'IF THEN' AND HORSE SHOE ('⊃') - A STRAWSONIAN ACCOUNT

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Are '⊃' and 'if then' of ordinary language interderivable? Logicians are somehow divided in their answer. Some of them hold that they are interderivable, while according to some others 'if then' entails '⊃', but not conversely.

Before considering the relation between 'if then' and '⊃', let us find out the standard or primary uses of 'if then' and the meaning of '⊃' sign.

The fundamental feature of the standard or primary use of 'if then' is that for each hypothetical statement made by the use of 'if', there could be just one statement which would be antecedent of the hypothetical and just one statement which would be its consequent. When we consider one statemenu as the ground for another and believe it to be true, we normally don't express ourselves in one 'if then' sentence; we rather say something of the form 'p, so q'. When, however, we consider one statement to be ground for another and we are not certain that the first statement is true, or over believe it to be false, we express ourselves by using the form 'if p then q'.[1] But not all standard or primary use of 'if then' exhibits all these characteristics. It is possible in particular, to show a use which has an equal claim to rank as standard and which is intimately connected with the use described, but which does not exhibit the fundamental feature of 'if then' mentioned above. Strawson calls them 'variable' for 'general hypothetic'; e.g. 'If ice is left in the sun, it melts'. The statement made by the use of this sentence has no single pair of statements, namely, its antecedent and consequent, which have been considered as the fundamental feature of the standard use of 'if then'. Moreover, this use of 'if then' is always non-truth functional.

On the other hand, the use of material implication is always truth functional. We determine the truth value of material implication rather mechanically. We say that any statement of the form 'p ⊃ q' is true if and only if it is not the case both that the first of its constituent statements is true and second false, and false if and only if the first of its constituent statements is true and the second false. That is to say the falsity of the first constituent or the truth of the second is equally a sufficient condition of the truth of a statement of material implication and the combination of the truth in the first with falsity in the second is the single necessary and sufficient condition of its falsity.[2]

Having considered the standard or primary use of 'if then', Strawson next proceedes to analyse the uncommon uses of 'if' in ordinary language and makes a comparison between it and '⊃'.

An uncommon use of 'if' is exemplified in "If he has passed his examination, I will eat my hat". Some may find here an identity of meaning between 'if' and '⊃' on the following supposed evidence.

i) There is no connection between antecedent and consequent.
ii) The antecedent is not to be fulfilled.
iii) The consequent also is not to be fulfilled.
iv) Therefore, "(p ⊃ q), - q" entails - p.

But Strawson maintains that this supposed evidence is misleading. For if the hypothetical statement expressed by 'if' sentence above were to be regarded as material implication, then it would not have been a quirkish oddity, which it is.

Another uncommon use of 'if' is exemplified in "if it rains, I shall stay at home". This does not express a statement which is either true or false, but only announces an intention. So the truth functional consideration does not apply to it.[3]

Strawson maintains that 'if p then q' entails 'p ⊃ q', but not conversely. Let use examine on what ground Strawson could make the assertion. Now, let us consider the following examples :

i) If the Germans had invaded England in 1940, they would have won the war.

ii) If it rains, the match will be cancelled.

The corresponding material implication sentences will have to be formulated by slight changes in the clauses concerned, thus as :

i) The Germans invaded England in 1940 ⊃ they won the war.

ii) It will rain ⊃ the match would be cancelled.

Now from the above examples, the similarity and the dissimilarity between '⊃' and 'if then' can easily be pointed out : If it does not rain, then this fact is sufficient to verify the material implication statement expressed by 'It will rain ⊃ the match will be cancelled', for if the antecedent is false, the material implication is true. But it's not raining is not regarded as a sufficient condition of the truth of the ordinary hypothetical statement expressed by 'If it rains, the match will be cancelled'. This hypothetical statement is ordinarily taken to be verified when both the antecedent and the consequent are found to be true.

