Indian Philosophical Quarterly, VOL. XIX No.1
January, 1992

ANALYSIS AND SYNTHESIS IN THE GEOMETRY OF LOGIC

"Human reason is by nature architectonic."
(I. Kant, *Critique of Pure Reason*, p. B502)

The words "analysis" and "synthesis" are among the most widely used *and* misused terms in the history of philosophy. They were originally used in geometrical reasoning during the age of Euclid to describe two opposing, but complementary, methods of arguing (roughly equivalent to deduction and induction). Since then philosophers have used them not only in this way, but also to refer to distinctions of various sorts between types of judgement or classes of propositions. To some they are regarded as defining differences of kind, while others regard them as defining differences of degree. Moreover, they have been connected in numerous different ways with other distinctions, such as "a priori vs. a posteriori" or "necessary vs. contingent". Some philosophers have become so frustrated at the ambiguity attached to the various uses of the terms "analytic" and "synthetic" that they have given up all hope of assigning a coherent meaning to this distinction.

In this paper I have no intention of reviewing or attempting to clarify all of these ambiguities. Nor do I intend to say much about the various traditional ways of construing the meanings of the terms "analysis" and "synthesis".[1] Instead, I will introduce another way in which the distinction can be applied. My hope is that, rather than adding yet further ambiguity, my treatment will provide a well-grounded handle for grasping the other legitimate ways of using these terms, by exploring the connection between these opposing terms and some fundamental logical flaws. In this process I will call attention to a set of fixed logical patterns which correspond not only to the nature of analysis and synthesis, but also to the logical structure of certain simple geometrical figures. The method I will use in this article of "mapping" logical relations on to geometrical figures with the same structure, which I have developed in more detail elsewhere, I refer to as "The Geometry of Logic"[2]. By returning these terms to their geometrical source, but in a

Received : 22/8/1990

new light, I hope not only to breathe new life into an old distinction, but also to encourage philosophers to take advantage of the logical patterns which are built into human reasoning.

A good example of a philosopher who recognized the importance of the structural patterns implied by analysis and synthesis is Immanuel Kant. Unfortunately, Kant's stress on what he calls the "architectonic" character of reason is often either ignored or thoughtlessly rejected by his critics and commentators, who assume that his architectonic plan only distorts the clarity of the arguments he develops in his critical works. The distortion is thought to be caused by Kant's supposed insistence upon forcing his subject-matter to fit certain preconceived patterns. Thus, it is believed that the true extent of the validity of Kant's arguments can be determined only by freeing them from these plastic patterns.

What this common approach ignores, however, is that Kant's use of an architectonic plan is an inseparable aspect of his a priori approach. In other words, Kant's assertion that there are certain necessary conditions for the possibility of any human experience just *is* the claim that human reason operates in accordance with a fixed *order*. If *reason* fixes this order for us, then philosophers ought to do their best to understand and follow this order whenever they adopt any a priori persepctive in their philosophizing. The major problem in interpreting Kant is, I believe, not that his love of architectonic distorts his vision, but rather that he never explains in clear and unambiguous terms just what this plan *is*. If, instead of undermining the basis of the a priori approach at the outset, interpreters concentrated on gaining a deeper understanding of the rational structure of reason's architectonic plan— in other words, on making explicit what for Kant was always only implicit— then I believe they would stand a far greater chance of understanding what he, and other philosophers like him, were really trying to say.

The present study, however, has nothing directly to do with Kant.[3] Rather, it will be devoted to the task of examining the way in which the architectonic structure of human reason can be seen to arise directly out of certain fundamental (fixed) "laws of thought".

