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CRITICAL THINKING AND REASONING

The subject of this paper is critical thinking and good reasoning. The problem is: what are the basic principles of critical thinking and good reasoning? Views range from 1) only general principles specifically provided by formal logic to 2) no general principles, only subject-specific ones with a variety of views in between.

Principles of good reasoning, according to the Formal Logic Approach are provided by formal logic only. Formal logicians believe that any principles that are not reducible to logic are not needed at all. There is another group of philosophers who think that no general principle can cover all fields of study and subject-specific principles are required for good reasoning in each field.

Our position is that 1) abstract general principles of logic hold good in all fields; 2) abstract principles of logic are not adequate for good reasoning in all fields; 3) in every fields of study there are unique subject-specific abstract basic principles of reasoning that are good in that particular field only.

We will start the paper with a discussion of formal logic and various ways of defining formal logic. Then we consider how abstract general principles of logic apply in all fields. In order to make our case that there are abstract general principles of good reasoning that hold good *only* in specific subjects, we must develop further the concept of subject-specific principles of reasoning (SSPR) that hold good in specific subject areas or fields.

The Formal Logic Approach claims, and does so correctly in our view, that some principles of reasoning are not subject-specific at all, that they are all objective and independent of the subject matter

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or the content of reasoning. These principles of critical thinking may be applied to many subjects irrespective of their subject matter. The applicability of the principles of reasoning, according to the Formal Logic Approach, is not limited to any subject matter in particular. These principles are true not with respect to any context but without reference to any particular subject.

According to our view of critical thinking every subject has its own methodology of correct thinking and some of it may not be applicable in determining the correctnesss of arguments in any other subject. There may be some principles of reasoning that are useful in ethics which may be not very useful or may not even be applicable at all in thinking critically about legal matters. In some cases we do need some subject-specific principles of reasoning to think critically in different subjects over and above the general principles of formal logic. But it should be kept in mind that the pressence of these subject-specific principles of reasoning does not, by any means, weaken the great theoretical and practical value of principles of reasoning within formal logic.

The general principles of formal logic and informal logic and the specific principles of problem solving and decision making are equally important in this view as necessary ingredients of critical thinking. The fact of the matter is that many of the problems in various specialized disciplines and also problems we always face in our concrete real life, such as social, educational, and cultural problems that are roughly called common sense everyday problems, cannot be solved if attention is restricted to the concepts, principles, rules and structures provided by formal logic. Not only are there subject-specific truths------ there are subject-specific rules of reasoning.

Real life problems are not always in ideal logical situations. For example, legal philosophy accepts all fundamental laws of logic; nonetheless, legal reasoning is controlled by legal principles and procedures, the principles and procedures that are not principles of logic as such, in order to achieve the fundamental objectives of a legal order in a given situation. There are many situational and domain specific constraints in various disciplines and in our every day life that may not be considered in formal logical systems because the formal logical rules simply do not apply to them.

Merrillee Salmon expresses her concern about the twentieth century narrow treatment of logic. Salmon maintains that in the past, 17th century Port Royal Logic (The *Art of Thinking*) had only one fourth of the book covering what we now call logic and the rest of the book covered materials that fall under the scope of what we now call reasoning. She is not quite happy with the modern scholarly usage of the term 'logic' which narrowly limits itself to "investigating and framing general principles concerning the relation between premises and conclusions of correct arguments."[1] Logic in this sense, according to Salmon, covers only a part of reasoning. Here is how Salmon describes the scope of reasoning:

> Reasoning consists of many different skills: the abilities to think coherently, to comprehend instructions and advice, to understand the difference between unsupported claims and arguments, to recognize when unsupported claims need support, and to marshall support from general background knowledge or from new investigations. Reasoning also includes formulating problems and figuring out their solutions, drawing conclusions from premises, designing through experiments or reall experiments that can test claims, formulating and using principles to evaluate arguments, seeing the force of counterexamples, making judgements of information's relevance, as well as surveying and assessing possible outcomes of decisions and plans.[2]

Our conception of critical thinking includes logic as well as reasoning in the sense that Salmon uses the term 'reasoning'. In the following pages we would like to address the issue of subject-specific principles of reasonning (SSPR) with some examples from different fields.

Subject-specific Principles of Reasoning (SSPR)

Everyone agrees that there are truths of logic and also, there are truths in other subjects that are not and cannot be deduced from logical truths. But that is not the issue in this study. The issue is whether there are any subject-specific principles of reasoning (SSPR) that are unique to each different field and, do not hold good in general-principles of reasoning that are good in one field and not good in another. This author believes and argues for, throughout this dissertation, the following theses:

1) Logic provides us principles of reasoning that hold good in all fields and all subjects in determining the validity or invalidity of arguments, and the logical truth of statements.