Moreover, the implicative statement expressed by "It will rain the match will be cancelled' is ordinarily taken to be consistent with the implicative statement "It will rain ⊃ the match will *not* be cancelled". So the formulea 'p ⊃ q' and 'p ⊃ - q' are consistent with one another and their joint assertion is equivalent to - p. That p ⊃ q and p ⊃ - q imply - p could be demonstrated thus:

1)	(p ⊃ q) . (p ⊃ - q) / ∴ - p.	
→ 2)	p	
3)	p ⊃ q	1, simp.
4)	q	3, 2. M. P.
5)	p ⊃ - q	1, simp.
6)	- q	5.2, M. P.
7)	q . - q	4,6, conj.
8)	p ⊃ (q . - q)	2 - 7 C. P.
9)	- p (By rule of absurdity)	

On the other hand, the statement expressed by "If it rains, the match will be cancelled", is ordinarily taken to be inconsistent with the statement expressed by "If it rains, the match *will not* be cancelled and hence their joint assertain in the same context is self-contradictory."

Strawson maintains that it would be wrong to identify the material implication sign '$\supset$' with 'If - then' of ordinary language. He says that there is a strong temptation to identify the English sentence --

a) "If he is a younger son, then he has a brother" with the material implication sentence --

(a_1) "He is a younger son $\supset$ he has a brother".

This tendency, however to identify (a) with (a_1) is the same as the tendency to identify 'if-then' with '$\supset$'. But Strawson remarks that this tendency must be resisted. Indeed, (a) is a disguised second-order sentence, and so is not equivalent to (a_1), but to.

(a_2) "He is a younger son $\supset$ he has a brother' is logically necessary.

So, it is clear that (a_1) is a part of (a_2) and so does not express the full force of (a). Hence they are not identical.

Someone might claim that like a material implication both the antecedent and the consequent of the hypothetical statement is true. Therefore, '$\supset$' can be identified with 'if then'. But this arguement however, is not tenable. Following Peter Alexander we may refute this suggestion.[5] Let us consider the following examples'.

i) Since '$p \supset q$' is true whenever p and q are both true; we can write:-

"Paris is in France $\supset$ Ireland is a republic" because both statement happen to be true. Now, if we simply replace '$\supset$' by 'if then' we obtain :

"If Paris is in France, then Ireland is a republic".

which is not the sort of thing we should normally regard as sensible to say. Because this is not the sort of connection between the two components which we useally expect when 'if then' is being used.

So it is clear that a statement of the form 'p ⊃ q' does not entail the corresponding statement of the form 'if p then q'. For a sufficient condition of the truth of the former need not be a sufficient condition of the truth of the latter. However, a statement of the form 'If p, then q' does entail a statement of the form 'p ⊃ q'. For one who makes a statement of the form 'if p, then q' must be prepared to deny the conjunction of the antecedent with the negation of the consequent. This can be explained with the help of the notion of entailment.[6]

S_1 entails S_2 = Def. ($S_1 \supset S_2$) is logically necessary, and

($S_1 \cdot - S_2$) is self contradictory.

For example : ''If he is a bachelor then he is unmarried'' is a necessary statement. Because if a man is bachelor then he must be unmarried and its negation is self-contradictory. To say that something is logically necessary is to say that its negation is impossible. It could be expressed by the use of modal operator thus :

$$L\ (p \supset q)$$
$$= -M - (P \supset q)\ \cdot$$
$$= -M\ (p \cdot - q)$$

- M (p · - q) is the stronger meaning of ''if p then q'', which means that it never be the case that p is true and q is false. From the stronger meaning of ''if p then q'' we can validly infer its weaker meaning. That is to say, we can infer ''it is not the case'' from ''it can't be the case''. Symbolically, we can infer - (p · - q) from - M (p · - q). Because - M (p - q) means ''it cannot be the case that p is true and q is falsẽ - M (p · - q) means'' it is not the case that p is true and q is false''. So the weaker meaning of ''if p then q'' is ''- (p · - q)''. This is similar to 'p ⊃ q', because 'p ⊃ q' is logically equivalent to - (p · - q). Strawson says that this is the minimum similarity between ''if p then q'' and ''p ⊃ q''. so it can be said that 'if p then q' entails ''p ⊃ q''.