Without a doubt, the most influential of all suggested "laws of thought" down through the history of philosophy has been Aristotle's famous "law of noncontradiction".[4] This law can be stated in a wide variety of different ways, but perhaps its most common *informal* version is: "A thing cannot both be and not be in the same respect at the same time". The *formal* versions of the same law are now normally expressed in propositional terms, such as $P \longrightarrow \neg\ (-P)$. (That is, if a proposition is true, then it cannot be the case that its negation is also true.) An even simpler version, however, can be derived by regarding the law as referring to the bare relation between two *concepts* — in particular, between a concept (A) and its opposite (—A). Aristotle's fundamental law of all consistent thinking can then be expressed $A \neq -A$, or (informally) "Black is not not black", "I am not not I', etc. The simplest of all versions of this law would be one which abstracts the furthest from all content (i.e. from the "formal content" implied in any use of *P* to stand for any *proposition* or *A* to stand for any *concept*). This can be done by merely removing the "A" from both sides of the equation, thus leaving a kind of bare mathematical relation: $+ \neq -$ (positivity is not negativity). Hereafter, whenever I refer to the "law of noncontradiction", it is this latter version of the law which I will have in mind.

The law of noncontradiction is the absolute formal basis of all analytic reasoning. "Analysis" in this sense essentially refers to the process of dividing a thing, or concept, or proposition into two opposite parts. For example, the twenty-four hour period of time we call "one day" can be divided into "daytime" and "not daytime" (or "nighttime"). Of course, most analysis, both in philosophical and in ordinary discourse, does much more than simply divide one idea into two subordiante categories. For instance, there are certain times during the day when we hesitate to say whether it is "daytime" or "nighttime"; and as a result, a further level of analytic division is made between "dusk" and "dawn". Analyses of this type can and do go on to form quite complex patterns of relations between groups of concepts.

An interesting and largely unexploited parallelism exists between formal logic and the structure of certain simple geometrical figures. Taking advantage of this parallelism by using such figures as logical "maps" can help us to gain a clear understanding of some of the more intricate patterns of what I shall call "analytic relations".[5] The simplest, twofold analytic relation, as defined by the law of noncontradiction, can best be mapped onto the two (equal and opposite) endpoints of a line segment, as in Figure 1. This is what I will call the standard map of a "first-level analytic relation"(1LAR).

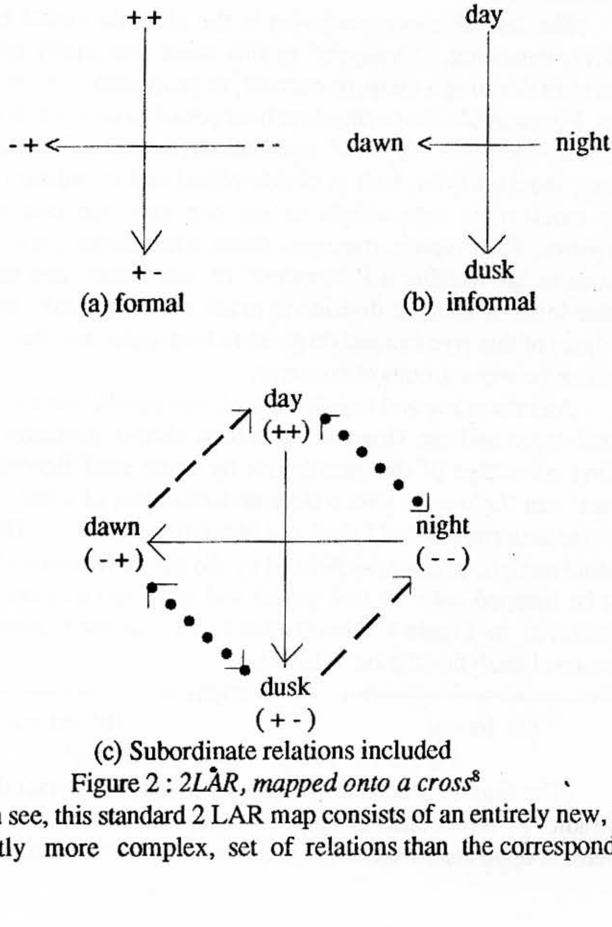
— <——————————> + night <——————————> day

(a) formal (b) informal

Figure 1 : *The standard 1 LAR map*[6]

(The double-headed arrow should be taken to depict the fact that contradictory terms, such as + and —, repel each other : they point us, as it were, in opposite directions.) On its own, of course, such a simple map is of little use, since it puts in a simple form something which is already simple enough. However, the same does not hold true when we apply the same principles to higher levels of analytic relations.

