RATIONALITY, PROOF AND PARADOX

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Can an impossible proposition feature in any discourse which has claims to rationality? That is the question. Can we use an inconsistency or contradiction as a *premise* of an inference : as something *from* which we draw rational deductions? Can we derive, rationally deduce, an impossible proposition as a *conclusion* from given data? What is the structure and function of a *Reductio Ad Absurdum* type of proof in which contradictions play a crucial role? The theorem of C. I. Lewis (that an impossible proposition entails arbitrarily *any* proposition whatever) - is it, if true, merely a 'logical freak', a paradox thrown up by our quest for formalization, or is it an idea which has a significant *practical* point, and can be given a *pragmatic* justification? These form a cluster of questions which form the theme of this essay.

I : Rationality

Deviation or lapse from rationality can occur in different ways. The ideal of rational thought may be flawed in at least these two ways : (a) failure to recognize an inconsistency in one's system of ideas, propositions, beliefs, (b) failure to eschew recognized inconsistency. The first describes the case of the thinker who would readily eschew such inconsistencies as are discovered and recognised by him, but, who, in a particular instance, fails to recognize inconsistency in some area of thought, and hence, unwittingly, entertains the unsuspected intellectual incoherency; his thinking, then, it would be fair to judge, is flawed, irrational. Human thinking, like other human functions and activities, being characteristically imperfect, this form of irrationality is far from being uncommon, and ranges from trivial incoherencies to serious, fundamental lapses in the heart of a scientific theory, capable of undermining the very foundations

of a system. Raymond L. Wilder observes : There are examples in mathematical literature of cases where considerable material was published concerning systems which later were found to be inconsistent! Until someone suspected the inconsistency and set out to prove it, or (in some cases) submitted upon it by chance, the systems seemed quite valid and worth while' (*Introduction To The Foundations of Mathematics* p. 24)

The second form of irrationality mentioned above describes the attitude of a thinker who discovers conceptual incoherency, recognizes it for what it is (a contradiction), and yet will not reject or exclude it from his system. Moreover, (let us suppose) this conscious and deliberate accommodation of inconsistency, for this person, represents a general attitude - a policy. Plainly, far from being a common phenomenon, this would seem to be a bizarre attitude, an aberration. Nevertheless, skirting the doubt whether this attitude is psychologically possible, this form of irrationality will be considered here for two reasons : one, since it throws up some interesting logical and epistemic issues; secondly, in order to evaluate certain arguments of Karl Popper who discusses this possibility. The failure to eschew recognized contradictions, the toleration and accommodation of discovered inconsistency, I shall refer to as 'primitive irrationality' The contrary, and corrective attitude, the decision to eschew in thinking all contradictions, I shall call 'minimum rationality'. Of course, normally, 'minimum rationality' is simply an implicit unconscious state of readiness or willingness to steer clear of inconsistency, and assumes the status of a decision only at an advanced level of reflection. The point behind the appelations, ('primitive irrationality' and 'minimum rationality') will emerge in the course of the discussion.

Reacting to Prof. Putnam's suggestion that 'human beings are machines, but inconsistent machines', J. R. Lucas ("Minds, Machines and Godel'' from *MINDS AND MACHINES*, ed; Alan Ross Anderson pp. 43-59) declares that '.... when a person is .... prepared to contradict himself without any qualm or repugnance, then he is adjudged "to have lost his mind''.' (p. 53). Considering the question whether men (as compared with cybernetical machines) are inconsistent, Lucas makes a pertinent distinction between inconsistencies as 'mistakes' and as 'set policies', and he maintains that human inconsistencies are to be viewed as the former, not as 'set policies', Lucas writes :

'They (human inconsistencies) correspond to the occasional malfunctioning of a machine, not its normal scheme of operations. Witness to this that we eschew inconsistencies when we recognize them for what they are. If we really were inconsistent machines, we should remain content with our inconsistencies, and would happily affirm both halves of a contradiction'. (p. 56).