Peter Alexander agrees with Strawson in this regard. He says that the material implication is a less complex relation than the implication expressed by ''if then'', and is not identical with it. But this is not to say that there is no connection between the two notions. He says that the material implication is the "weakest" form of implication, and it can be regarded as giving minimum conditions for any implication.[7]

Finally, Strawson points out that the parallelism and non-parallelism between "if" in its standard employment and which can be best indicated by the following laws :

a) The following laws which hold good for '$\supset$' but not for 'if then, can be considered as non-parallel.

$- p \supset (p \supset q)$

$- p \supset (p \supset - q)$

$q \supset (p \supset q)$

$q \supset (- p \supset q)$

$- p = (p \supset q) . (p \supset - q)$.

b) The follwing laws which hold good for '$\supset$' -

$[(p \supset q) . p] \supset q$

$[(p \supset q) . - q] \supset - p$

$[(p \supset q) . (- q \supset - p)]$

$[(p \supset q) . (q \supset r)] \supset (p \supset r)$

have, subject to some reservations, their parallels in the case of "if - then".

(if p, then q; and p) $\supset$ q.

(if p, then q; and not q) $\supset$ not p.

(if p, then q) $\supset$ (if not q then not p).

(if p, then q; and if q then r) $\supset$ (if p then q)

The above four laws are valid patterns. Besides these, Strawson points out some invalid laws which hold for '$p \supset q$'; These are :

$(p \supset q) . q \therefore p$.

$(p \supset q) . -p \therefore - q$.

with certain reservations in the case of 'if - then' the following parallels hold good :

(If p, then q; and q, therefore p),

(if p, then q; and not - p, therefore - q).

In the light of the above discussion Strawson concludes that "⊃" is not identical with "if then". Its close parallel in ordinary language is rather "not both ... and not ..."[8]

In his paper "Ifs and Hooks' Clark criticises Strawson. He raises several objections against Strawson.[9] We shall however, consider only one of them. Strawson maintains that (p ⊃ q) and (p ⊃ - q) are not incompatible with each other, though "if p then q" and "if p then not -q" are. M. Clark claims that there is no guarantee that in each and every case the conjunction of 'if p then q' and 'if p then not -q' is incompatible. To establish his own view, he cites the following examples :

C) The match won't be cancelled.

D) Whenever it rains on the day of a match the match is cancelled.

E) If it rains the match will be cancelled.

F) If it rains the match won't be cancelled.

There is not incompatibility or inconsistency between C and D in ordinary sense. C entails F and D entails E, But E and F are incompatible with each other. Now, we can show that (E . F) which are incompatible with each other can be validly derived from (C . D) which are compatible. This can be shown by the following derivation.

1)	(C ⊃ F) . (D ⊃ E)	
	/ ∴ (C . D) ⊃ (E . F)	
→ 2)	C . D	
3)	C	2, Simp.
4)	D	2, Simp.
5)	C ⊃ F	1, Simp.
6)	F	5, 3, M. P.
7)	D ⊃ E	1, Simp.
8)	E	7, 4, M. P.
9)	E . F	8, 6, Conj.
10)	(C . D) ⊃ (E . F)	2 - 9 C.P.