For example, the second-level analytic relation (2LAR) can be derived by performing a first-level analytic division on each side of a simple, 1LAR — — i. e. by splitting both the + and the — into opposites by adding another + or — to each. This gives rise to four logical combinations of terms (hereafter called "components") :[7] - -, - +, + -, + +. The new level can be mapped by adding a line segment perpendicular to the one which appears in Figure 1. Accordingly, in Figure 2 the first term in each component represents the line segment itself in its relation to the other line segment, whereas the second term represents the opposition between the two endpoints of the line segment in question.



(a) formal (b) informal

(c) Subordinate relations included

Figure 2 : *2LAR, mapped onto a cross*[8]

As we can see, this standard 2 LAR map consists of an entirely new, and significantly more complex, set of relations than the corresponding

1 LAR map. Each higher- level relation will include two components whose terms are identical. These "pure" components (+ + and - - in 2 LAR) will always play an important part as the "primary" or "root" components out of which the other, "impure" components (+ - amd - + in 2 LAR) arise. Therefore, the direction of flow on the main axes of the standard maps of higher-level analytic relations (i.e., 2LAR and up) will always go from the pure to the impure combinations, regardless of their positivity or negativity. The axes of the cross represent the two dominant relations in the standard 2LAR map. Subordinate relations can be included by connecting the - + and + - poles and the - + and + + poles with diagonal arrows (see the dashed lines in Figure 2 (c)). (Note that these subordinate relations, being the opposites of the dominant relations, flow from the impure to the pure combinations.) The only relations in a 2 LAR which are absolutely contradictory (i.e., the only ones in which the two components being related do not share one common term) are the relations between the + + and - - poles and between the + - and - + poles. These pairs of components repel each other in a way which none of the other pairs do (hence the double-headed arrows). The reason is that they represent not second-level relations as such, but the two first-level relations out of which the 2LAR is derived.

The secondary or derivative nature of the impure components can now be made clear by including diagonal lines (dotted in Figure 2 (c) between these two sets of contradictory poles (i.e., between the poles corresponding to the two pure components and those corresponding to the two impure components). If we follow the 2 LAR arrows in Figure 2 (c), we can see the natural *cycle* which we all actually experience each day: day (+ +) points towards dusk (+ -), which in turn, drives us back into night (- -); night points to dawn (- +), which in turn ushers in a new day (+ +) ; and the cycle begins again.

Not all 2LARs follow this standard cycle so perfectly. This is not the place to examine the many variations which can arise. But one variation is worth mentioning here. Analytic relations from the first to the sixth level play a crucial role in determining the structure of the ancient Chinese book of wisdom, the *I Ching* (or *Book of Changes*). This book consists of sixty- four "hexagrams" (i.e., six-term components) which come together to form a perfect sixth-level analytic relation (6LAR) . Each hexagram represents a certain situation, and by casting yarrow sticks (or flipping coins) two of the sixty-four components are chosen to represent a person's current situation. The *change* from one component to the other (i.e., the logical *relation* between them) is then used as a symbol to help a person understand the potential benefits and dangers of the situation in question. The point of interest for our purposes is that whenever the hexagrams are grouped into sets of four and mapped onto a cross (for 2LAR), or grouped into sets of eight and mapped onto a double cross (for 3LAR), the contradictory pairs of components are always placed at opposite ends of the same axis of the cross. In other words, the two relations which I have depicted above as the first-level roots of the properly second-level relations are treated as themselves being the most significant relations, as in Figure 3.

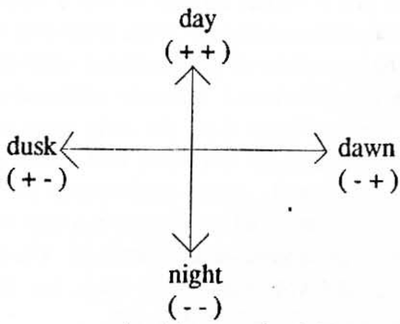


Figure 3 : *The non-standard (or standard Chinese) 2 LAR map*

Without going into detail to account for the differences between this form of 2 LAR and the form I have chosen as standard, it will suffice to say that the Chinese map emphasizes the (apparently) unresolvable tensions (which real life often presents to us as tasks to be resolved), while the standard map in Figure 2 explicates and emphasizes the polarities (i.e., pairs of components with one common term) which make it logically possible to resolve such fundamental tensions.