Another philosopher who comments upon what I have called, 'primitive irrationality' and 'minimum rationality', is Karl Popper. He sets out to attack ("What is Dialectic" *MIND*, 1940) a kind of view which implies not just tolerance and complacency in the face of contradictions, but a positive glorification and commendation of a contradiction as something which enriches human thought and leads to intellectual progress. Popper, attributing this view to those whom he refers to as, 'dialecticians', strongly deprecates 'that loose way in which dialecticians speak about contradictions'. Popper observes : 'the delecticions sometimes emphasise that such a contradiction between a thesis and an antithesis is extremely productive, is the very promoter of progress, and that we are therefore quite wrong in assuming, and that logic is quite wrong in teaching, that contradictions are something always to be avoided. They even go so far as to say that a contradiction is something which, quite naturally, occurs everywhere in the world' (p. 407).

Of course, Popper will have none of this. Popper (the logician) joins issues with the Mystical Delectician and defends 'our resolve, not to agree to contradictions' (p. 407), i.e. Popper sets out to defend what I have called 'minimum rationality'. And although I agree with Popper's attack on the irrationalism sponsored by the Dialectician, and with Popper's insistence upon our adopting the attitude of 'minimum rationality', I believe that his critical arguments against the Dialectician are not, and cannot be effective, I wish to show that Popper's whole line of attack is misconceived : that the massive irrationalism of the Dialectician cannot but escape unscathed and unshaken by Popper's polemical arguments.

Defending and 're-analysing' the arguments of his 1940 MIND article against certain criticisms, Popper ("Are Contradictions Embracing" *MIND*, 1943 pp. 47-50) explicates the basic aim and thrust of his argument as follows: (a) to show that contradictions are 'embracing, i.e. that every sentence can be

inferred from it' (p. 47.), (b)   to show that 'the embracing character of contradictions is a *practical* reason for not admitting them'. (p. 47).

The second aim, thus, is to show that contradictions are undesirable *because* they permit the deduction of arbitrary propositions. Hence that the Lewis Theorem, far from being freakish and paradoxical, provides a pragmatic justification and motivation for one's adoption of 'minimum rationality'. Popper's argument for this conclusion may be reconstructed as follows : (All subsequent references are to the *MIND* 1940 article). 'Once a contradiction were admitted all science, would collapse' (p. 410). Question : How would the admission of a contradiction result in the collapse or breakdown of all science? Popper's explanation : by using some 'trivial' rules of inference ('Addition', 'Disjunctive Syllogism', etc.). from a pair of contradictory sentences we can deduce 'any sentence whatsoever'. From, "The sun is shining" and "The sun is not shining" we can deduce an arbitrary sentence like, "Caesar was a traitor".

Thus, Popper concludes. 'if two contradictory sentences are admitted any sentence whatsoever must be admitted'. Question : But why is arbitrary deducibility from a contradiction undesirable, and to be eschewed? (It must be kept in mind that Popper's critical thrust is directed against the Mystical Dialectician who has to be cured of his primitive irrationalism). Indeed, would not the mystically inclined Dialectician be inclined to welcome universal deducibility as something which promotes the fertility of ideas? How would Popper show that such arbitrary deductions contribute to 'a complete breakdown of science'? Popper's explanation : if just any arbitrary proposition, *B*, can be logically deduced from a contradiction *A & -A*. then by the same principles of inference, the negation of the arbitrary proposition,. viz., - *B*, can also be logically deduced from *A & -A* - as such, nothing informative has been achieved. To quote Popper : 'if we can deduce any sentence whatsoever, (from a pair of contradictory sentences), then, clearly, we can deduce any negation of any sentence whatsoever : It is clear that instead of the sentence, "Ceaser was a traitor", we can, if we wish, deduce "Ceaser was not a traitor". In other words, from two contradictory premises we can logically deduce anything, and its negation as well. We therefore convey with such a contradictory theory - nothing. A theory which involves a contradiction is entirely useless, because it does not convey any sort of information. .... Once a contradiction were admitted, all

science would collapse' (p. 410).