2) Logic does not deal with all the principles of good reasoning in all fields, simply because like all other subjects logic has a subject matter of its own and, it does not include all the principles of good reasoning in all specific subjects. It simply deals with principles of good reasoning that are good across the disciplines or fields. Formal logic deals with syncategorematic terms e.g., 'and', 'not', 'all', 'if..... then....' which are used in all fields.

3) In every discipline or field, over and above the formal logical principles that are good in all fields equally, there are many subject-specific principles of good reasoning. To say that there are subject-specific principles of reasoning that are not deducible from or reducible to logic is different from saying that the factual truth of premises are subject-specific. There is no disagreement that the factual truth or falsity of premises in different subjects are not determined by logic. Beyond that, it is also true that there are principles of reasoning in each subject that are unique to that subject and, are not deducible from or reducible to logic. Also, those subject-specific principles of reasoning may not be good for reasoning in some other subject or subjects.

Those subject-specific principles of reasoning are provided by the field and these are unique in that subject, and may or may not hold good in other subjects at all. Knowledge of a particular field provides those principles of good reasoning in that particular field, and logic does not and cannot do that.

This is why we have deontic Logic ('It ought to be a voluntary action in order to be a moral action'), Logic of belief and knowledge ('P *knows that* Q iff P believes that Q, Q is true, and P has undefeated justification for Q', 'P *believes that* Q iff Q is true or false'), etc. These extensions of logic have different scope and functions, and are significantly different from formal logic. These logics have their truths about particular facts, and truths about general principles that are clearly different from one another's. But the question is: do these disciplines and, others like them have their own unique subject-specific rules of reasoning that are neither deducible from nor are they reducible to the principles of reasoning in logic?

In order to answer this question we have to show that activities of reasoning well in a particular discipline are governed by abstract principles other than the principles of logic. It is not merely showing that there are subject-specific truths in those subjects that are unique to those subjects only. True statements about particular entities or about general concepts in a subject are not the principles of reasoning in that subject. In history, for example, it is true that Socrates was not born before Buddha. This statement is a statement of a particular event. On the other hand, there is a factually true general statement in history---"No queen had ever been a cobbler before she became a queen". But, neither of these statements are principles of reasoning, although these statements may be used as premises in arguments.

SSPR in Mathematics

Let us consider a problem from elementary mathematics:

"What number must be added to the numerator and to the denominator of the fraction 1/4 to give the fraction 2/3?"

In order to solve this problem we must first understand that the numerator is the number on the top and denominator is the bumber on the bottom of a fraction. Thus in this fraction 1/4, 1 is the numerator and 4 is the denominator. This is pretty simple truth in mathematics. What is not so simple is to find what is or are the principle(s) of reasoning that will help us in getting to the conclusion, the correct number in this case.

One sort of rule for reasoning in such problems is to represent the unknown number by x. Then: one formulates the problem as $x+1/x+4 = 2/3$ and focuses on trying to find the mathematical value of x. The reasoning proceeds by steps, perhaps as follows:

Step 1) $x+1/x+4 = 2/3$ [from the statement in the problem]

Step 2) $3(x+1) = 2(x+4)$ [from 'a divided by b equals c divided by d' *infer* 'a times d equals b time c']

Step 3) $3x+3 = 2x+8$ [from 2 based on 'if $x(y+z)=w(u+v)$, then $(xy+xz)=(wu+wv)$]

Step 4) $3x-2x = 8-3$ [from 3, based on 'if $x+y=z+w$, then $z-x=w-y$']

Step 5) $x = 5$ [from 4, based on $3x-2x=x$ and $8-3=5$]

Notice here that the problem cannot be solved by setting the fractional equation at the beginning of this solution by merely setting $x+1 = 2$ and $x+4 = 3$. Clearly, the steps of reasoning by which we arrived at the conclusion are steps of good reasoning that led us to the correct conclusion. Selecting or finding out some correct steps or reasoning as opposed to many others is a matter of judicious choice. Making this judicious choice is an integral and unique characteristic of reasoning.

Let x, y, z, u, v, and w represent some mathematical number. The universal truths of mathematics that have been used here are the following :

1. (x) (y) (z) (w) (If $x/y = z/w$, then $xXw = yXz$) - [used in step 2)]
2. (x) (y) (z) (u) (v) (w) (If $x(y+z)=w(u+v)$, then $(xy+xz)=wu+wv$) - [used in step 3)]
3. (x) (y) (z) (w) (If $x+y=z+w$, then $z-x=w-y$) - [used in step 4)]

These postulates that have been used above are unique in mathematics, and are not reducible to any principle of inference in logic. These postulates are universal and general in the sense that they are subject-specific in the sense that they are applicable only in the field of mathematics and are not reducible to principles of logic.

