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Discussion

A REFUTATION OF JAGAT PAL'S DEFENCE OF ARISTOTELIAN SQUARE OF OPPOSITION

There is a long-drawn-out controversy between the traditional (Aristotelian) and the modern logicians concerning one of the fundamental doctrines of syllogistic logic, namely, the doctrine of square of opposition. The dispute seems to revolve solely around the tricky notion of "existential import" of propositions. Jagat Pal (Hereafter referred as JP) in a recent paper[1] has attempted to uphold the traditional interpretation of categorical propositions and the various relations involved in the square of opposition. For this purpose he has argued extensively against the modern logicians. He tries to show that the modern interpretation of general propositions, far from refuting the traditional square of opposition, supports it.

The purpose of this paper, in the present context, is to examine the veracity of some of the claims made by JP against the modern interpretation and to see how far he has succeeded in refuting this thesis. We desire to concentrate our examination mainly on two points: (i) According to JP, A and E propositions can be true together as claimed by the modern logicians only if they assume the domain of discourse as empty. But he adds in the same breath that for the modern logicians to be fair and consistent with tneir thesis it is not possible to assume the domain of discourse to be empty, and (ii) JP contends that a

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propositional function, even after being quantified, turns out to be neither true nor false, whereas for the modern logicians a quantified well-formed-formula is either true or false. If we warily reflect on and meticulously scrutinize these two points we can show that the interpretation undertaken by JP for the cause of the Aristotelian logic is altogether improper and utterly misleading.

JP holds that the modern logicians should presuppose an empty domain of discourse in order to refute the traditional square. According to him, the well-formed-formulas $(x)(\emptyset x \supset \psi x)$ and $(x)(\emptyset x \supset \sim \psi x)$ can be true together only if x in $\emptyset x$ refers to nothing. But it can be possible only if the domain is empty. However, JP endeavours to show that the modern logicians cannot admit an empty domain because the truth conditions of the quantified formulas are specified in terms of the substitution instances of the propositional function which has been quantified and a substitution instance contains individual constants which refer to individuals in the domain. In a word, in JP's opinion, modern logic cannot admit empty domain without involving inconsistency. As a result, the traditional square stands unrefuted. We shall argue against JP to establish that for refuting the traditional square by way of showing that A and E can be true together an exponent of modern logic need not admit an empty domain of discourse.

Many logical thinkers right from J. S. Mill consider the traditional A and E propositions as conditional, rather than categorical, in nature. So the forms or the logical structures of these two propositions are represented in predicate logic by the following formulas.

Propositions	Logical Forms or Structures
A	$(x)(\emptyset x \supset \psi x)$
E	$(x)(\emptyset x \supset \sim \psi x)$

As a result, whenever general propositions like A and E are used they do not categorically assert any thing about any individual. On the contrary, they can be said to be uniquely specifying truth conditions of propositions.

JP rightly maintains (on p. 306) with the modern logicians that A and E propositions of the above form can be true together only if the antecedent, i.e., $\emptyset$x is false in both the cases irrespective of the truth values assigned to the consequent ψx as they are propositional functions of the forms $\emptyset x \supset \psi x$ and $\emptyset x \supset \sim \psi x$. But his interpretation in falsifying $\emptyset$x seems quite implausible. He claims that $\emptyset$x becomes false if and only if x in $\emptyset$x refers to nothing, i.e, to an empty class–a class devoid of any member. But we would like to submit, in this context, that this interpretation is not consistent with the modern logic in question. Or to put the matter differently, for the modern logicians it is not desirable to assume empty domain for explaining their position of the relation holding between A and E propositions. Since it appears that if $\emptyset$x does not have any true substitution instance, it is because of the fact that the predicate expression $\emptyset$ here refers to an empty class: thus whatever may be the referent of x, it fails to belong to the class ϕ (ϕ does not apply), and, accordingly becomes false. Professor I. M. Copi has rightly observed that, save contradiction, "...none of the relations in the square array...holds for the traditional A, E, I and O propositions, *even if we assume that there is at least one individual in the universe*" [2]. Thus it is evident that our interpretation of the falsity of (x) ϕx is in conformity with the modern approach.

It might, however, be argued against us that if a non–empty domain is presupposed then it would no longer be possible to deny the existential import of A and E propositions. We may dispense with this objection with reference to George Boole's treatment of the forms of general propositions. It is conspicous that George Boole interpreted general propositions in his class-theoretical terminology which represented a particular proposition as asserting the non-emptiness of the product of one class with another class or its complimentary and a universal proposition as asserting the emptiness of the same. Thus, A and E propositions such as "All S is P" and "No S is P" can be interpreted as $S\bar{P}=O$ and $SP=O$ respectively where the 'O' symbol stands for 'null-class'. Both the propositions thus represented are *categorical* about the emptiness of the *product class*, viz., $S\bar{P}$ and SP respectively. But what is striking to note is that they are *non-committal*[3] about the existence of members of the classes denoted by *S* and *P*. That is why the existential import is denied of A and E propositions in general. Or, to put the matter alternatively, that to deny the existential import it is not necessary to deny the existence of individuals in the domain but to be non-commital about their existence.