My criticisms of Popper will be best understood against the following schematization of his argument :

(1)   A contradiction *A* & *-A* is intellectually undesirable because.

(2)   *A* & *-A* permits (by means of elementary logical rules) the deduction of an arbitrary proposition, *B*, and also permits the deduction of *-B*.

(3)   Thus contradiction, *A* & *-A*, permits the logical deduction of contradiction, *B* & *-B*.

(4)   But *B* & *-B* is uninformative, useless for the purpose of science.

(5)   Therefore, since *B* & *-B* is deducible from *A* & *-A*, the latter is intellectually undesirable, and must be rejected, since it leads to the collapse of science.

A critique of Popper's Argument. (1) In the first place the argument can be short circuited I mean by this that since Popper's basic and ultimate rejection of a contradiction is grounded in its uninformativeness, his reason to show that the (original) contradiction, *A* & *-A*. ("The sun is shining and the sun is not shining") should be eschewed, should be : as a contradiction, it is uninformative. Steps 2 to 5 seem altogether superfluous. Why is it necessary to show that *A* & *-A* entails *B* & *-B* which, as a contradiction, is uninformative and useless for the development of science? If *B* & *-B* is uninformative, so also *A* & *-A*. If the uniformativeness of *B* *-B* can be read off directly (as seems to be implied by Popper), so also, surely, the uniformativeness of *A* & *-A*, There seems to be no cogent reason why the uniformativeness of *A* & *-A* has to be inferred, to be shown indirectly with the help of the Lewis Theorem.

(2)   It is difficult to see why a primitive irrationalist, (the Mystical Dialectian), should be expected to mend his ways merely by being shown that from the contradiction, *A* & *-A* (which he is not prepared to renounce), contradiction, *B* & *-B* can be logically deduced. If, cheerfully, he tolerate *A* & *-A* why should one expect him to be intellectually disconcerted to be shown that *B* & *-B* can be deduced from his premise?

(3)   Is it sufficient to damn contradictions only because they are uniformative? Does not the malaise lie deeper? After all, it is commonly assumed

that expressions of the form A or $-A$, ''Either the sun is shining or it is not shining'', are uniformative. But can they be charged with being irrational? They are tautologous, but not impossible - they do not defy reason. It makes perfectly good sense to say to someone. ''Either you take the exams or you don't'', and to go on to show the person the implications and consequences of each alternative. But ''the sun is shining and it is not'' is to be resisted not simply because of its uniformativeness, but because in the most fundamental, primitive, and indefinable way it strikes at the root of reason, at any exercise of the most rudimentary act of thinking - without its elimination reasoning and thinking grinds to a halt.

(4) It seems to me that Popper has misrepresented the thrust of his own argument, or, at least, that he places the stress in the wrong places. In summing up the aim of his argument (*MIND* 1943) he stresses that the Lewis Theorem (the principle of universal, arbitrary deducibility from contradiction) explains why contradictions are undesirable and inadmissible : 'The embracing character of contradictions is *practical* reason for not admitting them.' But the analysis of his argument reveals that the thrust, actually, is not the consideration that *any*thing and *every*thing can be deduced from a contradiction, or, at least not this consideration alone - it is not merely this which make contradictions inadmissible and which leads to the breakdown of scientific enterprise. The real and ultimate consideration (in Pooper's analysis) seems, rather, the contention that since any and everything can be deduced from a contradiction, a *fortiori*, B & $-B$ a contradiction, can be deduced from a contradiction. To re-quote Popper on this point, he writes : 'If we can deduce any sentence whatsoever, (from a pair of contradictory sentences), then, clearly, we can deduce any negation of any sentence whatsoever.... In other words, from two contradictory premises we can logically deduce anything, and its negation as well' (p. 410). For some curious reason Popper fails to stress the significance of this, namely, that the 'Anything and its negation as well', (which is deduced from two contradictory premises) amounts to a contradiction. The following is a fair summary of Popper's argument: contradiction, A & $-A$, is undesirable and should not be admitted into our system, since from it (by means of the Lewis Theorem) we can logically deduce the contradiction, B & $-B$, which is undesirable since it is uninformative, and hence, useless as science. From this it seems clear that, in the characterization of his own argument and its aims, Popper has placed the

stress in the wrong place. He ignores the vital importance of the point that what is deduced from a contradiction is also a contradiction; he only draws attention to the idea of the arbitrary character of what is deduced from a contradiction as constituting the real and ultimate ground for the rejection of contradictions.

