

WITTGENSTEINIAN APPROACH TO PARADOXES¹

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"Something surprising, a paradox, is a presdox only in a particular, as it were defective, surrounding. One need to complete this surrounding in such a way that what looked like a paradox no longer seems one." --

LUDWIG WITTGENSTEIN

Remarks on the Foundation of Mathematics,
(3rd edn., Oxford : Basil Blackwell, 1978. P. 410.)

In the light of Wittgenstein's above remark, my objective in this paper is to consider some of the paradoxes and place them in their proper surrounding so that what looked like paradoxes no longer seems to be so.²

Let us take the following sentence for our consideration:

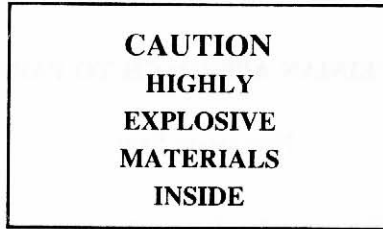
The sentence inside this box is false

This sentence is said to be paradoxical, because if what the sentence says is true, then it is really false and if what the sentence says is false, then it is really true.

The assumption of the truth of sentence leads to its falsity and the assumption of its falsity leads to its truth. As a result, the philosophers and logicians find here a paradox?

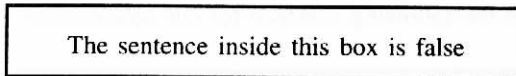
But is it really a paradox?

Let us consider the following sticker :



Now the grammar of this sticker demands that it should be pasted on the box containing highly explosive material. Only if the sticker is pasted on the outside surface of the box containing explosive materials, the sticker is meaningful. It would serve its purpose only if it is pasted on the box in such a manner that it would draw the attention of the human eyes. But if it is put inside the box, then the purpose of the sticker is lost. ___ It becomes meaningless. Here one goes against the grammar of the sticker.

Similarly, in the example :



we go against the grammar of the sentence. How we go against its grammar can be made clear if we bring in the distinction between *matters-of-fact-sentence* and *value-sentences-about-matters-of-fact-sentences*.³

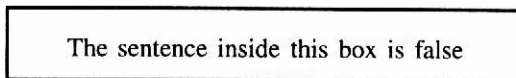
"The cat is sitting on the mat"

is an example of a matters-of-fact-sentence and

"The cat is sitting on the mat is true"

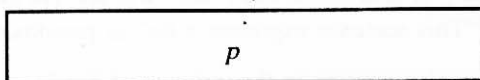
"The cat is sitting on the mat is false"

is an example of a value-sentence-about-a-matters-of-fact- sentence.



This sentence which is a value-sentence-about-a-matters-of-fact-sentence is confused with the grammar of a matters-of-fact-sentence and an attempt is being made to determine its truth-value. As the grammar of a value-sentence-about-a-matters-of-fact-sentence is confused with the grammar of a matter-of-fact-sentence, if the confusion is got rid of, the matter will stand thus :

The sentence inside this box is false



where p is a matter of fact sentence.

Now the sticker is at the right place in accordance with its own grammar. --- No more a paradox. It is a judgment on p and it does not need any judgement on itself.

The understanding of the grammar of the word "this" is likely to throw more light on this so-called paradox.

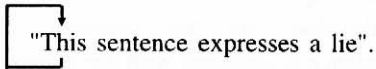
1. The sentence inside this box is false.
2. This sentence is not true.
3. This sentence expresses a lie.
4. This statement is false.

All these sentences are considered to be paradoxical in nature, for, it is said, if one assumes them to be true, they are false and if one assumes them to be false, they are true. But if we think so, we forget the grammar of the word "this". The word "this" is a pointing-word. The grammar of "this" is to point out something other than itself. It is not an auto-referential,⁴ but a hetero-referential word. Having in mind our distinction between matters-of-fact-sentences and value-sentence-about-matters-of-fact-sentences, we can say :

"This sentence expresses a lie"

↓
"p"

but not

 "This sentence expresses a lie".

The grammar of the word "this" is the same as the grammar of the word "that" except the difference of the situation of the thing pointed to. --- Whether the object pointed to is situated relatively near or away from the speaker. Our understanding of the grammar of the word "this" will prevent us from considering such sentences as "This sentence expresses a lie" as paradoxical sentences.