A traditional logician might argue that the above proof simply shows that the above process of reasoning is nothing more than a proof of deductive logic using rules of inferences in the system. She will say that the above proof can be formulated in deductive logic the following way :

1) x+1/x+4 = 2/3 [*Premise* 1, from the statement of the problem]

1.1) (x) (y) (z) (if x/y=z/w, then xXw=yXz) [*premise* 2, a postulate or universal truth of arithmetic]

1.2) (if x+1/x+4 = 2/3, then 3(x+1) =2(x+4) [from 1.1) by *UI* four times, replacing 'x' by x+1', 'y' by 'x+4', 'z' by '2' and 'w' by '3']

2) 3(x+1) = 2(x+4) [by 1], 1.2) *Modus Ponens*]

2.1) (x) (y) (z) (w) (v) (if x(y+z)=w(y+v) then xy+xz=wy+wz) [*premise* 3, a posstulate of arithmetic]

2.2) if 3(x+1) = 2(x+4) then 3x+3 = 2x+8 [from 2.1) by *UI* five times, replacing 'x' by '3', 'y' by 'x', 'z' by '1' and 'w' by '2' and 'v' by '4']

3) 3x+3 = 2x+8 [by 2), 2.2) *Modus Ponens*]

3.1) (x) (y) (z) (w) (if xy+xz=wy+wz then xy-wy=wz-xz) [*premise* 4, a postulate of arithmetic]

3.2) If 3x+3 = 2x+8 then 3x-2x = 8-3 [from 3.1) *UI* four times, replacing 'x' by '3x', 'y' by '3', 'z' by "2x' and 'w' by '8']

4) 3x-2x = 8-3 [by 3), 3.2), *Modus Ponens*]

4.1) (If (x) (y) (If 3x-2x = y then x=y) [*premise* 5, a theorem of arithmetic]

4.2) If 3x-2x =8-3, then x = 8-3 [from 4.1) *UI* twice, putting 'x' for 'x', and 8-3 for 'y']

5) x = 8-3 [4), 4.2), *Modus Ponens*]

6) x = 5 [from 5, and the subtraction table : 8-3 =5]

A traditional logician, therefore, will claim that in solving the above problem of mathematics she had to use some postulates of mathematics, the rules of inference *Modus Ponens*, and *UI* (Universal Instantiation), and nothing else. Therefore, she will conclude that reasoning involves nothing else but the rules of logic and postulates in the subject.

This view of reasoning confuses the process of reasoning with the *product* or *by-product* of reasoning.[3] The above logical proof was drawn only as a *by-product* of the reasoning that took place to solve the problem. The steps of reasoning were stated earlier above. Before the reasoner could state the above logical steps she had to reason first to make sure that she has justification for all her steps of proof. Reasoning is an activity to figure out something that is not already known to the reasoner.

Reasoning involves the reasoner's initial ignorance of the solution to the problem or even uncertainty of the way out. It is like the Socratic method of dialectic to find out what is initially unknown. Logic, on the other hand, does deal with the product of the reasoning-----the argument, and analyze to evaluate if it is a good argument.

Good reasoning requires the reasoner to be judicious in choosing the steps of reasoning toward solving the problem—the reasoner must know how to define or figure out the problem, recognize the diversity of choices for trial and error, where to start and how to proceed with the steps of reasoning toward an unknown conclusion. In a formal proof of the validity of an inference we do not have to figure out what the problem is, we already know the given premises and the conclusion, and we have to simply figure out the unknown steps that are to be taken to reach the conclusion. We also have to provide justification for those steps through the rules of inference that are also given in logic. Here is a problem for your consideration :

> The plan to open a four-year coeducational college has just been put into operation with a freshman class of 500, mostly females. Each year another class will be added. The Director of Admissions is told that she must raise the ratio of male students in the next two years so there will be no more than three females for every two males; but she must do this without discharging any students, and she must admit men and women in equal numbers. The first freshman class

has 400 female students and 100 male students. What is the minimum number of men and of women the Director of Admissions must admit in the next two years to meet this objective?[4]

This is essencially the same problem that we have just solved except for the change of numbers from ones to hundreds. The mathematical formulation of this problem will look exactly like the initial step of the previous problem that we have solved----- what is x, if (x+1)/(x+4) = 2/3 (where the x this time will be a number in hundreds that will be the number of male students and of female students that must be admitted). But it will be hard if not impossible for most people to formulate the problems. Thus, defining a problem is sometimes much difficult than solving the problem. Solving the problem in this case may seem to be much easier once it has been defined precisely.