(5) My last comment concerns Popper's confident assumption that he (the logician), addressing himself to the Mystical Dialectician (who is not fazed by contradictions), can succeed in 'showing' that contradictions are embracing, that any, and all, propositions can be logically derived from a contradiction. Popper sets out to achieve this aim by marshalling the so-called, 'Independent proof' of Lewis, as follows :

1)	A & -A	
2)	A	(1, Simpl.)
3)	-A & A	(1, Comm.)
4)	-A	(3, Simpl.)
5)	A v B	(2, Add.)
6)	B	(4, 5, D. S.)

Addressing himself to the Dialectician, Popper, in effect, says : Look, if you are prepared to admit, *A* & -*A*, a contradiction, you *must* admit just *any* old proposition. Why? the Dialectician is bound to ask. In reply, Popper has to start with the given contradiction as premise (line 1) and run through the successive steps, at each step appealing to the relevant rule of logical inference. Thus, he would have to start the deductive process by saying to the Dialectician: Now, if you accept *A* & -*A*, then, first of all you must accept *A*. Why must I? The Dialectician asks. And this question poses the critical obstacle which prevents the deduction from taking off; because the only adequate reply possible for Popper is to appeal to the pain of contradiction : is you accept *A* & -*A*, and do not also accept A, you are contradicting yourself. But surely, this appeal must prove to be futile : contradiction is no pain to our friend, the Mystical Dialectician. The same frustration faces the logician, the person who enjoys 'minimum rationality', at every step and line of the 'proof'.

To sum up. The conclusion must be faced : 'primitive irrationality' cannot be cured or corrected by *logical* means. The entire Popperian enterprise is

misguided and self-defeating. The Lewis Theorem may, very well, be impeccable from an abstract, logical point of view; it may, also, perhaps, be shown to have some pragmatic value and justification in some epistemic setting (a possibility to be discussed later), but it is a mistake to suppose that the paradoxical principle can be invoked as a justification of 'our decision, our resolution, not to agree to a contradiction' (p. 407) against some thinker whose attitude to contradictions ranges from complacence, or passive tolerance, to positive glorification : as something 'extremely productive, ... the very promoter of progress'; someone who maintains that 'we are therefore quite wrong in assuming, and that 'logic is quite wrong in teaching,' that contradictions are something to be avoided. .... a contradiction is something which quite naturally occurs everywhere in the world'. (p. 407).

II. Non-Recognition of Inconsistency

Consider the intellectual defect described earlier. Consider a person who possesses 'minimum rationality' (i.e., a person who will not tolerate inconsistency once it has been identified), but who does not recognize the inconsistency in a given set, Γ of his beliefs (ideas, hypotheses, assertions). This failure is clearly different from the 'primitive irrationality' discussed in the last section' - and not infrequent. The question is, can this sort of intellectual defect or lapse be corrected? Can it be corrected by *logical* means?

What is it to declare of a set Γ of propositions that is an inconsistent set? The following distinction is, I submit, relevant to our discussion, and needs to be elaborated. Consider the following two propositional formulas, both of which are inconsistent or impossible.

(1) Q & -Q

(2) (Q ---> -R) & - (-Q v -R)

The first propositional form, I shall call SURFACE-INCONSISTENCY (S-INCONSISTENCY), and also, EXPLICIT-CONTRADICTION. The second propositional form will be referred to as DEPTH-INCONSISTENCY (D-INCONSISTENCY), or IMPLICIT-CONTRADICTION.

