern logician in the case of existential propositions but also in the cases of universal proposition without assuming the non-emptiness of the subject and the predicate terms of the propositions. And to presuppose the non-emptiness of the universe of discourse is not to assert that there are certain things in it. Because presuppositions are not assertions.

They are mere theoretical postulates which are made to explain and understand certain things. Here one might ask the question : why did the modern logicians do it? No doubt, there might have been good reasons behind their interpretation. But one of the reasons which comes to my mind is that they must have thought that truth-conditions for general propositions always require some universe of discourse non-empty which makes the admissionability of true and false predication possible. However, whatever reasons there might have been, the fact remains that an assertion of existence does not follow from the presupposition of the non-emptiness of the universe of discourse which can be given as a ground for the association of an existential proposition with a conjunction opposed to a universal proposition.

Still one might argue that when the modern logicians associated a universal with a conditional and an existential with a conjunction, they did not do it on the ground of their assertions of non-existence and existence but on the ground of the utility of their applications. The universal quantifier is a powerful expression and if it is combined with another powerful expression a conjunction. it would give us a sentence that would likely have no use for it. For example, if we symbolie "All humans are mortal" as (x) (Hx . Mx). We would have an absurd proposition that everything in the universe is both a human and a mortal. On the other hand, if we use a conditional with an existential quantifier, we would end up with a sentence that would be practically useless. For example, if we symbolize "some girls are honest" as ($\exists$ x) (Gx $\supset$ Hx) we would have proposition that there is at least one thing in the universe such that if it is a girl, then it is honest. which would not serve our purpose. It would be virtually useless. It is because of this reason the modern logicians associated a universal with a conditional and an existential with a conjunction and not the other way round. But this argument does not seem to hold good. Because when we symbolize "All humans are mortal" as (x) (Hx . Mx)) and interpret it that everything in the universe is both a human and a mortal, we do not make an absurd statement, The absurdity arises only when we take the universal quantifier, 'everything in the universe', in isolation and not in connection with the subject class of the proposition with which it. occurs. But when we take it in connection with the subject-class of the proposition which is expressed by the symbol of an individual variable, the absurdity does not arise. It disappears

because the universe of discourse gets determined but not by the use of a universal quantifier. It gets determined by its association with the predicate term of the proposition. It is the predicate term that determines the universe of discourse, not the subject term of the proposition. The mistake is committed when we identify a particular universe of discourse with any universe and start interpreting a universal proposition in it and that should not be done. A universal quantifier has no meaning in itself. It acquires its meaning in use and in use it always goes with categorematic words. And when it goes with categorematic words, it indicates in what range the subject categorematic word is to be taken in its extension. Moreover, the consideration of whether universal and existential propositions do have a genuine application in a particular context or not are extra-logical and hence are not important from the logic point of view. The modern analysis of a general proposition, whether universal or existential, is obstructive in character. It does not rest upon its applicability. The point of departure of modern logic is an artificial language, not a natural language and its method is constructive. To say all this does not mean that the modern analysis is useless. It only means that utility is not the formal ground of their analysis on the basis of which the analysis of a general proposition can be justified. Because there could be a context in which we might want to interpret a universal proposition not by associating it with a conditional but by associating it with a conjunction, for example, in the cases where it is used to assert existence rather than conditionality. So is the case with an existential proposition.

Whatever I have said so far may be rejected by saying that the contrast which I draw between the implicational explication of the universal quantifier and the conjunctive explication of the existential one is meaningless because both implication and conjunction can always be written in terms of each other. There is no doubt that both implication and conjunction can be written in terms of each other but it can be done only with the help of the negation sign. Without the negation sign implication and conjunction cannot be expressed in terms of each other and that makes a lot of difference. Because to say that implication can be written in terms of negation and conjunction is not to say that it can be written in terms of conjunction alone. If it were so, modern logicians would have not reinterpreted the inner logical structure of universal and existential propositions in terms of two different logical forms; they would have interpreted

both either in form of implication or in the form of conjunction alone, without the help of negation sign which they never do. Modern logicians, in fact, cannot refute some of the Aristotelian theses of the square of opposition, which they do, following the expressibility of implication in terms of conjunction alone, without the help of the negation sign and that shows that they do draw the contrast between the implicational explication of the universal quantifier and the conjunctive explication of the existential one. If this be so, the arguments of mine hold good.

In view of the above discussions, thus, we can say that the modern analysis of A, E, I and O propositions in two different logical structures is not logically well grounded. If A and E propositions are interpreted by associating them with a conjunction, then they cannot both be true but they can both be false. The truth of A could imply the truth of I and the truth of E would imply the truth O. If I and O propositions are interpreted by associating them with a conditional, then the truth of a universal proposition would imply the truth of its corresponding existential proposition: and I and O propositions can both be true. But in either case, the laws governing the relationships between universal and existential would be different from that of Aristotle's and modern logic.