Secondly, when the problem has been understood and defined, the reasoner looks for a method to follow to solve this problem. There could be various different methods of reasoning to follow to solve the same problem. In many cases it becomes a matter of trial and error to get to a correct method or the correct method of reasoning. There is no principle or rule of formal logic that will guide the reasoner to adopt one method or the other in the above case, nor can formal logic supply the principles of reasoning that were used in solving this problem. Moreover, these rules of reasoning cannot be deduced from or be reduced to the rules of inference in formal logic.

Thirdly, when we have tried to determine what the steps of reasoning would be in order to get the unknown number x, no theorem or rule of inference of logic determines which one of many possible steps to be taken to make it a good effort (good reasoning) to solve the problem.

Good reasoning involves or consists in choosing the best among many alternative steps to choose from towards solving the problem. For example, in an attempt to solve the algebraic problem on pages 219-20,[5] we have reasoned it out following the definition of the problem in step 1). We found out certain steps of reasoning to get to our goal of finding the value of x when x+1/x+4 = 2/3. We tried to derive a sequence of equations which result in an equation of the form "x = ..." while the dots are being filed by some function of mathematics (in this case '8-3'). But how we get that equation. The reasoner has to make decisions as to what the next best step should be in reaching the goal directly.

We can think of many derived algebraic equations and auxiliary rules of reasoning that could be used as steps of reasoning in this problem. It is essential that the reasoner is aware of those algebraic rules of reasoning. The reasoner must know how to use those rules to do correct reasoning in solving algebraic problems. Some of the algebraic rules of reasoning are as follows :[6]

a) If you have gotten 'x=y', then infer 'x+z = y+z'
b) If you have gotten 'x/y = z', then infer 'wXx/wXy =z'
c) If you have gotten 'x/y = z', then infer 'wXx/y = wXz'
d) If you have gotten '(x+y = z+w)', then infer 'xXw = yXz'
e) If you have gotten '(x+y = z+w)' then infer '(z-x = w-y)
f) If you have gotten 'x(y+z) = w(u+v)', then infer (xy+xz = wu+wv)

and many others. These are auxiliary hypothetical imperatives. All of these are backed up by, but not the same as, certain universal mathematical truths, e.g., f) is backed up by the universal mathematical truths numbered 2. on page 220.

In each step of the problem solving in this case the reasoner could use any one of these rules of reasoning a) through f). All would be sufficient to arrive at a logically valid or mathematically sound conclusion. But what we are looking for here is not *any* or *a* valid conclusion, but *the* correct solution to the problem. Therefore, the reasoner has to pick the most appropriate rule of reasoning at every step of reasoning to get to the goal. In solving the problem on pages 219-20, a decision to use a) or c) or e) or f) in moving to the next step would have been a poor choice because that would take the reasoner off the track, indicating that the reasoner has poor knowledge of and/or poor ability to reasoning well in algebra.

The result actuallly given as step 2) of the reasoning process on page 219, 3(x+1) = 2(x+4), was not merely a logically valid or mathematically sound step, it was the result of making a good choice (as well as mathematically sound, of course) among many alternatives at hand, in achieving the goal. In algebra classes it is the duty of instructors to teach people how to choose the best rule of reasoning at a given time toward achieving a given goal. Thus, algebraic reasoning is very much goal-oriented, and not just a matter of following mechanical algorithms leading to some sound mathematical or valid logical conclusions.

Thus, good reasoning in algebra consists of inculcating the major over-all strategic rules and methods, together with awareness of auxiliary rules and suggestions about how and when to use them. The subject-specific rules of reasoning involve much more than theorems or rules of logic alone.

It is not logic that is the best guidance towards making the best choices among many alternative ways to solve a problem. The rules of reasoning are chosen by the reasoner, and are not dictated by logic. Rules of reasoning are not *categorical imperatives,* they are *hypothetical imperatives.* They are not forced upon the reasoner as required rules or principles, they are simply suggested or presented to the reasoner as possible steps of reasoning.

Fourthly, in reasoning we do not have any actual truth to begin with. We only have assumptions or premises to start the reasoning towards problem-solving. The equation "(x+1)/(x+4) = 2/3" that we began with the previous problem is not a truth, it is a sentential function. Each step of equations following this equation was also a sentential function. Sentences are either true or false, sentential functions are neither true nor false. If we replace 'x' by the same number at every step of our reasoning then that would make each step of reasoning true.