We may advance another argument to fortify our thesis that for refuting the traditional square a modern logician need not presuppose an empty domain. Consider, for example, the proposition $(x)\ (\phi x \supset \psi x)$. A modern logician would admit it to be perfectly possible that even if the propositional function ϕx does not have any true substitution instance, ψx can have true substitution instance. But this can never be possible if ϕx fails to have any true substitution instance only on the condition that the domain is empty. Or, in other words, if ψx has true substitution instance there must be some individual of which it is true. In that case if ϕx fails to have true substitution instance it cannot be due the absence of individual in the domain but it must be due to the inapplicability of the predicate expression. And thus our contention seems to be valid.

Another important point that demands to be highlighted in this context is that, contrary to JP's supposition that in an empty universe $(x)\ \phi x$ would be false, a good number of modern logicians would hold that it would come out true on this condition. W. v. Quine, among others, makes this point explicitly vivid. He maintains that "... '$(x)Fx$' is bound to come out true for the empty universe (there being no objects for 'Fx' to be false of) ...".[4] The same point has also been stressed by Patrick Suppes. In his opinion the well-formed-formula '$(x)\ \phi x\ \&\ \sim(\exists x)\ \phi x$' is inconsistent if the universe is assumed to be empty.[5] It is palpable to see that if the first conjunct of this formula comes out false in an empty universe (as has been assumed by JP) then the formula as a whole fails to be consistent even in empty universe, but if an empty universe makes $(x)\ \phi x$ true, then only the whole well-formed-formula can be consistent in that universe.

Thus it follows from the above that according to modern interpretation the well-formed-formula $(x)\ (\phi \supset \psi x)$ and $(x)\ (\phi x \supset \sim \psi x)$ would both be true if the extension of the predicate expression ϕ in the propositional function ϕx is empty. To be more precise, what a modern logician needs to presuppose for refuting the doctrine of traditional square is not an empty universe but an empty predicate (i.e., a predicate whose extension is an empty class). In fact, the controversy regarding the acceptability of the doctrine hinges on this issue. While the traditional logicians hold that every term in a categorical proposition stands for something which exists, the modern logicians claim that there *can be* terms designating empty classes. And on this point the stand taken by the moderns seems to be more reasonable than that of the Aristotelians. For, in the first place, the modern approach tallies with the semantic intuition of the speakers. Consider the speech act performed by a speaker in uttering the sentence. "All tresspassers will be prosecuted". Obviously, the speaker does not intend, in this particular context, his hearer to believe that there, in fact, are tresspassers; instead he is actually non-committal about the existence of tresspassers, but one can be non-committal about the existence of things only if one admits the possibility of both the existence and non-existence of the things in question : in admitting the possibility of having empty terms in general propositions modern logic thus reflects the application of the native speaker's intuition. Secondly, modern approach does not require us to impose unnecessary restriction on the scope of logic. Since the admission of the possibility of having empty general terms in general propositions is not inconsistent with the speaker's semantic intuitions, there is no reason as to why the scope of logic should be restricted only to such propositions consisting only of non-empty terms. To quote from Kneale and Kneale, "To modern logician this seems a rather curious restriction on the scope of logic. For, it is not at all difficult to construct a general statement in which at least one of the terms lacks application...".[6]

It is important to note that the modern analysis of general statements is not inconsistent even with that logico-semantic intuition of Aristotle which led him to admit that all general terms must have some application. On Aristotle's view a statement is true or false according as the predicate applies to the subject or not (as referred to by JP as the predicate criterion of falsity). Accordingly, this view requires that a subject term must refer to something about which the question of the application of predicate can arise at all. Now, if we shed some light on the modern analysis of the general statements we find that the general term, which appears as subject in Aristotelian statement, occupies the position of predicate expression in a statement of predicate logic, and, accordingly, even if these terms do not refer to any existent entity, the statement of predicate logic would not fail to be true or false. In the notation of predicate logic a variable stands for the subject of the general statements, and in so far as a modern logician assumes the existence of a non–empty domain of discourse, a variable always gets some or other object of domain as its value. So, we notice that the Aristotelian intuition that the subject term must have some reference is upheld by the modern logicians also. Consequently, it can ligitimately be claimed that modern analysis of general proposition is much more comprehensive than the Aristotelian approach.