Regardless of the way in which the relations are mapped, one good way to explain or understand any 2LAR is to see it as arising whenever two sets of yes-or-no questions are asked side by side. For example, if we want to know what the weather is like outside, we might ask " Is it raining ?" and "Is the sun shining ?" , to which there will be four possible answers (i.e., four components):

+ + Yes it is raining, and yes the sun is shining.

+ - Yes it is raining, but no the sun is not shining.

- + No it is not raining, but yes the sun is shining.

- - No it is not raining, and no the sun is not shining.

If all the components in such an analytic relation represent real possibilities, then I call such a relation "perfect". If one or more components describes an impossible situation, the resulting relation is called "imperfect". (The above example is a perfect 2LAR, because all four possibilities sometimes occur in the real world: + + in this example is uncommon, but it occurs whenever a rainbow appears.)

Because any true 2LAR can be regarded as arising out of two sets of yes-or-no questions, any time we map four concepts which we believe are related together in this way, we should be able to specify their logical relationships by defining them in terms of two sets of contrasting words (i.e., words contrasted with their negations). Thus, all possible weather situations implied by the two questions given above can be depicted in terms of the simple 2LAR map in Figure 4.

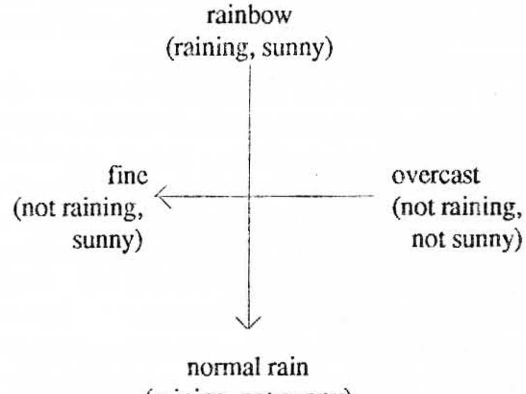


Figure 4 : *The weather, mapped onto a 2LAR cross*

Indeed, one of the greatest benefits of mapping analytic relations in this way is that it forces us to uncover the logical patterns which actually underlie the relationships we *thought* we understood between familiar concepts. Sometimes, when we attempt to do this, we find that the concepts we thought were related are not in fact related, or not related as simply as we had believed.

The process of twofold division can be carried on indefinitely to higher and higher levels of analytic relations, each of which can be represented by components which combine + 's and -'s in more and more complex ways, which in turn can be mapped onto more and more intricate geometrical figures. The formal apparatus adopted above can be used to express such higher levels by repeatedly dividing each of the previous level's components into two opposite parts by labelling each component with an additional + or -. Accordingly, the formula for deriving the number of terms and components for any level of analytic relation is quite simple:

$C = 2^t$,

where "C" refers to the total number of possible *components* ; and "t" refers to the total number of *terms* in each component.

(Note that "t" is also identical to the *level* of analytic relation under consideration.)

Thus, for example, the *I Ching* contains *64* different components because it is based on a sixth-level analytic relation (2^6=64). But of all types of analytic relation, 2LAR is probably the most important, and certainly the most frequently occurring in philosophical arguments. (It is, for example, the only type of analytic relation needed to understand the logical structure of Kant's table of twelve categories.) Therefore, it will not be necessary to discuss the higher levels here.