This paradox also appears in the writings of Tarski in his discussion of the Semantic Theory of Truth.⁵ There he makes an attempt to show how we get the paradox. But for the sake of simplicity, we shall quote Max Black's exposition of Tarski's position. Max Black says,

"One feature of Tarski's procedure which we must be careful to imitate is a rigorous observance of the distinction between an "object" and a "meta" language. The need for this, or an equivalent, is easily shown by an argument making use of the following figure :

The statement printed within a rectangle on this page is false

To save tiresome verbiage, let "*c*" be agreed to be an abbreviation for the words "The statement printed within a rectangle on this page." If the reader will consider the meaning of "*c*" and then *examine this page*, he should be led

(1) *c* is identical with the statement "*c* is false."

On the other hand, it seems hardly possible to deny :

(2) "*c* is false" is true if and only if *c* is false.

From (1) and (2) there follows :

(3) *c* is true if and only if *c* is false.

which is a self-contradiction. From an empirical truth (1), and a statement apparently true by definition (2), a contradiction has been deduced.

"This paradox arises through ambiguous use of the term "statement" and may be resolved by introducing an appropriate distinction. If statements containing the term "true" or "false" are systematically labeled "secondary" to distinguish them from the "primary" statements, from which those terms are absent, no paradox will arise. For the rectangle on this page must now be supposed to contain the words "The *primary* statement printed within a rectangle on this page is false", which themselves constitute a *secondary* statement, say *s*. Since no primary statement is in fact printed within the rectangle, it is easily seen that *s* is false but not self-contradictory."⁶

Max Black's view is that a statement like the one stated within the rectangle turns out to be self-contradictory and paradoxical. But if we bring in the distinction between *primary* and *secondary* statements, this will no longer appear as self-contradictory or paradoxical. We shall treat the sentence given in the rectangle as a *secondary* statement which is asserted about a *primary* statement. We look for this *primary* statement. If we do not find a *primary* statement, then the secondary statement is *false* but not self-contradictory or paradoxical.

On analysis, however, whether from the point of view of *object-language* and *meta-language* or of *primary* and *secondary* statements or of *matters-of-fact-sentences* and *value-sentences-about-the-matters-of-fact-sentences*, one finds that such sentences are neither paradoxical *nor false but nonsensical*.

Let us examine how we enter into this paradoxical wilderness.

To save tiresome verbiage we accept :

(i) *c* = "The statement printed within a rectangle on this page."

But we object to Max Black's following equation "

(ii) *c* is identical with the statement "*c* is false".

For, this second identification only leads us into the paradoxical wilderness by giving another equivalence for "*c*". The meaning of *c* gets changed. In (i) it means :

c = "The statement printed within a rectangle on this page"

and in (ii) it means :

c = "The statement printed within a rectangle on this page *is false*".

So when Max Black says :

" c is false" is true if and only if c is false.

he is perfectly correct for here c in body of its occurrences is used in the sense of (i), but when he says :

c is true if and only if c is false

we say that the meaning of the so-called subject --- " c " --- in its first occurrence has changed over to the meaning given in (ii) above whereas " c " in its second occurrence has the sense given in (i). So in this very sentence the first occurrence of " c " means "the statement printed within a rectangle on this page is false," whereas the second occurrence of " c " means "The statement printed within a rectangle on this page". Even then

c is true if and only if c is false

would be correct and non-contradictory, for it would, in fact, means :

" c is false" is true if and only if c is false.

but here the use of the first " c " is quite confusing. For the schema

c is true if and only if c is false

may mean :

(I) "The statement printed within a rectangle on this page is false" is true if and only if c is false.

which is perfectly correct and conforms to

" c is false" is true if and only if c is false.

But it may also mean :

(II) "The statement printed within a rectangle on this page" is true if and only if c is false.

which exactly conforms to the schema :

c is true if and only if c is false.

And this looks quite paradoxical. How we reached this paradoxical position, let us note. Let

c (I) = "The statement printed within a rectangle on this page"

and

c (II) = "The statement printed within a rectangle on this page is false".

And now when Max Black says,

c is identical with the statement " c is false"

he actually says that

c (I) is identical with c (II).

And this unwarranted identification leads to the paradox. For in the schema

c is true if and only if c is false

the predicate "is false" which was the immediate predicate of the first c is lost sight of and in its place the predicate "is true" is now thought to be its immediate predicate. If we correct the schema it would stand thus :

c (II) is true if and only if c (I) is false.

And given this schema it is impossible to see any paradox here.

Or given the abbreviation " c " for *the whole sentence* within the rectangle and the given schema :

_____ is true if and only if _____

we can have

" c " is true if and only if c

and here there is no paradox.

After making the distinction between primary and secondary statements, towards the end of the quoted passage Max Black says.

"For the rectangle on this page must now be supposed to contain the words "The *primary* statement printed with a rectangle on this page is false," which themselves constitute a secondary statement, says S."

