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WHITEHEAD'S METHOD OF EXTENSIVE ABSTRACTION

In ordinary parlance as well as in mathematics we have both intervals of space and (magnitudeless) points such that the extremities of some intervals, viz., the line-segments, are points. Now, there are three *prima facie* possibilities with regard to the question of primacy between intervals and points: (i) that the points are given and the intervals arise from them in some way (such as by summation or fluxion); (ii) that the intervals are given and the points arise from them in some way (such as by division or abstraction); and (iii) that both intervals and points are given independently of each other though certain relations hold between them as of necessity.

Alfred North Whitehead (1861–1947) has been much concerned with this question, but he has not actually stated in so many words as to why he had adopted one of these views and rejected the others. However, from the hints he drops here and there, we may say that his reasons must have been somewhat as follows.

A little consideration suffices to show that the third view does not really present a viable alternative. If a given relation holds between two terms as a matter of empirical fact, then it may be that the two terms are given independently of one another and of the relation that subsists between them. But if a relation is supposed to hold between two terms as of necessity and the relata are supposed to be indefinables or to be definable independently of each other and of the relation that subsists between

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them as of necessity, then such a relationship can be asserted only in the form of a synthetic a priori proposition and only on the basis of intellectual intuition. Intuitionism has been a powerful movement in philosophy and mathematics, but, it seems that Kant was the last among notable philosophers to subscribe to such a view. The "synthetic *a priori* proposition" can now hardly be regarded as a proposition, and "intellectual intuition" can hardly be regarded as a source of knowledge. In any case, the third view not only evokes Ockham's Razor (if one of the other two views be substantiable) but actually involves a circularity as any attempt at directional analysis of the deductive system we call geometry will show. The second seems to have been the most widespread view among the Greeks, some taking an interval as constituted of a finitude of "extended" points, some regarding it as constituted of an infinitude of magnitudeless elements, and some as arising from the fluxion or motion of a point. The discovery of the incommensurability of the diagonal of a square with its side nipped the possibility of extended elements in its bud. The view that an interval consists of an infinitude of elements gave rise to a dilemma : if the elements have magnitude, no matter how small, then the interval must be infinite in magnitude; if the elements have no magnitude, then the interval has no magnitude either. The first horn of the dilemma, if all the elements have the same magnitude, has never been controverted. The second horn also remained unanswered until the 19th century when mathematicians developed the theory of transfinite numbers and it became possible to see how magnitudeless elements could give rise to an interval of positive magnitude. However, one should not accept as a datum something which was not among the primary data of sense perception. Four-dimensional spatio-temporal events are the ingredients of which the universe consists and which are our primary perceptual data. Hence, one had to arrive at the point from the four–

dimensional continuum of sense perception by some rational process of abstraction. The view that an interval arose by fluxion has the same drawback of taking something as a datum which is not among the primary data of sense perception. The possibility that the interval was given and the point arose from it by continued division had long ago been disproved by Zeno. If you divide a line–segment into two halves and then each of the halves in their turn into two halves and so on, then you never can arrive at the point. That the point arises from the interval by abstraction must be the true view, but while the Greek genius had intuitively jumped at the notion of a point, no one had ever actually abstracted the point from the interval—it had simply been assumed that the process of simplification led to the point.

It seems to me that in view of some such considerations, Whitehead put himself to the task of abstracting the point from the four dimensional spatio–temporal event or region with the help of a few indefinable notions embeded in sense perception and a number of reasonable axioms, and thus of endeavouring to establish geometry on a more secure epistemological foundation than had Euclid or even modern mathematicians.

I

Whitehead has presented his method of extensive abstraction in four of his works :[1]

1. "La théorie relationiste de l'Espace", *Revue de metaphysique et de morale,* XXIII (1916), pp. 423–54.
2. *An Enquiry Concerning the Principles of Natural Knowledge,* Cambridge, 1919, Part 3.
3. *The Concept of Nature,* Cambridge, 1920, Chapter 4.
4. *Process and Reality,* New York, 1929, Part 4.

