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II

ANOTHER WINDOW INTO ZENO'S ANTINOMIES

Joseph W. Smith, in an astute article in the *Indian Philosophical Quarterly,* January 1990, discussed the important question whether there are really perennial philosophical disputes. As an example of his perspective on that question, he probed the range of comments which have been offered, over the centuries, in regard to Zeno's paradoxes.

I should like to present, in this paper, a way of viewing Zeno's paradoxes which, although it is not entirely new, is, I think, worth considering now, since it has not hitherto been unfolded explicitly, with an adequate exposition of its pregnant ramifications.

1. I shall try to show, in this paper, (a) that Zeno's most famous conclusions which defy common sense are based, or (in order to be valid) would *have to be based,* on a certain presupposition, which I formulate in paragraph 2 below, and (b) that that presupposition is unacceptable. If I succeed in these efforts, it will follow that Zeno's arguments (purporting to prove that motion is impossible and even that a plurality of existents is impossible) *are invalid.*

2. Zeno's arguments are valid only if the following presupposition is correct:

> If you multiply an infinitely large number by an infinitely small number (that is, by an infinitely small fraction), the product of the two will be (indifferently) either infinitely large (because you have done an infinite number of aggregations) or infinitely small (because no matter how often you multiply an infinitesimal, you will never get anything but an infinitesimal result).

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3. Our intuitions, I think, lean easily toward accepting the first possible result (that the product will be infinitely large), and less easily toward the second possible result (that the product will be infinitesimal and therefore insignificant). But that is because, over the course of intellectual history, we have in general talked more about infinity than about the infinitesimal. (This does not apply, however, to mathematicians who have dealt with the infinitesimal calculus.)

4. Something close to my contention—that, in Zeno's view the product which results from multiplying infinity by the infinitesimal is either infinity or the infinitesimal—is found in the following passage in Kathleen Freeman's *The Pre-Socratic Philosophers; A Companion to Diels, Fragmente der Vorsokratiker* (Cambridge, Mass.: Harvard University Press, 1946), p. 156, where she is paraphrasing Zeno:

> "... you are bound to start with either a Nothing or an Infinite, and by ... [means of multiplication] you get only what you start with, either a Nothing or an Infinite."

5. Bearing in mind Freeman's paraphrase, I now want to show that Zeno's arguments *depend* on the presupposition expressed in paragraph 2 above. (Later, I will try to show that that presupposition should be replaced by a better one.)

6. Take, for example, the arguments against the reality of motion and, in particular, the strongest such argument. Accordintg to that argument,

> 6a. If you want to move from A to B (which is, let us say, a distance of forty feet), you must first accomplish the task of moving *one-half* the distance from A to B (that is, you must succeed in moving twenty feet).
>
> 6b. But in order to move the just-mentioned distance of twenty feet, you must first accomplish the task of moving one-half of *that* distance (namely, ten feet).
>
> 6c. Similarly, in order to move the just-mentioned distance of ten feet, you must accomplish the task of moving one-half of *that* distance (namely, five feet).
>
> 6d. And so forth. That is carrying this senquence of requirements to its logical conclusion, you must carry out *an infinite number* of tasks in order to move from A to B.

6e. But you can never complete an infinite number of tasks.

6 f. Therefore, motion is impossible.

7. But what does the infinite number of tasks amount to? It amounts to your multiplying the following two items together: (a) an infinite number of acts of motion and (b) an infinitesimal distance. (An infinitesimal distance is what you will get down to in the smaller and smaller required distances which you must succeed in travelling.)

8. Actually, instead of saying that you can never complete an infinite number of tasks (which Zeno could have difficulty in showing), he would have been on better ground if he had said:

> Even if you succeeded in completing an infinite number of motions, the distances which you would have succeeded in thus traversing are all *infinitesimal* and their sum is infinitesimal too, because no matter how many times you multiply an infinitesimal distance, you will get only an infinitesimal (insignificant) result.

Another way of observing that the infinitesimal is *insignificant* is to think of it as the *limit* (a "point" having no magnitude at all) of the process of constantly reducing a distance.

9. Thus, in this case, the product of multiplying infinity by the infinitesimal would be taken by Zeno to be infinitesimal .

10. I now need to show that sometimes Zeno tskes the producr of multiplying infinity by the infinitesimal to be *infinity*, in accordance with the first possibility which is listed in the hypothesis set forth in paragraph 2.

11. I shall show this by quoting a passage from Simplicius's commentary on Aristotle's *Physics*, where Simplicius says that Zeno argued as follows *against a plurality of existents;*

> "[If things are a many, and are divisible into parts,] each ... [part] must have a certain magnitude and a certain thickness, ... and the same may be said of what is in front of it; ... and something will [always] be in front of it ... [ad infinitum]." Translated by John Burnet in *Early Greek Philosophy* (London: A. & C. Black, fourth edition), p. 315.

In Simplicius's summary, what Zeno said is that a plurality of divisible existents requires an *infinity* of existents (which he deemed unacceptable, since there would be no room for an infinity of existents).

12. In the preceding few paragraphs, I have illustrated my contention that (a) Zeno sometimes held that infinity multiplied by the infinitesimal equals the infinitesimal and (b) he sometimes held that infinity multiplied by the infinitesimal equals the infinity. Now I wish to show that, in at least one place, Zeno mentioned both possibilities. He wrote:

> "so, if things are a many, they must be both small and great, so small as not to have any magnitude at all, and so great as to be infinite" Burnet, *op cit p.* 316.

13. I shall now offer an altenative to the assumption set forth in paragraph 2. I suggest that, if the infinite is multiplied by the infinitesimal, the product will be *a finite number*. Freeman came close to saying this when she wrote: "Any finite length is infinitely divisible, but it does not cease to be finite" (Freeman, *op cit,* p.161).

14. If the thought that "infinity times the infinitesimal equals a finite number" is acceptable, then Zeno's arguments fail, since they are based on the notion thar infinity times the infinitesimal equals *either* infinity *or* the infinitesimal.

15. A possible objection to my proposal that infinity times the infinitesimal equals the finite was voiced by A. J. Ayer in *The Central Questions of Philosophy* (1973; Pelican edition, 1976), p. 20, where he said:

> "It is no good our knowing that the sum of an infinite series can be finite if we are unable to explain how the series can never come into existence."

16. Ayer seems to want an explanation of how a series of infinitesimals "can ever come into existence." My comment on such a requirement is as follows: When mathematicians says things like "You

can't divide by zero" or "Any number multiplied by infinity will give you infinity," they are offering *proposed ideas* which they believe will be fruitful in as many contexts as possible. They are not offering propositions which are subject to the kind of explanation that Ayer seems to want. And my *proposed idea* that infinity times the infinitesimal equals the finite will, I think, be fruitful in many contexts. For example, it shows that Zeno's "proof" that motion is impossible does not work.

3077 Chestnut St., N. W.
Washington, D. C. 20015
(U.S.A.)

WILLIAM GERBER