It is true that while doing reasoning we use some rules of logic, but reasoning itself or the processes of reasoning is not logic. Principles of reasoning are different in nature from the principles of logic. Principles of reasoning are rules or principles that help find reasons for claims. Reasoning is a process that usually leads to a conclusion from some premise or premises. But reasoning is not always good or successful. Logic provides some rules or principles to justify only some steps of reasoning that are good and answer questions as to whether an argument is valid or a set of statements is logically consistent. The reasoning process, on the other hand, begins with a problem, the answer or solution of which is still unknown. Reasoning process leads to a conclusion and also tells us if the resultant argument it leads to is a deductive argument or not.

Thus, a rule of reasoning (SSPR) may be said to have the following form :

> If you have a problem----- i.e. (i) there is some unknown property, or relation, or subject, x, and (ii) you want to know, or find out, what that property, relation or subject, x, is, and (iii) you have available only information of sort z about the unknown property, relation or subject x, as you start *then* do A!

Angell gives a precise illustration of an SSPR of algebra in the following words. He analyzes SSPRs as 'hypothetical imperatives.' Here is a 'hypothetical imperative' that he considers a basic subject-specific rule of reasoning in algebra:

> *If* you are trying to solve a problem of arithmetic, a problem in which (i) a number is unknown and (ii) you want to find out what it is, and (iii) the statement of the problem gives you some characterization of the unknown number,
>
> *then* try to find the answer by finding a sequence of true equations, the first equation being gotten from the statement of the problem, each of the others derived from the equation preceding it, and the last one being an equation in which the unknown, x, is on one side and an arithmetic function of constant number is on the other side.[7]

This is an SSPR because it is *abstract* in that it uses terms like 'number' and 'equation', but is not tied to any specific number or particular equation and it is *general* in that it can be used for reasoning about an infinite number of possible algebraic problems of the sort presented in maths texts as well as in our practical life. It is also *subject-specific* in that it uses abstract concepts e.g. 'number', 'equation' that are specifically and precisely mathematical concepts. Also, it is a *principle* and not a universal truth (not a *categorical* impperative), because it is one of the many suggestions for reasoning out a solution to a mathematical problem of the above sort. A problem solver is expected to find a solution to the problem stated in the antecedent by producing a sequence of mathematical equations to be produced in the consequent.

But, it is still a basic principle of mathematical reasoning for solving problems. All principles of reasoning, regardless of their grammatical variations, are goal-conditioned 'hypothetical imperatives'[8]. But not all goal- conditioned 'hypothetical imperatives' are principles of reasoning. Many hypothetical imperatives are simply guides to overt, physical action. For example, if you want to cook some Mexican food, *then* pick up a Mexican cook book, get all the ingredients and follow the recipe and directions in it.

Also, hypothetical imperatives that suggest solving a problem directly from an authority or some other source, e.g., computer programms, books are not to be treated as principles of reasoning. Principles of reasoning must be usable by the individual who is doing the reasoning to solve a problem without reliance on outside sources. Thus, many mechanical rules are not to be treated as rules of reasoning. It must be remembered SSPRs are rules of reasoning that are abstract and general, but yet subject-specific, and not reducible to the subject-specific theorems or postulates or mechanical algorithms or to the principles inference of formal logic.

Many traditional logicians confuse reasoning with logic, and thus confuse the principles of reasoning with the principles of logic. They confuse, as John Dewey mentions in many of his writings,[9] the process with the product of reasoning. Not all reasoning lead to logic although all logic is a result of reasoning. In our examples of mathematical problem solving we used some principles of reasoning that are neither derived from logic nor reducible to logic. SSPRs are not principles of logic, they are principlles of reasoning. SSPRs are used not so much to evaluate arguments, but to initiate and guide one in solving specific kinds of problems.

In the algebraic example above, when we formulate the problem, the formulation of the problems does not yield a logically valid structure, how well we formulate it does not depend upon logic, it depends upon the subject-specific knowledge of the principles of mathematics. Thus, logic does not supply all the general, abstract principles of reasoning in all subjects----- subject-specific knowledge contributes to a lot of those general principles in the subjects that are peculiar to those subjects only.

One might say that these are examples of SSPRs, and ask for a description or a schema of an SSPR. In response to that it can be said that an SSPR is not simply a subject-specific descriptive general statement of fact which is true of elementary entities in the field. Since, according to our definition, reasoning is a kind of activity, a principle of reasoning is also a dynamic rule of action. Principles of reasoning facilitate transition from one assertion to another.

Thus, an SSPRs is a fundamental principle of reasoning in a field which is a general statement (not at all tied to particular entities or facts in the field) and which involves basic concepts in the field and which help reasoners draw new conclusions from some known or given statements. An SSPR in a particular subject is based upon the fundamental concepts of the field which differentiates it from other subjects. SSPRs are not necessarily derived rules of a subject matter, although they may be. They are neither deducible from nor reducible to principles of pure formal logic.