S-INCONSISTENCY may be described structurally, as the logical conjunction of an atomic proposition with the negation of that atomic proposition; example : "Today is Sunday and today is not Sunday", It may be pointed out that, unlike D-INCONSISTENT propositional formulas, S-INCONSISTENT formulas exhibit structural identity, and may, therefore, be designated by (the commonly used) constant, Λ. The description of D-INCONSISTENCY would be; a propositional formula which though not S-INCONSISTENT, is logically inter-derivable with some S-INCONSISTENCY. The second formula above satisfies the description, and it can be shown to be logically equivalent to Q & $-Q$ (an S-INCONSISTENCY), or to R & $-R$ (an S- INCONSISTENCY) (or even, arbitrarily to B & $-B$ - though reservations about its epistemic propriety will be expressed and discussed later). We have commented upon the structural identity of S-INCONSISTENT formulas; instances of D-INCONSISTENCY, on the other hand, may reveal different degrees of structural complexity Compare (2) with the following D-INCONSISTENCY :

(3) $- (-P \& Q) \vee - (-Pv - -Q)) \& (-R \& - Q).$

The above is interderivable with the SURFACE-INCONSISTENCY, Q &-Q. That D-INCONSISTENCY admits degrees of structural complexity has some bearing upon epistemic issues like the perspicuity or 'readability' of the impossibility is propositions - an issue with which the present section is concerned, and to which we now turn.

The question before us is : can a preson's non-recognition of the impossibility of Γ be corrected by logical reasoning? presuming 'minimum rationality' (the readiness to eschew recognized inconsistency). The conclusion for which I wish to argue is : logical proof and demonstration is rational procedure which enables a person to 'discover' the hitherto unrecognized impossibility of Γ , *only if* Γ represents some D- INCONSISTENCY.

REDUCTIO AD ABSURDUM (R. A. A.) is precisely that logical method which demonstrates the impossibility of Γ. The structure of R. A. A. may be set out as follows :

1) Γ

.

.

.

n) Λ

Thus the impossibility of the D-INCONSISTENCY of the second formula mentioned earlier can be demonstrated by the following R. A. A.

$\Gamma = (1)$	$(Q \rightarrow R)$ & $- (-Q \vee R)$	
(2)	$Q \rightarrow R$	1, SIMPL.
(3)	$(-Q \vee R)$ & $(Q \rightarrow R)$	1, COMM.
(4)	$- (-Q \vee R)$	3, SIMPL.
(5)	$- -Q$ & $-R$	4, De Morgan
(6)	$- -Q$	5, SIMPL.
(7)	Q	6, D. N.
(8)	R	2, 7, M. P.
(9)	$-R$ & $- -Q$	5, COMM.
(10)	$-R$	9 SIMPL.
$\Lambda = (11)$	R & $- R$	8, 10, CONJ.

Note : It would also be possible to derive Q & $-Q$ from Γ)

The aim of R. A. A. is to demonstrate the modal value of a set, Γ of propositions, to prove or demonstrate that Γ is *impossible*. The aim is sought to be realized by *logical* means : by a reasoning in which S-INCONSISTENCY (R & $-R$ in the example) is logically deduced from Γ, a D-INCONSISTENENT set. The whole exercise has an epistemic point. It is directed toward a thinker who either does not recognize the inconsistency and impossibility of Γ, or suspects impossiblity, but is not sure since the inconsistency is not on the "surface" to be read off directly and immediately (unlike the inconsistency and impossibility of R & $-R$). Of course the success or failure to "spot" D-Inconsistency, to recognize it immediately without recourse to R. A. A. demonstration, is person and topic - relative and also related to the degree of

complexity in the given system's structure, (e.g. consider the extremely complex depth-inconsistency in Frege's system exposed by the Russell-paradox or Contradiction). For the present purpose of depicting the epistemic point guiding R. A. A. it is presumed that the individual has no difficulty in recognizing the inconsistency and impossibility of Λ; also, that the individual has sufficient logical acumen to grasp the principle that if something impossible follows from Γ, Γ must be impossible. By the R. A. A., therefore, a person comes to *discover* and *know* of the modal value of Γ, that Γ is impossible. So, R. A. A. has an epistemic role and function.