It may, however, be urged that the Aristotelian logicians, talking about categorical propositions, not only deny the possibility of having empty terms in the position of subject, but they also deny this possibility in the case of predicate terms also, and accordingly it is not plausible to criticize this doctrine on the assumption that there can be empty predicate terms. Our submission on this point would be that the admission of the possibility of having empty predicates is consistent both with the semantic and logical requirements. We have already pointed out that there are cases in which semantic intuition of the speakers makes it plain that there are empty predicates. Now we are inclined to submit that even if we intend to confine ourselves to Aristotelian forms of propositions then also in some cases there would be a logical requirement for the admission of empty predicate. Consider, for instance, the statement : "No men are miser." A remarkable feature of this statement is that the predicate 'miser' here is a subset of the set designated by the subject, namely, 'man.' Now if we examine closely we find that this statement can be true. But the question arises : what constitutes the truth condition? The truth condition of this statement consists in that no member of the class denoted by the subject term should be a member of the class denoted by the predicate. But since the predicate class here is a subset of the subject class so whenever the statement is true, the class denoted by the predicate term must be empty. So an Aristotelian would have to admit either that this statement, viz, " No men are miser " is necessarily a false statement, or he would have to admit a paradoxical situation that if the statement is true then it is neither true nor false (due to the existence of the predicate that denotes empty class). But both of these consequences would be unwarranted. Hence, it can be claimed that for explaining the possibility of the truth of certain statements it is necessary to admit the existenee of empty predicates.

It may, however, be urged against our proposed defence that the well-formed-formulas of predicate logic that represent the forms of A and E propositions are neither true nor false, and, accordingly, the question of their being true together does not arise at all. This is in fact the other point in JP's thesis that we desire to contest. On JP's view, a propositional function is neither true nor false because the reference of the variables that occur in it is undetermined. He says, " since individual variables are undetermined expressions, $\phi x \supset \psi x$ and $\phi x \supset \sim \psi x$ are also undetermined expressions. As a result of it, they fall short of the categories of truth values " (p. 308). And he claims that even after quantifying the propositional function, the resulting well-formed-formula remains undetermined and so is neither true nor false. We agree with JP that both the propositional function and its variables are undetermined expressions, but in our opinion this in no way implies that the quantified well-formed-formula is also undetermined. The reason why a propositional function fails to have any truth value is simply that it is not possible to specify the truth conditions for a propositional function. It is true that a propositionol function the subject term should be a member of the class denoted by the predicate. But since the predicate class here is a subset of the subject class so whenever the statement is true, the class denoted by the predicate term must be empty. So an Aristotelian would have to admit either that this statement, viz, " No men are miser " is necessarily a false statement, or he would have to admit a paradoxical situation that if the statement is true then it is neither true nor false (due to the existence of the predicate that denotes empty class). But both of these consequences would be unwarranted. Hence, it can be claimed that for explaining the possibility of the truth of certain statements it is necessary to admit the existenee of empty predicates.

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NOTES

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1. Pal, Jagat, "Modern Analysis of Syllogistic Logic : A Critical Reflection", *Indian Philosophical Querterly*, Vol. XVI, No. 3, July 1989, pp. 303-317.
2. Copi, I. M.. *Symbolic Logic*, 5th Edn. (New York : Macmillan Publishing Co. Inc., 1979). p. 68. Italics ours.
3. This point has also been emphasized by W. v. Quine with reference to diagrammatic representation of categorical statements in whieh overlapping circles are used to represent the two terms of a categorical statement. The significance of shading of a particular region positively asserts *emptiness* whereas whiteness of a region signifies *lack of information*. Thus, in A and E propositions emptiness of the product classes S*P* and SP respectively is unequivocal but indicate mere lack of information about other unshaded regions. Please see W. v. Quine, *Methods of Logic*, 2nd Edn. (London ; Rutledge & Kegan Paul, 1962), pp. 69-70.
4. Quine, W. v., *op. cit.*, p. 96.
5. Suppes, Patrick ; *Introduction to Logic*, (New Delhi : Affiliated East-West Press Private Limited. 1957), p. 68.
6. Kneale, W. and Kneale, M. *The Development of Logic* (Oxford : Clarendon Press, 1962), p. 60.

DR. S. R. KAWLE

We are profoundly sorry to report the sad and untimaly demise of Dr. S. R, Kawale (13-9-1930 — 31-1-1990), Professor and Head of the Department of Philosophy, S. P. College, Pune-30. Apart from being a conscientious, dedicated and well-known teacher of Philosophy for the last thirty-four years, he was a humane administrator and above all an understanding and co-operative esteemed senior colleague. He was associated with Philosophy Department of Poona University in number of ways, and participated in various activities undertaken by it with utmost sincerity. Not only was he associated with the *Indian Philosophical Quarterly* since its inception but was also a member of its Board of Consulting Editors from volume VI The Quarterly was privileged to be profited by his timely suitable advice and encouraging counsel. In his passing away we have lost a senior respected colleague and the Quarterly has lost a well-wisher.

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