Instead, we can now turn our attention to another, quite different type of logical relation. Analytic relations are based, as I have explained, on Aristotle's law of noncontradiction. Indeed, the whole system of classical logic (and much of modern logic as well) is based on this most abstract of all analytic laws. (For this reason, I will refer to conventional logic as "analytic logic".) Many, perhaps most, philosophers assume a kind of absolute validity for this law: "Nobody will disagree that A and non—A are simultaneously impossible".[9] However, not all thinkers — not even all philosophers — have agreed with this consensus (Aristotelian) view that words can carry a meaning only if they are noncontradictory. Heraclitus, for example, takes as a first principle of his philosophy the notion that "Opposites are the same".[10] And Hegel, of course, raises this principle of synthesis to the status of a universal law, capable of describing the movement of all historical consciousness.[11] Moreover, the self-contradictory statements which can be found in the Scriptures of every major world religion are taken by many people to have a very profound meaning.[12]

How can any statement be meaningful if it breaks the fundamental law of thought? The full answer to this question would require an entire treatise on metaphysics. For our present purposes a rather dogmatic answer must suffice. The law of noncontradiction should be regarded not as the fundamental law of *all* thought, but rather, as laying the foundation for one *type* of thought, namely, the type which is the subject-matter of *analytic* logic. *If* contradictory expressions can in fact carry some meaning, then they must do so by depending on some other type of logic. I will refer to this "logic of paradox" as *synthetic* logic.

Since contradictions are not forbidden in synthetic logic, there is actually no way to talk coherently about it, except in terms of its analytic (noncontradictory) counterpart. Trying to describe synthetic logic on its own terms would give rise to a totally unintelligible bundle of contradictions which would not be anchored in anything concrete. The best way to *talk* about synthetic logic is to regard it as the *opposite* of analytic logic. So if analytic logic is based on the law of *non*contradiction ($+ \neq -$), then synthetic logic must be based on the law of *contradiction*, which we can express as $+ = -$. (This new law defines what it means for two words or propositions to stand in a contradictory relation to each other.) Likewise, just as the analytic law of noncontradiction is closely related to the law of *identity*, $+ = +$, so also the synthetic law of contradiction is closely related to the law of *non*-identity, $+ \neq +$. These four laws actually form a perfect 2LAR, so they can be mapped onto a cross, as in Figure 5.

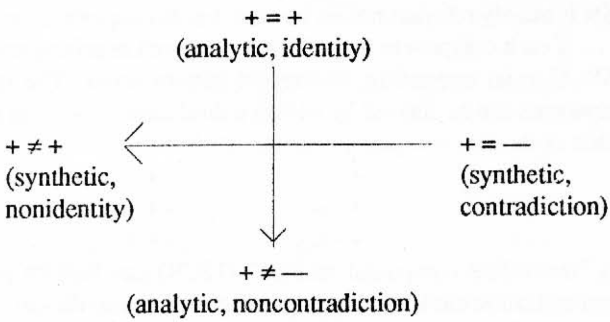


Figure 5 : *Four basic laws of logic, mapped onto a 2LAR cross*. The two questions which give rise to this 2LAR are : (1) Is the law analytic (rather than synthetic)? and (2) Does the law define the nature of (non) identity (rather than the nature of (non)contradiction)?

Because synthetic relations do not obey the laws of analytic logic, they cannot simply be mapped straightforwardly onto line segments, the way analytic relations can be. Moreover, synthesis is not a process of division, or opposition at all, but a process of *combination* or *integration*. Consequently, it takes a minimum of *three* terms to describe a synthetic relation. If we again use analytic logic as a basis, then this threefold process can be described as the process whereby a pair of opposites (+ and -) are regarded as the same — that is, the two are *synthesized* into one whole. If we choose the symbol "x" to represent this paradoxical third step, then we can conveniently map a first-level synthetic relation (1LSR) onto a *triangle*, as in Figure 6.

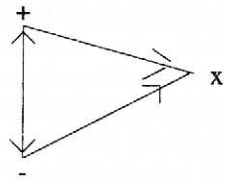


Figure 6: *1LSR, mapped onto a triangle*

Like analytic relations, synthetic relations can be combined with each other to produce higher levels of logical relations. For synthetic relations, the formula which determines the number of terms and components at each level is :

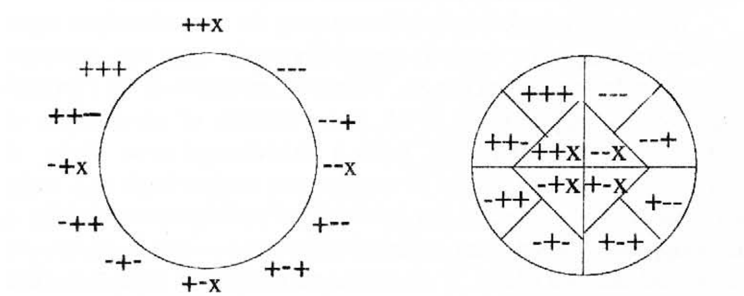
$$C = 3^t$$

However, for the limited purposes of this introductory sketch, a discussion of 1LSR will suffice. This is particularly true because, as we shall see, 1LSR is mainly relevant not on its own, but in conjunction with 2LAR.