The above analysis of R. A. A. highlights the point that the ability to recognize a S-INCONSISTENCY is a necessary condition of the effectiveness of R. A. A. It follows from this that, in the unlikely case of extreme mental aberration, where an adult thinker cannot recognize A & $-A$ as inconsistent, self-contradictory and impossible, the employment of R. A. A. to correct this intellectual failure would be a pointless exercise.

Another observation thrown up by the study of R. A. A. will be reinforced in the next section, and it is the following : Λ (S- INCONSISTENCY) in a proper reasoning with an epistemic point occurs *only* as a *conclusion* inferred from premises - it can have no meaningful role as a *premise* from which a conclusion may be inferred.

III. Does R. A. A. Continue Through the Contradiction ?

Does R. A. A. proof go *through* a contradiction (S- INCONSISTENCY)? Or does it to *back* from a contradiction? The former view is favoured by I. M. Copi who is supported by Charles H. Lambros against criticisms raised by Donald Scherer. The implications of the conclusions arrived at in the foregoing sections, I shall argue, tend to vindicate Scherer's contention and some of his criticisms of Copi.

Donald Scherer (''The Form of *Reductio Ad Absurdum''*, *MIND*, 1971) complains that though R. A. A. is an argument form which is plainly valid, 'Yet logicians tend in their writings either to ignore it or to treat it in a confusing and confused way'. In particular, Scherer takes exception to Copi's proposal, (*SYMBOLIC LOGIC*, p. 69.) that 'instead of viewing *reductio ad adsurdum*

proof as proceeding only *upto* the contradiction, we can regard it as going on through the contradiction to the conclusion of the original argument'. Scherer sets out Copi's formalization as follows :

$$(1) \ P \qquad / \therefore q$$

$$(2) \ -q \qquad\qquad\qquad\qquad (n + 7 - -q \ v \ q \quad (n+6, \ Impl)$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad (n + 8) \ q \ v \ q \qquad (n+7, \ D.N.)$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad (n + 9) \ q \qquad\qquad (n+8, \ Taut)$$

$$(n) \ r \ \& \ -r$$

$$(n + 1) \ r \qquad\qquad (n, \ Simp.)$$

$$(n + 2) \ -r \ \& \ r \qquad (n, \ Comm.)$$

$$(n + 3) \ -r \qquad\qquad (n + 2 \ Simp.)$$

$$(n + 4) \ r \ v \ q \qquad (n + 1 \ Add.)$$

$$(n + 5) \ q \qquad\qquad (n + 3, \ N + 4, \ D. \ S.)$$

$$(n + 6) \ -q \to q \qquad (2 - n + 5 \ C. \ P.)$$

Scherer attacks Copi's form of R. A. A. on the following grounds : 1. It 'fails to manifest the basis upon which *reductio ad adsurdum* is informally conceived to rest' (p. 247). 2. It 'is less than intuitive' (p. 248). 3. It 'is both epistemologically and formally impossible' (p. 249).

Charles H. Lambros ("Scherer on *Reductio Ad Absurdum*" *MIND*, 1973, pp. 581-585), seeks to defend Copi against all the three charges levelled by 'Scherer against him. In terms of the conclusions drawn from my own analysis of the role of a contradiction in reasoning, I offer a substantial vindication of Scherer's critique (I do not offer any comment on the question of the formal impossibility of Copies form).

I believe that Scherer is perfectly sound in his representation of the form of the R. A. A. proof when he maintains that, 'the ordinary conception, ... presents the reasoning as going back from the contradiction, which is denied, ... to negate the assumption which is taken to have generated the contradiction' (p. 248). Thus, Scherer believes that the proof is formalized 'most naturally' as follows :

(1) P   / $\therefore$ q

(2) - q

      .