There are SSPRs even in formal logic. For example, statements of the form "If P, then Q" are not necessarily principles of reasoning. They are simply conditional statements. On the other hand, *Modus Ponens* is a principle of reasoning which goes as follows : "Given a statement P which is true, and a statement (If P then Q) which is true, infer Q is true". Thus, an indicative statement is static, an SSPR is not — an SSPR is dynamic — it helps make moves from something known to some unknown facts.

We have given some examples of SSPRs in mathematics. But, some people, following Russell and Frege, might want to hold that maths *is* deducible from purely formal logic. They might say that Russell and Whitehead wrote all of the three volumes of *Principia Mathematica* to show that. Contrary to this view of Russell and others, our view is that Mathematics *is not* logic. Following Quine, we hold that theorems of formal logic are statements that are true solely by virtue of their logical structures.[10]

Strictly speaking, Quine in his *Elementary Logic* holds, and we think he does so rightly, that formal logic is quantification theory.[11] Following Quine, we hold that strictly speaking, formal logic is nothing else but sentential logic and qualification logic. Set theory is not part of formal logic, it is subject-specific discipline that is beyond logic.[12]

Pure formal logic deals with statements that cannot possiblly be false. Like many other people, Quine in his early writings (*Mathematical Logic,* 1940) shows a great fascination for Russell's theory. Later on, Quine realized that he had made a mistake and revised his theory on the scope of logic (in *Elementary Logic,* revised in 1965) and held that logic does not include relations of identity and class membership (set theory). Mathematics is not reducible to set theory; rather set theory is used to develop mmodels for mathematical statements.

Since it is obvious that set theory is a requirement for the development of mathematics from logic, and since set theory is not part of formal logic, it follows logically that mathematics cannot be deduced from pure logic. One might wonder what then is the relationship between mathematics and set theory. According to many, set theory does not really yield mathematics. It simply provides some complicated set—theoretic models, a set of true statements which can be made to stand in one-to-one correspondence with true statements in mathematics.[13]

It is at least highly debatable, if not absolutely false, that the complicated definitions and concepts of logic that are offered as definitions of natural numbers were created before people started counting numbers, or adding and subtracting numbers. The truth is that long before the invention of these definitions and concepts, people started using natural numbers to count, add, etc. Set theory *does not* analyze what is really contained in the notion of numbers. Mathematics cannot be deduced from pure logic since the development of mathematics from logic requires set theory and, set theory is not part of logic.

It must also be noted that in order to solve many mathematical problems like the one on algebra earlier in this essay it is not necessary that we use the rules of reasoning used in developing the set-theoretic models of mathematics. Can someone who has the mastery only in the methods and proofs of set theory solve the problem ? The answer seems to be far away from a "yes". It is not at all clear if there is any necessary connection between the rules of reasoning in algebra and the rules of reasoning in formal logic in the sense in which the principles of reasoning in algebra could be derived from the principles of reasoning in formal logic. Mathematics is related to logic onlly when 'and', 'all', 'not', and 'if........then....' are involved.

Moreover, set theory is not *all* of mathematics. Mathematics cannot be reduced to set theory. Set theory is used only to depict some system of set theoretical models which are analogous to mathematical theorems. But that must not lead us to believe that mathematics is all set theory. Mathematics is much more than mere set-theoretical models. In order to show any correspondence between mathematical truths like '(2+2) = 4' and theorems of logic in set theory, one must first understand what '2' and '+' and '=' mean and how and why '2+2 = 4'. Primitive notations of set theory which contains *no* mention of numbers would not help at all in solving the understanding *why* and *how* this is the case that '2+2 = 4'. This analysis could be extended to other disciplines.

SSPR in Chemistry

Speaking of the distinction between logical equivalence and mathematical equality reminds us that there are still other kinds of equalities, an equation in chemistry, for example, gives us information about chemical reaction between two or more simple or compound chemical elements. In a chemical equation, on the left hand side of the "=" (equal) sign we put the *reactants* (elements that disappear at a result of the chemical reaction) and, on the right hand side of the equation sign we put the *products* (elements that appear as a result of the chemical reaction).

While in a mathematical equation we are dealing with equal numbers on both sides of the equation, and in logical equivalence we are concerned with the same truth value on both sides, in chemistry, however, we have to look out for the satisfaction of three conditions.[14] A chemical equation is valid iff: 1) it is consistent with the relevant experimental facts and also, gives a precise statement of the elements that disappear (reactants) and the elements that appear (products); 2) no element in the mass gets destroyed and we can account for *every* atom in order to maintain the law of indestructibility of mass; and 3) the equation is consistent with the conservation of energy.