Clark argues that it is deniable that C and D are compatible. Further more, he claims that C entails F; it follows from the statement that the match won't be cancelled, that the match won't be cancelled (or at least whatever else) in fact happens. It is important to notice that the antecedent in F means "if in fact it rains" not if it was to rain" (in which case the consequent would have read ... won't read ...). It also seems clear, Clark thinks that D entails E. Now if C entails F and D entails E, We may say that C and D entail the conjunction E and F. Since C and D entail E and F and C and D are mutually consistent, E and F must also be consistent. For if C and D were mutually consistent and E and F were self-contradictory, it would be logically possible for C and D both to be true and E and F to be false. but then C and D would not entail E and F. Therefore, Strawson's contention that if p then q and if p then - q are incompatible does not seem to be tenable.

J. A Faris in the last part of his monograph "Truth - Functional Logic"[10] tries to show the interderivability of "if - then" and "⊃". To show that "if - then" and "⊃" are interderivable, we shall have to show (i) "if-then" entails "⊃" and (ii) "⊃" entails 'if-then'. Regarding (i) there is no disagreement among the logicans. But all disputes centre round the question whether 'if-then' is derivable from "⊃". Faris attempts to show that "⊃" entails 'if - then'.

Faris says that someone might claim that the use of "if p then q' is determined by some kind of necessary connection existing between p and q. But there is no need to admit such type of necessary connection so far as $p \supset q$ is concerned. So they are not interderivable.

Faris, however, enquires into the nature of the truth of 'if p then q' which is supposed to be necessary condition. It might appear at first that the connection is 'q is derivable from p'. For example, if 'p' stands for 'No Frenchmen were saved' and 'q' stands for 'No one who was saved was a French-man, then for some cases this certainly works :

(i) If no Frenchmen were saved then no one who was saved was a Frenchmen.

Where q is derivable from p. But Faris maintains that there are many cases where this procedure does not hold. Suppose 'p stands for' 'Smith is taller than Jones' and 'q' stands for 'Smith is taller than Robinson', then we have the proposition of 'if - then' form -

ii) •If Smith is taller than Jones then Smith is taller than Robinson.

in which 'q' does not certainly follow from 'p' simpliciter. To obtain 'q' from 'p' we must take the help of an additional proposition like :

(iii) Jones is at least as tall as Robinson.

Faris, then, says that someone might wrongly claim that (iii) is the necessary as well as the sufficient condition for the truth of (ii) If so, Faris however, does not accept this suggestion. For he holds that the truth of (iii), however, though a sufficient, yet is not a necessary condition of the truth of (ii). Because if (iii) is a necessary condition of (ii), it can not be the case that (ii) would be true, whereas (iii) would be false. But it is possible for us to show that (iii) can be true even if (ii) is false. Let us consider the following propositions.

(iv) Every member taller than Jones is red-haired.

(v) Every red-haired member is taller than Robinson.

(vi) Robinson is a member.

Now, if the above three propositions are true then the proposition (ii) must be true. And it is also important to note Faris says, that (iii) is false, even though the set of propositions consisting of (iv) to (vi) could well be true. For example, suppose Smith and Jones are two members of which Smith is red - haired but Jones is not. And further suppose that Smith, Robinson and Jones are respectively 6 ft., 5 ft., and 4 ft., tall, then the set of propositions consisting of (iv) to (vi) can be shown to be true, whereas (iii) is false. So it is proved that (iii) through sufficient, is not a necessary condition of the truth of (ii). But we are in search of a connection which is necessary condition of the truth of 'if p then q'.

Following the above examples, Faris, however, suggests a more promising essential condition (namely, condition E), in which he says that there is a set of true propositions (though unspecified) such that q is inferrible from p together with S[11]. The condition E, Faris maintains, will be staisfied in the case of true as well false propositions. Thus condition E is satisfied in the case of (iii) on the ground that if (iii) is a true proposition, then there is a set of true proposition, namely, the set consisting of (iii) itself, such that q is a fair inference from p together with S. Similarly, condition E is also satisfied in the case of (iii) on the ground that if (iii) is false, then there is a set of true propositions, namely, the set consisting of (iv) to (vi) such that q can be inferred from p together with S.