The method presented in all these works is essentially the same although there are some differences among them in matters of detail. It would in itself be of interest to study these differences and to trace the evolution of Whitehead's thought on and technique for extensive abstraction from his *Organisation of Thought* published in 1917 to the *Process and Reality* published in 1929. Some scholars have discussed some of these differences, but, I am not aware of any detailed study of those differences. However, in this article we propose to study the essential elements of Whitehead's method, since our main purpose here is to evaluate it, and shall therefore confine ourselves to only one of the four works, *Process and Reality*, which is his *magnum opus* and contains his most mature attempt at extensive abstraction.

Whitehead takes ***region*** and ***extensive connection*** as indefinable terms and explains his usage concerning these two terms from which we learn that the former is at least a four–dimensional continuum and the latter means any kind of relation that any two regions can have to one another. He first defines the concepts of inclusion or whole–part relationship, overlapping, dissection of a region (i. e., a set of mutually exclusive and collectively exhaustive parts), intersect of two regions (i.e., a region in which two regions overlap), unique and multiple intersection of two regions (if there are two or more non–contiguous intersects of two regions then two regions have multiple intersection and if they have only one intersect then there is unique intersection), externally connected (i. e., contiguous), tangentially included (i. e., so contained that part shares in the 'surface' of the whole) and non–tangentially included (i. e., so contained in the interior that the part in question is completely surrounded by another part of the given region), and then introduces the notion of an abstractive set as a set of regions any two of which are such that one of them includes the other non–tangentially and none of which is included in every other member of the set. Thus, he presents the notion of convergence to a geometrical entity, point, line, surface and solid, *without* postulating any of these entities. That is, we begin with a region R of any size and then take as a member a region M which is non-tangentially included in the given region, i. e., a part of region R which is surrounded on all sides by another part of R having some thickness so that the upper surf ace of M is not connected with any region not included in R. By taking smaller and smaller such parts of R as the members of our set we obtain a set of regions none of which is the smallest member and the regions converge to a solid, surface, line and point. (Even in the case of converging to a region, that is, to a four-dimensional continuum, it is clear that such a region is not a member of the set but lies beyond 'all' the members of the set, just like the *omega-plus-one th* tmember of an infinite convergent series, the members of the set approaching it more and more closely as we move down the converging end of R.)

Whitehead introduces the notion of one abstractive set *covering* another abstractive set (i.e., that of every member of one set non-tangentially including some member of the other) and that of 'equivalence' of abstractive sets, or in ordinary parlance, the notion of *sameness* of convergence. A geometrical element is now defined as a complete group of equivalent abstractive sets, equivalent to one another and to no other abstractive set outside the group. Then the notion of one geometrical element being incident in another geometrical element is introduced : when every member (abstractive set) of a geometrical element a covers every member of another geometrical element b, but a and b are not identical then b is said to be incident in a (i.e., to be contained in a). And now we reach the 'point' as a geometrical element in which no other geometrical element is incident.

Whitehead points out that this definition is to be compared with the Euclidean definition of a point as that which has no part.

We may now introduce the notion of being prime in reference to assigned conditions by which Whitehead means that no other geometrical element satisfying those conditions is incident in the given geometrical element. Whitehead points out that a point is an absolute prime in the sense that no other point or geometrical element can be incident in it. He is now in a position to define a segment as a geometrical element between points p and q in which p and q are incident and in which no geometrical element is incident in which also p and q are incident; p and q in such cases are to be called the end–points of the segment.

Whitehead now introduces the notions of a point being situated in a region and in the surface of a region : a point is situated in any region which is a member of one of the abstractive sets composing that point, and a point is situated in the surface of a region x when all the regions in which that point is situated overlap with x but are not included in x.

A complete locus of points can now be defined : A complete locus of points is a set of points that compose all the points situated in a region, or in the surface of a region, or all the points incident in a geometrical element. The volume of a region is a complete locus consisting of all points situated in that region; a surface of a region is a complete locus consisting of all the points situated in the surface of that region; and, a linear stretch between two end–points is a complete locus consisting of all the points incident in the segment between those two points.