      .

      .

n) r & -r

(n + 1) - q $\longrightarrow$ (r & -r)

(n + 2) - (r & -r)

(n + 3) q

Scherer is right when he observes : 'The intuition upon which a *reduction* rests is that since a contradiction can never be true, whatsoever validly implies a contradiction must be false' (p. 248).

Comparing the two forms, that of Copi and of Scherer, there is justice in the latter's claim that the underlying intuition of the *reductio* is 'clearly manifested' in his form especially in the segment from lines, n + 1 to n + 3. Scherer comments : 'Copi's form, on the contrary, at no point involves denying the contradiction. The line -(r & -r) simply does not occur'. (p. 249).

Arguing for the 'epistemological impossibility of Copi's form, Scherer focceses upon Copi's line n, viz. r & -r. He objects : 'it is impossible to suppose rationally that both r and -r are true' (p. 249); he further points out that 'the supposition is always necessarily irrational and thus never an adequate foundation, ground of basis upon which to build any knowledge' (p. 250).

Lambros raises the following challenge : 'which lines in a derivation it must be possible to suppose as true. They must be just the premises of the original argument plus the hypothesis line introducing the I. P., or else *every* line at every stage of the proof. Scherer does not say which. But Copi's form .... the form reported by Scherer himself, has a contradiction neither in the premises nor in the initial line of the I. P., and so does not on that score suffer the defect. But then if it is supposed to be possible that *every* line in the proof can rationally be supposed to be true, then Scherer's form .... having r & -r as one of its line is itself epistemogically impossible. In fact any conceivable form of *reductio*. involving a contradiction at *some* point, would be epistemologically

impossible in this sense, but 'it seems unlikely that this would be a good reason for doing away with the *reductio* altogether'. (p. 583).

It must be conceded to Lambros that a *reductio will* involve a contradiction (Surface Inconsistency) at *some* point, and, hence, that the occurrence of a contradiction as line in the proof, by itself, does not constitute epistemological impossibility. However, logically, and rationally, the *only* point at which a contradiction should be permitted is as a *conclusion*, as a logical consequence *inferred* from an inconsistent set, Γ, and which is in virtue of being so inferred, demonstrates the impossibility of Γ. Once a contradiction, Λ, is inferred from Γ, it (i.e. Λ) has served its purpose, and there is no epistemic point (beside being epistemologically impossible) in using it as a *premise* line *from* which something is sought to be deduced or inferred. Thus, considering the epistemic function of the *reductio*, viz., to discover and demonstrate the modal value (impossibility) of Γ, the contradiction has a significant role *only* as something ded*uced* from previous lines - ultimately *from* Γ; to use Λ as something from which further deductions are made, is epistemically pointless, and, what is more serious, involves a lapse into irrationality, which Scherer rightly criticises.

So, although it is true, as Lambros retorts, that the offending line, r & -r, features in Scherer's form of *reductio* as much as in Copi's, the line has a different logical role and status in the two forms; and, in virtue of this difference does involve an irrationality (making line *n* epistemologically impossible) in Copi's form, but not in Scherer's form. The difference in logical status may be explicated as follows, In Copi's form of the *redctio*, line n, r & -r, is deductively infer*red* from previous lines, *and also* is a line *from* which subsequent lines are deductively infer*red*. Thus, in Copi's form,. line n features in two sorts of deductive relationships : (1) as a *conclusion* inferred from previous premise line(s), *and* (2) as a *premise* from which subsequent lines are inferred. Now, in the first deductive relationship, line n, r & -r, does not have the status of something asserted or supposed to be true; rather, it is to be regarded as a logical consequence of something supposed to be true (ultimately of lines 1 and 2) - something which *follows* from a given supposition, something which *would* have to be accepted, *if* one granted truth to Γ. And, in this deductive setting, the occurrence of r & -r as a line in the derivation, as a *conclusion* drawn from previous lines, suffers from no irrationality and makes perfect

epistemological sense. Indeed, it represents the characteristic form of the *reductio* as employed in human thinking, scientific and popular, where we do not assert, or even, suppose, the contradiction drawn from our hypothesis, but regard it as something logically implied by what might have been mistakenly asserted or tentatively supposed. But in the second deductive relationship in Copi's form, when we consider the segment from line n upto line n + 5, line n, r & -r, is made to function as a *premise* : something *from* which deductions are made; and as such it has the status of something *supposed* as true - *this* is irrational and epistemologically impossible.