In order to have a *balanced* equation in this case we must fulfill the second and third conditons mentioned above. A balanced equation must have the same number of atoms of different kinds on both sides of the sign of equation "=". Moreover, a balanced equation has to have equal amount of net charge on both sides. A chemical equation shows that the reactant and the product are in a state of equilibrium. For example, to show the equality of the *reactants*, one molecule of gas S(g) with one molecule of gas T(g) to form the *products*, molecules of gases V(g) and W(g), chemists use the following equation :

$$S(g) + T(g) = V(g) + W(g)$$

It is important to note here that this chemical equality is very different from mathematical equality on the one hand, and logical equivalence or even identity on the other. '(5 X 6)' is mathematically equal to '(15 X 2)'. But it does not make sense to say that '(5 X 6)' or (5 groups of 6) and '(15 X 2)' or (15 groups of 2) are one and the same thing. Chemical equality is still another kind of equality different from mathematical equality, logical equivalence or identity.

The chemical equality in the above case demonstrates that the molecules of gases S and T react to form molecules of gases V and W. So long as these two gases S and T are present, this reaction continues. As soon as an appreciable number of V and W molecules form, they react with each other to produce S and T. This process goes on as long as the molecules are present.

The scientific method that use in critical thinking or reasoning in chemistry contains the following steps: (1) empirical observation; (2) classification and analysis of the empirical date to formulate a generalization from those observed facts; (3) verifying or checking the rules or patterns to which the observed facts conform; and (4) checking the generalization by setting up new experiments, and refine it or change it in the light of further evidences.

The process of checking and rechecking generalizations (scientific generalizations are commonly called *hypotheses*) in a wide variety of ways in order to refine or reject them goes on indefinitely. It must be noted here that there is no strict uniformity among scientists in using these steps in the process of checking and rechecking hypotheses. Although there is no single way of interpreting the data collected through scientific observation, the observations themselves should be beyond dispute.

It is not only the methodology that is unique and different in chemistry than in logic, the general principles of reasoning in chemistry are also unique in chemistry and may not be reduced to logic. These are not general principles that can be deduced from or reduced to mathematical equality or logical equivalence, yet they are general principles, subject-specific general principles of chemistry.

There are other general principles of reasoning specific to individual subjects that are not reducible to logical principles in the narrow sense of logic. For example, the so-called logic of knowledge and belief has to do with the subject-specific knowledge of epistemology ----what exactly is meant by 'knowledge', what exactly is meant by 'belief', what is the difference between knowledge and belief----- these are questions that are to be answered not by logic (in the strict sense of *formal logic*), but they are to be answered by epistemology. The so-called logic of probability and statistics has to do with the subject-specific knowledge of the relationship between events and their possibility of occurrence.

The bottom line is : in solving many problems in various disciplines and in daily life, we cannot simply ignore the semantic content of the question which poses the problem. This does not, by any means, weaken the value of formal principles to determine the validity of arguments. Of course, the rigorous principles of logic are useful in seeing if it is possible to have a counter example to a particular logical form of an argument. Logic also offers strong techniques to determine the logical truth of statement forms. The very important point that we want to make here is that, although very useful for our purposes of reasoning, it simply does not provide sufficient tools for reasoning with a view to solving any problem other than purely logical ones completely.

Critical thinking, the way we understand it, goes beyond logic in that it not only evaluates the logical validity of arguments or the logical truth of statement forms, it also helps us to get into the issue raised by the argument. Critical thinking has as its goal the analysis of actual arguments, not just the logical structures of arguments.

SSPR in Euclidean Geometry

Now, let us switch to another subject, Euclidean Geometry where we can make use of subject-specific principles of reasoning to solve a problem (or answer a typical mathematical question that may appear in Euclidean Geonmetry).[15] *Problems:* 1) How many regular convex solids are there? and 2) Why is it that there are five, and only five, regular convex solids? In order to answer these questions we must first ask ourselves if we have the background knowledge in Euclidean Geometry. Suppose we have the background knowledge of the axioms, postulates, and definitions of the three dimensional Euclidean Geometry. So, we know that a regular convex solid, by definition, has equilateral plane figures as its faces, and the angles at any vertex will add up to less than three hundred and sixty degrees.

From the background knowledge of the system of three-dimensional Euclidean Geometry that we have, along with the help of a set of reasons or *grounds*[16], as Toulmin calls them, *with strict geometrical necessity*, we can draw the conclusion that there are five and only five regular convex solids. This conclusion (which is a theorem in the three-dimensional Euclidean Geometry) holds true because of the whole system of Euclidean geometry. No general principles of logic will help us to draw this conclusion.

