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UNDERSTANDING THE SYSTEM CONSTRUCTION IN LOGIC

The aim of this paper* is to understand the mechanism that lies behind the system construction in logic. In other words, it explains this mechanism by explaining the aim and method of logic. Further it reflects on the nature of system construction as a whole and suggests that the system construction in logic is no way different from that of theory construction in any branch of knowledge. This, in turn, shows that the nature of "Logical truths" and "truth" in different fields of sciences are not different in kind but on the contrary they are on the same par in this sense that the method of verification as well as their nature are same.

In constructing a system of logic, logicians are interested in characterizing a family of meta-logical notions such as logical truth (L-truth), logical falsity (L-falsity), Logical satisfiability (L-satisfiability), Logical unsatisfiability (L-unsatisfiability), Logical consistency (L-consistency), Logical inconsistency (L-inconsistency), Logical consequence (L-consequence) etc. and thereby are in a position to formulate a theory of demonstration. The problem of characterization of all the notions reduces to the characterization of any one of them. Because, all these notions are interlinked with each other in an interesting way. For example, we define all the above notions by L-truth as follows. A formula K is called L-inconsistent if and only if-K is L-true. A set S of formulas implies K (or K logically

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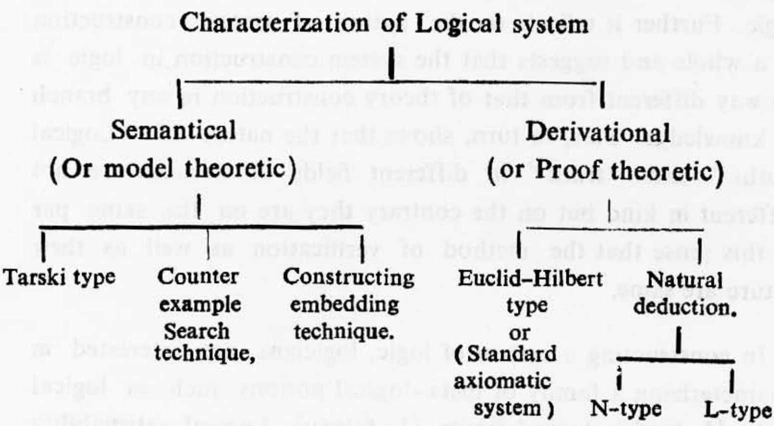
follows form S) if and only if S U { –K } is L–inconsistent. Similarly, K is called L–unsatisfiable if and only if – K is L–true and K is satisfiable if and only if K is not L–unsatisfiable. In brief, the same can be done by use of any other above notions chosen as our starting point. Thus, the characterization problem for a system of logic may be stated as a method by which the class of logical truths and valid inferences may be isolated or delineated. Historically we have different methods[1] of characterization which may be summarised in the following table –



Table–1

The characterization of any system of logic cannot be achieved without a precise theory of forms which can be thought as a bed–rock on which a system can be developed. It states how the artifical (or formal language) for a system of logic can be constructed. In otherwords, the aim of artifical language (say L) is to exhibit the class of all possible Logical form. The formulas in it are simply the logical x–rays of sentences of the corresponding ordinary language related to L. The process of formalization can be viewed as explicating the hidden forms or structures of sentences while suppressing its content. Of course, there is nothing as *the* Logical form because logical forms are relative to a given family of concepts. For example, the logical forms of propositional modal logic is different from that of classical propositional logic as the family of base concepts differ, namely the former has "negation", "conjunction", "disjunction", "implication" and "biconditional" as its base concepts while the latter has, in addition to all these, the notion of "necessity" and "possibilility". Thus, the theory of forms (or logical forms) presupposes a suitable family of concepts as its base. But the problem is how to choose such a family of concepts ? A reasonable solution would be that our choice of base concepts should be determined by our intention for the construction of the system in question. For example, Aristotle has suggested that such a family of concepts should be global in natute. Because, for him, logic plays an instrumental role in his theory of Science. A body of true sentences, in order to be called Science, should be deductively systematized i.e., starting with the first principles (i.e., Axioms), by use of rules, all and only true statements can be generated. Thus, it is highly important to know when and only when a given statement follows from the axioms or not. According to Aristotle logic aims at solving this problem. Since this is indispensable for any discipline, logic serves a global role. In other words, though logic is instrumental,[2] in its function yet in itself is global. Thus, Aristotle's suggestion is that a scientist should train himself in logic before he studies system of sciences "and not be enquiring into them while they are listening to lectures on it".[3] The theory of demonstration that it (i.e., logic) gives not only works in a particular field but also the same principles of demonstration will work in every field. Thus, Aristotle thinks as evident from his works that the family of base concepts for the subject matter of logic should be global (or logical) as distinguished from local (or non-logical) concepts. The following classification of concepts in this connection may be instructive.