Coming to Scherer's form, it is, indeed, true (as pointed out by Lambors) that r & -r appears as a line in the derivation - again as line n. But the examination of Scherer's form reveals that in it (unlike in Copi's form) r & -r in line n does not enter into the dual deductive relationship described above. In Scherer's form r & -r only occurs as something infer*red* from previous lines; as argued above, this involves no irrationality, since it does not have the status of something supposed to be true, but only of something which follows as a consequence of what is supposed or assumed, of something which one *would* be reduced to accepting as true, proving the absurdity of the hypothesis in question, from which it is derived.

The credibility of Scherer's form is further enhanced when we note that his form contains as explicit denial of the contradiction (see line n +2) - a *going back*' from the contradiction, 'to negate the assumption which is taken to have generated the contradiction' (p. 248). This is, again, precisely what rationality demands. Capi's form, on the other hand, fails altogether to denounce or deny the offending contradiction at any stage, and, instead, 'going on through the contradiction', attempts to reach the desired conclusion.

Conclusions :

To review the principal theme and argument of the essay : can inconsistent and impossible propositions feature in any reasoning which has claim to rational discourse? If so, can impossible propositions feature as premise(s) *from* which conclusions may be derived or inferred? Can an impossible proposition occur as a conclusion inferred from other propositions as

premises? Can the Lewis Theorem be used intelligibly in an epistemological setting? What is the role of a contradiction in R. A. A.? Drawing upon the conclusions arrived at in the foregoing discussion the following answers are offered.

(1)   A Surface-Inconsistency (contradiction) can have no place as a *premise* in any reasoning which is epistemically significant i.e., no epistemic purpose is, or can be, served by attempting to deduce or infer propositions from a Surface-Inconsistency.

   a)   Deductions *from* a contradiction or Surface-Inconsistency are epistemically pointless if the aim of the deduction is to correct *primitive irrationality* and to 'justify' our decision, our resolve to reject contradictions ('minimum rationality'). (Refer critique of Popper's argument in section I.).

   b)   Deductions *from* Surface-Inconsistency are equally useless in a *reductio* proof designed to demonstrate the impossibility of some Depth-Inconsistency,   (see   previous   section).   But   a Surface-Inconsistency can, and does, intelligibly feature as a *conclusion* inferred from other propositions as premises : the *reductio* is precisely the type of reasoning in which this happens, and it possesses great epistemic significance and value.

(2)   Within an epistemic setting can there be rational discourse with an impossible proposition as premise? In an epistemically directed reasoning, an inconsistent and impossible proposition features as a premise only when such inconsistency is of the Depth variety; and the epistemic point of such reasoning is, precisely, to demonstrate and expose the impossibility   of   the   premise   (by   deducing   from   it   a Surface-Inconsistency). This, again, is the *reductio*.

(3)   Thus, the *reductio* represents a form of reasoning where impossible propositions feature *both* as premise *from* which propositions are deduced, and as conclusion deduced from other propositions. But in the former case, it is only the Depth variety which is encountered, and, in the latter, always the Surface variety.

(4)   Finally the Lewis Theorem :

A & -A ⊢ B

As brought out in the criticisms of Popper and Copi we must conclude that even if this principle of Lewis is true it cannot be defended and given a pragmatic justification (as attempted by Popper), nor can it be given a significant role in the *reductio* (as proposed by Copi). In other words little can be done to dispell the highly paradoxical flavour of the theorem.

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