In keeping with our discussion about the sticker above, we would like to say that even this modified sentence cannot be there within the rectangle. Its rightful place is somewhere outside the rectangle.

A proposition (or statement) is either true or false. As such "true" and "false" are said to be properties of propositions. And whenever we ascribe such properties to propositions we write " p is true" or " p is false". We could very well have had a convention in which (T) p and (F) p would express whatever is expressed by " p is true" and " p is false" respectively. This proposed convention would have saved us from a lot of confusion generated by the "is" of "is true" or "is false".

When we say " p is true" or " p is false", p is a matters-of-fact-sentence to which we ascribe a truth-value---*true* or *false*. If we say that " is true" and " is false" are functions, then the arguments for these would be p, q, r which are complete and well-formed matters-of-fact-sentences. But we find that in the sentence "The statement printed within a rectangle on this page is false", the expression "The statement printed with a rectangle on this page" is not a complete and well-formed sentence (not to speak of its factual content). It is incomplete without a verb and the "is" in "is false" cannot function in the role of an ordinary verb here. As a result, it cannot be reduced to " p is false".

Thus, we may conclude that "is true" or "is false" can be predicated of p , where p is a complete and well-formed matters-of-fact-sentence containing its own verb ---not just a phrase. And if "is true" or "is false" occurs in such statements as "The statement printed within a rectangle on this page is false" we must be able to reduce such a sentence into " p is false" where p is a complete and well-formed matters-of-fact-sentence. If we are not in a position to reduce such a sentence into " p is false" or in other cases where such a sentence is asserted to be true into " p is true" then such sentences are *neither true nor false nor paradoxical, but nonsensical*.

When we say " p is true" or " p is false", we simply mean that p has the truth-value property true or false, but for the determination of the truth-value of p we have to see whether a corresponding fact is there or not (where p is a matter-of-fact-sentence). But in case of sentences that are said to be paradoxical

in nature, no such attempt can be made to see whether a corresponding fact is there or not. *The truth-value of such sentences are resorted to be determined by assumption only.* Moreover, the “is true” and the “is false” in “*p* is true” and “*p* is false” do appear to be parts of sentences “*p* is true” and “*p* is false”, but really they are not parts of these sentences. --- They merely indicate the possible truth-value property of *p* which with a new convention we could have very well expressed without any confusion as (T) *p* or (F) *p*.

What I mean to say by saying all this is : “truth” and “falsity” are truth-value properties of propositions; they are not parts of propositions. If there is a proposition, *p*, then there is some fact which will enable us to say on comparison between the fact and the proposition that “*p* is true” or “*p* is false”. If these truth-value ascriptions are challenged, then one has to go back to the very same fact⁷ which was compared with *p*. There is no other *value-fact* which can be compared with the value-sentences “*p* is true”, “*p* is false”, for the contents of

- (i) “*p*”
- (ii) “*p* is true” and
- (iii) “*p* is false”

do not differ. They have just one content---i.e., what *p* says. And only one fact will enable us to give truth-value ascriptions to them. If on challenge we go back to the fact again in order to confirm our initial value-ascription and find the value ascription to be correct, then (ii) will become :

- (iv) “*p* is false” is true

or (iii) will become :

- (v) “*p* is false” is true

and (iv) is equivalent to “*p* is true” and (v) is equivalent to “*p* is false”. If we say of subsequent truth-ascription “T- operation”, then whatever might be the number of T-operations the original truth-value remains unaltered. For example,

$$(Tn...(T3 (T2 (T1 (p \text{ is true})))))) = p \text{ is true}$$

or

$(Tn...(T3 (T2 (T1 (p \text{ is false})))))) = p \text{ is false.}$

And if we call the subsequent falsity-ascription "F-operation", then every operation alters the last truth-value. Logically the F-operation is more important than the T-operation. But the Tarski schema is a case of T-operation. So

"*c* is false" is true if and only if *c* is false

is equivalent to

"*c* is false" if and only if *c* is false.

Even from this angle the paradox disappears.⁸

"I am a liar."

"What I am saying is false".

"What I am saying now is a lie."

"I now tell you a lie."

"I am lying now."

All these sentences are said to be paradoxical. But, again, are they really paradoxical?