Concepts

Logical (or Global) — Non-Logical (or local)

Logical (or Global): Indisputably Global — that is not so

Table 2

A concept is called *Logical (or global)* (Aristotle calls them " Koina archai ") if it plays a role in every scientific discipline, otherwise it may be called *Non-logical* (or Local). For Aristotle, the notion of " Subject," " predicate " and " the relation between them " as expressed in form of copula are logical. In our modern logic the concepts like " negation ", " conjunction ", " disjunction ", " implication ", " biconditional", " some", "all" etc. are global, while the concepts of " number ", " point ", "line", matter etc. are not global. There may be disputes among logicians with respect to the globality of a concept For example, a number of logicians including C. I. Lewis [4] are of the opinion that the modal concepts like " possibility ", " necessity ", " impossibility " etc. are logical in the sense that the notion " logically follows " (or logical implication) cannot be characterized faithfully without the modal notions. In brief, for defining " logical implication " we have to consider the modal notions in addition to our truth functional connectives; otherwise we will have the paradox of material implication. On the other hand, according to W. V. Quine they (modal concepts) are not global in the sense that for all scientific purposes alethic modalities (i. e. " possibility," " necessity " etc.) are dispensable. Hence, they are not logical concepts. Whatever may be the case, atleast indispensable role of base concepts is evidently clear.

Once we identify the suitable family of concepts as our base, we may then go on to define the artifical language in an usual way. Thus, the difference in base concepts results a difference in the class of logical forms (i. e. the class of formulas). After developing the theory of logical forms, logician proceeds to characterize the notion of L–truths i. e. he isolates the truths of logic. This can be either semantically or derivationally as shown in our table 1 given above. If one tries to characterize them semantically he should preferably characterize L–satisfiability which intuitively means true in some logically conceivable (or L–conceivable) situation. Then L–truth (or laws of logic) would be characterized as true in all logically conceivable situations. Similarly, in an usual way the notion of L–valid inferences can be defined. Thus, the entire burden of characterization of the meta–theory for a system of logic falls on the characterization of L–conceivable situations. Hence, for different logics we will have different characterization of L–conceivable situations. For example, the well–known notion of Boolean valuation of propositional logic serves as the formal counter–part of the informal notion of L–conceivable situation and for predicate logic, the notion of first order valuation serves the purpose. On the other hand, if one is interested in developing the system of logic in derivational manner, his course would be to define the notion of proof in the artificial language. This may be done in the way described in table 1. Once the notion of proof is formalized we say that a formula is a truth of Logic if and only if it is provable i. e. all provable formulas are truths of logic.

In general, a system of logic should be described as an ordered triple S_f, = (TLF_f. TLT_f, TL Inf_f) Where "S", "TLF",

" TLT " and " TL Inf " means respectively as the logical system, theory of logical forms, theory of logical truths and the theory of logically valid inferences. Finally, the subscript " f " indicates the chosen family of global concepts. If we would be little economical, we may reduce these above three co-ordinates to two by considering the laws of logic as a special kind of inference in which it follows from the empty set of premises. It may be noted that two systems may have the same theory of logical forms but they may have different sets of logical truths. For example, the classical logic and intuitionistic logic have the same theory of logical forms but their body of logical truths differ. This is because though logicians agree on the theory of logical forms yet they disagree about the meaning of the base concepts such as " negation " and " disjunction ".

So far we have explained what is a system of logic ? What does it aim at ? and how can it be constructed ? Now in the following paragraphs let us reflect philosophically about the system itself.

System construction in iogic, like the theory construction in any branch of knowledge, should be backed by necessary pre-theoretic analysis of the chosen family of concepts. Pre-theoretic analysis (or conceptual analysis) though indespensable would not by itself permit us to uncover various properties of the concepts which may lie at varying depths from the surface level of the discourse. Theoretically significant properties of concepts are not given to us like items in the sunlight. On the other hand, the field of our intuition is more often than not like a twilight zone. Our intuitions may be sharp concerning some surface level features of the concepts but they may very well fail us when we work at a relatively deeper level. For example, the relation between logical necessity and truth. What must our intuition dictate, will at least be represented by a law such as whatever is logically necessary is true i.e, "CLpp". But we are not so sure whether whatever is necessary is also necessarily necessary i.e. "CLpLLp". Thus, in constructing system of logic we are clearly modelling upon the pre-theoretic insights of a given family of concepts, their relations among themselves and so on.