What is required of us to think critically about this problem is the knowledge of the subject-specific abstract principles of reasoning unique to solid geometry. Since we do not have any specific knowledge about solid geometry, we do not know exactly what those general principles of inference in solid geometry are, but someone who does know the subject would know what those general principles of inference are that are essential for formulating arguments in solid geometry that are valid and, perhaps, sound as well. What this example demonstrates is that in order to develop the ability to think critically in a subject, one needs to know the relevant subject-specific principles.

Our examples of subject-specific principles of reasoning in various fields, e.g. chemistry, geometry, ancestor-relationships, and algebra do indicate that there are principles of reasoning which are subject-specific yet abstract and general in their own subjects. These principles are neither universal truths themselves nor are they theorems or rules of inference from formal logic. These examples also demonstrate that there are rules of reasoning that must be used appropriately in order to lead reasoning in a good way.

However, this discussion by no means indicates that principles of logic are not applicable in those fields at all. Of course, in addition to the subject-specific abstract principles of different fields, there are general principles of logic that provide techniques of good reasoning in all disciplines. This account has been an attempt to remove some of the basic and very important conceptual confusions that traditional logicians have as to the principles of reasoning and the principles of logic.

In conclusion, our claim is that this account clearly shows that (i) there are subject-specific rules or principles of reasoning (SSPR) that are abstract and general in the given field; (ii) SSPRs are not in themselves universal truths although sometimes their acceptability might depend upon the support of general truths; (iii) SSPRs are neither theorems of logic nor are they principles of inference of formal logic; (iv) good reasoning in a field depends upon choosing the best rule (hypothetical imperative) among many available alternatives; and (iv) good reasoning involves choosing rules of reasoning that are appropriate and relevant to the objective of the problem solving.

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NOTES

1. Salmon, Merrillee H., "Informal Logic and Informal Reasoning" in James F. Voss et al. edited *Informal Reasoning and Education*, p. 153.

2. *Ibid.*

3. This distinction between the *process* and *product* of reasoning was first made by Dewey in his *How We Think* published in 1933.

4. I am grateful to Angell for bringing this problem to my attention.

5. Angell helped me on this in our private conversation while I was revising this essay. I am grateful to him for that.

6. Angell helped me on remembering these rules of algebraic reasoning that I used in solving algebraic problems in high school.

7. Angell, *Philosophical Thoughts,* 11/7/1991.

8. I took this term from Angell's *Philosophical Thoughts,* 11/7/1991. This term was originally used by Kant.

9. John Dewey makes frequent mention of this process/product distinction in his *How We Think* and other philosophical writings.

10. Quine says that "... a sentence is logically true if all sentences are true which share its logical structure and "what I mean by the logical structure of a sentence at this stage is its composition in respect of truth functions, quantifiers and variables." (*Philosophy of Logic,* Prentice-Hall, 1970, p. 49)

11. We must note here that Quine revised or even changed his position on the scope of formal logic over the years. In 1940 when he wrote *Mathematical Logic* he followed Russell and Whitehead in holding the position that set theory is part of formal logic. But later in his *Elementary Logic* (Revised edition, 1965), Quine does not consider set theory as part of logic. He says, "Logic in its strictest sense is quantification theory, and a logical deduction in its strictest sense consists in establishing a quantificational implication." (p.116)

12. Quine recognizes this fact in the "Preface" of his 1981 revised eddition of *Mathematical Logic* (p.iii) where he writes about the 1940 edition of the same book that, "Like *Principia* it subsumes set theory under logic instead of recognizing it a mathematical discipline beyond logic". Quine acknowledges that he changed his view on the scope of logic in the "Preface" to the revised edition of his *Mathematical Logic* (1981).

13. Angell pointed this out to me. He gave me some examples from Quine's *Mathematical Logic* where Quine deals with the concepts of the numbers 0, 1, 2, etc. to demonstrate that.

14. A pretty old college textbook on chemistry, *Chemistry* by Mitchell J. Sienko and Robert A. Plane, Second Edition, (McGraw-Hill, 1961) was helpful in formulating these principles.

15. Stephen Toulmin cited this example on p. 126 of An *Introduction of Reasoning.*

We decided simply to analyze and comment on his example.

16. As grounds, Toulmin mentions the following: In the *tetrahedron*, the faces joining at each vertex are three equilateral triangles, with angles totalling 3x60 degrees=180 degrees; in the (eight-faced) *octahedron*, 4 equilateral triangles, totalling 4x60 degrees=240 degrees: in the (20-faced) *eicosahedron*, 5, totalling 5x60 degrees =300 degrees. In the *cube*, they are three squares, with angles totalling 3x90 degrees=270 degrees, and in the (12-faced) *dodecahedron*, they are three pentagons 3x108 degrees= 324 degrees. No other set of equal angles at the vertex of a solid adds up to less than 360 degrees. (p. 126).