Suppose a person comes to me and says, "I am lying now". And he does not say anything further. I shall be at a loss. For I shall be unable to make out what it is that he is lying about. The sentence "I am lying now" anticipates another sentence and unless that sentence about some matters of fact is uttered immediately following it, the sentence by itself is meaningless. The sentence is meaningful only if it is said about another sentence the truth or falsity of which can be determined in relation to some matters of fact. So it is a value-sentence about some sentence concerning some matters of fact. If that does not follow it, then it falls flat--just a pointless utterance. This itself is a value-sentence but if any further value is to be ascribed to it at all, then it can be done with reference to some fact, and that fact is missing here. All that we have to do in order to face a paradox is to *assume* a truth-value. But the grammar of the sentence demands that either prior to, or following, its utterance, there must be some matters-of-fact- sentence on which this is a value-sentence and if any truth-value operation is to be carried on at all, then it must be carried on

in relation to some fact, *not by assumption*. If we forget the grammar of the sentence, then a paradox appears out of it.

But in some other cases one sentence may be followed by another and these together may give rise to a paradox as in the following cases :

- (i) On one side of a card is written :

“The sentence on the other side is true.”

and on the other side is written :

“The sentence on the other side is false.”

- (ii) “The next sentence is true.”

“The previous sentence is false.”

- (iii) If one person is saying that what the other is saying is false when the other is saying that what the first person is saying is true.

All the three paradoxes are of the same kind. So we shall discuss only the first.

If we accept the distinction we have already made, then

“The sentence on the other side is true”

will not be anticipating

“The sentence on the other side is false”

but it will be anticipating

“*p*”.

Similarly,

“The sentence on the other side is false”

will not be anticipating

“The sentence on the other side is true”

but it will be anticipating

“*q*”

(Where “*p*” and “*q*” are matters-of fact-sentences).

The value-sentences-about-matters-of-fact-sentences:

(a) “The sentence on the other side is true”

and

(b) “The sentence on the other side is false”

are, by their own grammar, meant to talk about the matters-of- fact-sentences:

(i) “*p*”

and

(ii) “*q*”.

By their own grammar they are not allowed to talk about each other. Only when we forget the grammar of such sentences or we are confused of their grammar, paradoxes arise out of such sentences.

That, I hope, is the end of the paradox.

Thus, I have tried in my humble way to provide proper surroundings for only a few of the paradoxes. If I have failed in my attempt, then nothing is achieved and no further responsibility is imposed on us. But if any success is achieved, then we have to shoulder a great responsibility of meeting the innumerable number of paradoxes in their specific fields and try to provide proper surroundings to each one of them such that each one of them disappears and we see paradoxes no more.

NOTES

1. I express my gratitude to Prof. Georg Henrik von Wright, and to Prof Krister Segerberg, Department of Philosophy, University of Auckland, Auckland, New Zealand for their valuable suggestions.

I am also grateful to the Department of Philosophy, University of Rajasthan, Jaipur for giving me an opportunity to read an earlier version of this paper in one of the national seminars held in the department in the year 1987.

2. My objective in the paper is to use Wittgenstein’s insight in solving some of the paradoxes. It is not my objective to make an exhaustive analysis and critical

examination of the concept of truth. The discussion of the concept of truth is limited to paradigm cases (e.g., "The cat is on the mat") as it is generally done by philosophers like Tarski (e.g., "Snow is white"). Otherwise, in the background of complicating factors like space, time, generality, etc., the concept of truth is as complex as our thought is---our language is. The paradigm case (e.g., "Snow is white") is in the centre and it provides us with a solid ground to stand and to understand the concept of truth in its most primitive form. We can imagine ever-widening circles of propositions---the distance from the centre (paradigm) being determined by the presence of complicating factors---space, time, generality, etc. and to the extent they make the propositions involved. To meet various kinds of complicacy in the presence of these factors we have different criteria of truth. But the understanding of the nature of truth in its primitive form enables us to understand the other forms.

So the discussion on the concept of truth is limited to the context of the paradox. Our purpose is served if we focus our attention on the paradigm cases.

3. A proposition is a sentence which is either true or false--- i.e., propositions have truth-values. If p is a matters-of-fact- sentence, then it can be either true or false. So the corresponding value-sentence-about-the-matters-of-fact-sentence would be either " p is true" or " p is false". By "value- sentences-about-matters-of-fact-sentences" I mean the sentences of the form " p is true" or " p is false" or their equivalents. No other kind of value is intended here. In " p is true" or " p is false" the matters-of-fact-sentence, p , is a part of the value-sentence, but in other cases it may not apparently be so, e.g., in the value-sentence "That's true" or "What you said is true", the matters-of-fact-sentence is apparently not a part of it, but it can always be reformulated such that it can have the form " p is true" in which the matters-of-fact-sentence is a constituent part.

I could have made use of the already available concepts like Tarski's "object-language" and "meta-language" or Max Black's "primary statement", and "secondary statement", but I have not done so. Rather, I have used *matters-of-fact-sentences* and *value-sentences-about-matters-of-fact-sentences*.