This does not mean logicians are completely free in the sense that their modelling is whimsical. On the other hand, at each stage of system construction, the system builder's responsibility is to account for the data in question. In the present case, our data are the pre-theoretic insights. If the system does not explain the data or runs contrary to data, we should try to repair the system or if the system would be unrepairable, we should reject the system and try to construct a new one. For example, if one constructs a system of propositional modal logic by adding the formula CMpp (i.e. if p is possible then it is true (or actual) to the well known system T^4 then the new resulting system would not be inconsistent but would be counter-intuitive. In other words, in it, the intuitive distinction between actuality (i.e. truth) and "possibility" vanishes. Thus, we should at first try to repair the system if this is not possible; but if that is not possible, then the obvious course would be to reject it. Exactly this sort of situation has happened to Mally's system of deontic logic. This shows that the system of logic is not sacrosanct like the scientific theories

In this connection there is a close affinity between a system logic and theories of science. Thus, the nature of logical system and scientific theories is not diametrically opposite to each other as many philosophers of the past have thought. As for example, Carnap's work,[5] this fact is reflected in the form of a distinction between explication and explanation. He says that explication is closely related to the study of formal sciences, while "explanation", "prediction" etc. are related to empirical sciences. This dichotomy is pre-supposed by related dichotomy between analytic and synthetic statements. According to logical positivists in general and Carnap and Ayer in particular the statements of formal sciences viz. logic and mathematics are analytic, whereas the statements of empirical sciences are synthetic. Historically this dichotomy has come down to the logical positivists through the Hume's distinction between ideas of reason and ideas concerning matters of fact. Acceptance of this dichotomy poses a serious threat to the nature of formal sciences as it makes all formal sciences a closed and uninformative (in the sense of logical positivists) discipline. Thus, this distinction has recently come under attack by various philosophers.[6] Moreover, the method of verification of truths of natural sciences is no way different from the verification of truths of logic. Of course, this is more conspicuous when the system is developed semanticlly.[7] In this connection Hintikka writes, "For instance, natural laws are never established by first verifying all their instances. The logic of their verification involves an opposite process viz. the process of deducing simple observable consequences from the complex expression of the law in question. Somewhat in the same way, some of our most intuitive methods of dealing with logical formulas involve the transition from complex formulas to the simpler ones rather than a transition in opposite direction."[8] From this it is clear that the method of verification of natural laws and truths of logic is analoguous. In other words, the nature of truths of logic is not different from that of other sciences. Dagfinn Follesdal says in the context of epistemic logic that "...the truth of logic of knowledge need not be considered as differing radically in epistemic respects from, say, truth of physics"[9]

From the above considerations we may conclude that a system of logic should be viewed as a proposal (or a hypothesis) to explain and discover the hidden theoretically interesting properties concerning the chosen family of concepts. The adequacy of the system should be judged from its explanatory power in explaining the data in question. If the given system does not explain the data or is inadequate to do so, we should construct another system. If, on the other hand, the new one fails, we should construct still another and so on. For example, philosophically we should think that the classical propositional logic is a proposal to analyze our intuitive notion of " Logical consequence " or " Logical implication ". According to C. I. Lewis, it fails to achieve its goal because it contains the formulas " CpCqp " and " CNpCpq " as theorems which seem to be anti-intutive and hence paradoxical. Hence, Lewis rejects it and constructs the system of logic called " the system of strict implication " [10] that avoids the above paradox. But his system contains another kind of paradox called " the paradox of strict implication." Again Anderson and Belnap [12] reject Lewis' proposal in defining the notion of logical consequence by modal notion viz. " necessity " or " possibility " (of course in addition to the truth functional notions) and try to characterize it by use of the notion of relevance. Thus, they construct the system of Relevant logic. More examples of this type can also be available in the field of deontic [13] and epistemic logic. [14] Hence the moral we learn is that the theory construction (or system construction) in logic, like that of theory construction in Science, is an on-going process. Therefore, when we construct system of logic and prepare to modify it as demanded by the complexities of theoretical situations it is likely to prove more rewarding like theory construction in different branches of natural and social sciences.

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NOTES

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