If each component in the 2LAR is viewed as arising out of a prior 1LSR, then an interesting, twelvefold pattern arises. The twelve new components can be derived by adding a third term (-, +, x, in succession to each of the four components in the 2LAR.[13]

- - -	+ - -	- + -	+ + -
- - +	+ - +	- + +	+ + +
- - x	+ - x	- + x	+ + x

This "twelvefold compound relation" (12CR) can best be pictured by mapping it onto a circle. Accordingly, Figure 7(a) uses the circumference, and Figure 7(b) uses the area of a circle to map the structure of this important logical relation.[14]



(a) using the circumference (b) using the area of a circle

(Note that the *quadrants* now represent the original 2LAR. And in order to derive all twelve components from a set of questions, we would need to add a third, "yes, no, or yes-and-no", question to the two "yes or no" questions from which the 2LAR is derived.)

The explanatory power of this a… the other maps described above can hardly be underestimated, as lo… as they are not used haphazardly and without due attention to their logical structure (as is unfortunately the norm all too often on the rare occasions when scholars do resort to using maps). The 1ZCR pattern, for example, turns out to be precisely the pattern used by Kant in his Table of Categories. And light can be shed on the logical structure of many other systems, as diverse as the Western Zodiac and Jung's theory of psychological types, by using the same map as a guide. Such tasks, however, are beyond the bounds of my present goal (but see note 14), which has been to introduce the essential elements of the Geometry of Logic. Understanding the grounding of analysis and synthesis in the simplest of logical laws does not, of course, immediately dispel all the ambiguities concerning this distinction which have plagued philosophers over the years; however it should serve as a first step towards realizing the goal of presenting the distinction in a more coherent way, and in so doing, to encourage a proper appreciation of the architectonic structure of human thought.

Department of Religion and Philosophy **STEPHEN R.PALMQUIST**
Hong Kong Baptist College
224, Waterloo Road
Kowloon
HONG KONG

NOTES

1. I have discussed in detail the nature of the analytic-synthetic distinction and its connection to other related distinctions in: "Knowledge and Experience: An Examination of the Four Reflective'Perspectives' in Kant's Critical Philosophy", *Kant - Studien* 78.2 (1987), pp.174-183 ; "A Priori Knowledge in Persepctive: Mathematics, Method, and Pure Intuition",The Review of Metaphysics 41.1 (September 1987), pp.3-22; and "A Priori Knowledge in Perspective: Naming, Necessity, and the Analytic A Posteriori", *The Review of Metaphysics* 41.2 (December 1987), pp.255-282.

2. The implications of this method of "mapping" logical relations onto

geometrical figures with a corresponding structure are worked out in considerable detail, and applied to numerous patterns used in philosophy as well as a number of other disciplines, in *The Geometry of Logic* (hereafter, GL) (manuscript in preparation). An early attempt to apply this method to various patterns in Kant's philosophy can be found in "The Architectonic Form of Kant's Copernican Logic" (hereafter "AFKCL"), *Metaphilosophy* 17.4 (October 1986), pp.266-288.

3. I have applied the above principles of interpreting Kant in various articles and in my forthcoming book, *Kant's System of Perspectives*.

4. This law is usually called the "law of contradiction", presumably because it denies the possibility of two contradictory statements both being true. Calling it the "law of noncontradiction" has the advantage of describing the *positive* side of the law: it defines the basic requirement for consistency. Another important reason for choosing this name will become obvious shortly.