This I have done keeping in mind that Wittgenstein does not accept hierarchies of languages.

4. I have said that the word "this" is hetero-referential, not auto-referential. But someone may raise an objection and say that the word "this" can be used quite innocently for auto-referential purposes---for example, "This sentence has five words" and here one understands off-hand where to look and can see whether what is said is true or not.

As far as the use of the word "this" for auto-referential purposes in such sentences as :

This sentence has five words :

is concerned, I would say that this is also a misuse of the demonstrative "this". Even Wittgenstein, as we know well, does not use this word in *Remarks on the Philosophy of Psychology*, Vol. I, 65 (Oxford: Basil Blackwell, 1980). Rather, he introduces a new symbol "I" for auto-referential purposes. He says :

"In the language of a tribe there might be a pronoun, such as we do not possess and for which we have no practical use, which refers to the propositional sign in which it, occurs. I will write it like this : I. The proposition "I am ten centimeters long" will then be tested for truth by measuring the written sign. The proposition "I contain four words" for example is true; and so is "I do not contain four words". "I am false" corresponds to the paradox of the Cretan Liar. ---The question is: What do people use this pronoun for? Well, the proposition "I am ten centimeters long" might serve as a ruler, the proposition "I am beautifully written" as a paradigm of beautiful script.

"What interests us is: How does the word "" get used in a *language-game*? For the proposition is a paradox only when we abstract from its use. Thus I might imagine that the proposition "I am false" was used in the kindergarten. When the children read it, they begin to infer "If that's false, it's true, so it is false, etc." People have perhaps discovered that this inferring is a useful exercise for children.

"What interests us is: how this pronoun gets used in a *language-game*. It is possible, though not quite easy, to fill out a picture of a language-game with this word. A proposition like "I contain four words" might, for example, be used as a paradigm for the number four, and in another sense so might the proposition "I do not contain four words". A proposition is a paradox only if we abstract from its use."

Even if we allow the innocent use of the word "this" in the following:

- A. This sentence contains five words.

- B. This sentence does not contain five words.
- C. This sentence is beautifully written.
- D. This sentence is ten centimetres long.
- E1. This sentence is short.
- E2. This sentence which is written after the sentence No. E1 is long.
- E3. This sentence which compares itself with either the sentence No. E1 or the sentence No. E2 is longer than either.

We can agree with Wittgenstein and say that they are bound to different language-games and through their fulfilling certain purposes in the language-games they appear to be true. We can say:

- (a) A paradoxical proposition seems to say something, but, in fact, it says nothing; here we have to *assume* its truth-value, as there is no relation between such a proposition and any fact.
- (b) A proposition like "The cat is on the mat" says some thing and we have to determine its truth-value in relation to some fact.
- (c) In a definite language-game, a proposition in which there is an innocent use of the demonstrative "this" claims to *show* what it says; and if it, in fact, shows what it says, it is true, otherwise false.

The fact is that this "innocent" auto-referentiality of the demonstrative "this" is not due to the demonstrative itself. But it is put to such a misleading use by the word "sentence".

For in the case of the following sentences:

- (1) This sentence is short.
- (2) This book is short.

(1) would be treated as auto-referential but not (2).

So we can say that this misleading auto-referentiality of the demonstrative is conveyed not by the demonstrative itself, but by other word or words in a sentence or even by a particular context.

- 5. Tarski, Alfred, "The Semantic Conception of Truth and the Foundations of Semantics" in Herbert Feigl and Wilfrid Sellars (eds.), *Readings in Philosophical Analysis*. New York: Appleton- Century-Crofts, Inc., 1949, p.58.
- 6. Black, Max, *Language and Philosophy*. Ithaca, N. Y.: Cornell University Press, 1949, pp. 91-92.

7. Facts appear and disappear. Most of the facts, so to say, might not be there for two split seconds. Notwithstanding that I have said that if the truth-value ascription of a proposition p is challenged, we have to go back to the very same fact in order to confirm or disconfirm the said value ascription. Here I have in mind the paradigm cases (i.e., Snow is white or Taj Mahal is at Agra) which are relatively stable.
8. The paradox is also due to another reason. There is a confusion between the logical and the empirical. The expression "examine this page" in the quoted passage from Max Black makes what is logical dependent on what is empirical in the sense that in order to understand what is said to be a logical paradox we have to look again and again at the page. But Wittgenstein would say: "Logic must take care of itself". (*Notebooks 1914-1916*, Oxford : Basil Blackwell, 1979, p. 2.)