Thus it is clear that whatever proposition p and q may be, condition E is satisfied, so the condition E must have to be regarded as the necessary and as well as the sufficient condition for the truth of 'if p then q'. If 'p ⊃ q' is true then condition E. is satisfied on the ground that there is a set of true propositions, namely the set consisting solely of the proposition ''p ⊃ q' such that 'q' is inferred from 'p' together with S. Now, we can paraphrase the argument as follows :

i) If 'p ⊃ q' is true then condition E is satisfied.

ii) If condition E is satisfied then it must have to be regarded as a necessary as well as the sufficient condition for the truth of 'if p then q'.

iii) So 'p ⊃ q' is to be proved as the necessary as well as the sufficient condition of 'if p then q'.

iv) If 'p ⊃ q' is to be proved as the necessary as well as the sufficient condition for the truth of 'if p then q', then we must have to say 'p ⊃ q' entails 'if p then q'.

We have already established that 'if then' entails '⊃'. So it follows that the propositions 'if p then q' and 'p ⊃ q' are interderivable.

NOTES AND REFERENCES

1. Strawson, P. F. (1952) *Introduction to Logical* Theory, Oxford, P. 82.
2. *Ibid*, P. 82.
3. *Ibid*, P. 89.

4. *Ibid,* PP. 84-85.

5. Alexander, P. (1967) *Introduction to Logic,* London, George Allen & Unwin Ltd. P. 128.

6. Strawson, P. F. *op. cit,* P. 89.

7. Alexander, P. *op. cit,* P. 129.

8. Strawson, P. F. *op. cit,* P. 90.

9. Clark, M. (1975) "Ifs and Hooks", *Journal of Analysis.*

10. Faris, J. A. (1962) *Truth - Functional Logic,* London, Routledge & Kegan Paul. P. 109.

11. *Ibid,* P. 117.

establishment of prameya is pramāṇa dependent, then it amounts to committing the blemishes of interdependence and circularity. In view of this, Nāgārjuna, would argue that cognitivists' view about cognitive episode leads to more muddle and misunderstanding about pramāṇa, prameya and pramā (causal and justificatory grounds, knowables and valid knowledge). Nāgārjuna laughs at the cognitivists and says that if the validity of pramāṇas would be admitted inspite of all these visible and obvious defects, then there would not be any difficulty to assume that son is produced by the father and that father is produced by that son. But in this case who is it that gives birth and who is that is born''.[16] Therefore, pramāṇas do not lead us to establish anything (nirṇaya), a possibility of doubt always remains. Pramāṇa and prameya cannot validate each other. The criterion of mutual dependence rather shows that both pramāṇas and prameyas are devoid of any essence of their own (Śūnya). Since there is neither established pramāṇa nor established prameya the so called 'valid knowledge claim' of the cognitivists become unwarranted.[17] All views (dṛṣṭi) about the world, for Nāgārjuna, become systematically misleading and therefore, they are to be eschewed.

II

A Cognitivist Critique of the Nāgārjunian Critique of Pramaṇa Considered:

It has been seen that a cognitivist claims that it is possible on our part to know something with certitude and we can justify our claims by adequate supportive grounds. A Nāgārjunian sceptic only gives caution to these claims and shows flaws of antinomies in cognitivist's 'reasonings'. Let us now see how far the sceptical charges be answered from the cognitivistic viewpoint.

Vātsāyana would meet the sceptical charge of infinite regress by saying that it is not necessary that before functioning as an instrument a thing must be known first. For example, we become visually aware of something in front of us by our eyes, the sense of sight but we can not see the senses itself. We do not question or doubt about reality of our eyes. This shows that in practical experience, the establishment of pramāṇa does not arise and there is no scope for infinite regress, because their truth can be apprehended directly or immediately. A piece of cognition is said to be valid if practice based on the assumption of its truth leads to the attainment of desired end. What Udayana