5. In "AFKCL" I use the term "division" instead of "relation", because that is the word Kant uses in his important footnote on p.197 of *Critique of Judgment*. However, in synthesis, as we shall see, terms are not being *divided*, but *combined*; therefore "relation" can serve as a more general term to refer to the formal structure of both analytic division and synthetic combination (or *integration*, as I call it in *GL*).

6. The relative position of the - and + on such maps is purely a matter of convention. I have followed the rule, whenever possible, of placing the + to the right or on top of the -, because of the associations most people have with "top" and "right" being stronger, or more positive, than "bottom" and "left". However, a different set of rules could just as easily be chosen, without affecting the essential nature or value of the maps, which are themselves merely tools for visualizing logical relations.

7. I will use "term" to denote any *single* occurrence of a + or -. The word "component" will refer to any *set* of (two or more) terms which is used to represent *one member* of a logical relation. In "AFKCL" the word "variable" was used instead of "component", and the word "expression" was suggested in a footnote as a possible alternative. But "variable" could be taken (incorrectly) to imply that the logical status of each set of terms can vary when in fact it is fixed. And "expression" could be taken (incorrectly) to imply that each set of terms expresses some fixed *meaning*, when in fact it is an entirely abstract representation of a *logical possibility*. Although "component" is sometimes rather awkward, I think it is preferable to

these alternatives.

8. In "AFKCL" 277 I map a 2LAR onto the cross by putting "+ -" at the three o'clock position, "- +" at six o'clock, and "- -" at nine o'clock. I have since adopted the convention of beginning at three o'clock with the negative ("-") and proceeding clockwise through "+ -" and "- +" to the positive ("+ +") at twelve o'clock.

9. V. Y. A. Perminov, " On the Nature of Logical Norms", *Philosophia Mathematica* II.3.i (1988), p.37.

10. Frank N. Magill (ed.), *Masterpieces of World Philosophy in Summary Form* (New York: Harper & Row, 1961), p.12-13.

11. Hegel rejects the absolute authority of the traditional "laws of thought (see e..g., G.W.F. Hegel, *Phenomenology of Spirit*, tr. A.V.Miller (Oxford: Oxford University Press, 1977), paragraphs 299-300), putting in their place his law of "thesis, antithesis and synthesis". As a result, many of his claims appear to be straightforward contradictions. For example, in paragraph 808 of *Phenomenology* he describes "History" as a "a *conscious, self-mediating* process — Spirit emptied into Time", a process in which "the negative is the negative of itself."

12. I am thinking here of statements such as the following: "the ten thousand things are one" and "right is not right" (Taoism); "He who sees inaction in action and action in inaction, he is wise" (Hinduism); "the phenomena of life are not real phenomena" (Buddhism); " I and the Father are one" and "you are all one in Christ Jesus" (Christianity); etc.

13. In "AFKCL 280 I presented this twelvefold pattern as arising out of a simple synthetic relation, described incorrectly as consisting of an eightfold pattern of three-term components (called "variables" in AFKCL"). That eightfold pattern, however, is actually best regarded as describing the form of a third-level *analytic* relation (3LAR). Nevertheless, the idea of deriving the twelvefold form of a compound relation from an 8+4 pattern was correct. The mistake I made in "AFKCL" was to regard the eight three-term components as synthetic simply because they each contain three terms. "AFKCL" does correctly explain that the synthetic (threefold) aspect comes from combining pairs of opposites in the eightfold pattern to produce the fourfold pattern. In other words, although the eight three-term components themselves are not synthetic, the (4+4)+4 pattern as a whole does contain four separate synthetic, (threefold) relations. I have now attempted to make this more clear by introducing "x" as the third term in each of the four components which arise in such twelvefold relations. In "AFKCL" I

simply left off the third term in the four components derived from the synthetic operation. Taken together, these two factors explain why Figure 5 in "AFKCL" 281 looks so different from Figure 6(a) below. The two diagrams are actually based on the same formal structure, but the order and clarity of the symbolism is greatly improved in the map presented here in Figure 6(a).

14.The interrelationships and sub-patterns within a 12CR are examined in detail in GL, ch.6. The ways in which this pattern can be seen operating in Kant's Critical System are explored in *Kant's System of Perspectives,* chs.7, 8, 11.