Whitehead makes an important remark about the Euclidean definition of a straight line. He says that the weakness of this definition is that nothing has been deduced from it whereas the uniqueness of a straight segment between two points (i.e., there being one and only one straight segment between any two points) should be deducible from it. Consequently, in modern times, as Whitehead points out, a straight line segment has been defined as the shortest distance between two points, and shortest distance has itself been practically defined as the line which is the route of certain physical occurrences. Whitehead tries to remedy this gap in the classical theory.

Whitehead mentions a class of oval regions and says that it is to be defined. The only weapon that he finds for this definition is the notion of regions which overlap with a unique intersect. He says that evidently it is a property of a pair of ovals that they can only overlap with unique intresection, but, he says, it is equally evident that some non-oval regions also overlap with unique intresection. However, he says, the class of ovals has the property that any non-oval region overlaps some oval regions with multiple intersection. He admits that a single oval region cannot be defined but a class of oval regions can be defined inasmuch as a class can be defined whose members have to each other and to non-oval regions the properties ascribed by him to the class of oval regions. Such a class, he says, will be called ovate.

Whitehead proposes a preliminary definition; An ovate abstractive set is an abstractive set whose members all belong to the complete ovate class under consideration. He then defines an ovate class of regions as those which fulfil a certain group of non-abstractive and a certain group of abstractive conditions. The non-abstractive conditions are : (i) any two overlapping ovate regions have a unique intersect which also is an ovate region; (ii) non-ovate region overlaps some ovate regions with multiple intersection; (iii) any ovate region overlaps some non-ovate regions with multiple intersection; (iv) the surface of any two externally connected ovate regions touch either in a complete locus of points or in a single point; (v) the surface of a non–ovate region touches the surface of some ovate region externally connected with it in a set of points which does not form a complete locus (i. e., the two regions touch in a set of points which does not comprise a line segment, surface or volume); (vi) the surface of an ovate region touches the surface of some non–ovate region externally connected with it in a set of points which does not form a complete locus; (vii) any finite nnmber of regions are included in some ovate region (i.e., there is a sufficiently large ovate region to *contain* any given finite number of regions); (viii) if A and B be any two ovate regions such that A includes B then there is an ovate region C such that A includes C and C includes B, and (ix) there are dissections of every ovate region which consist wholly of ovate regions, and, there are dissections which consist wholly or partly of non–ovate regions. The abstractive group of conditions are : (i) there are ovate abstractive sets among the members of any point; (ii) if any set of two, or of three, or of four, points be considered, there are ovate abstractive sets prime in reference to the condition of covering those points; and, there are sets of five points such that no ovate abstractive set is prime in reference to the condition of covering those points. Whitehead points out that by reason of the definitions of the abstractive group of conditions, the extensive continuum in question is four–dimensional. An extensive continuum of any number of dimensions can be defined analogously. Whitehead asks us to notice that the property of being dimensional is relative to a *particular* ovate class in the extensive continuum (emphasis ours) : there may be ovate classes satisfying all the conditions except the dimensional conditions. He further informs that a continuum may have one number of dimensions relatively to one ovate class and another number of dimensions relatively to another ovate class. Whitehead opines that the physical laws which presuppose continuity, possibly depend on the interwoven properties of two or more *distinct* ovate classes. (emphasis ours).

Whitehead assumes that there is at least one ovate class in the extensive continuum of the present epoch which has the two groups of characteristics enumerated above. He selects one such ovate class and says that all [further] definitions will be made relatively to the selected ovate class. He assures us that there being an alternative ovate class is immaterial to the argument; if there be such an other one, the derivative entities defined in reference to this alternative class are entirely different to those defined in reference to the selected class. He now presents the theorem which is going to help prove the uniqueness of a straight segment : if two abstractive sets are prime in reference to the same two-fold condition of covering a given group of points and of being equivalent to some ovate abstractive set, then the two abstractive sets are equivalent. He offers an elegant proof.[9] It follows as a corollary that all abstractive sets, prime with respect to the same two-fold condition of this type, belong to one geometrical element.

We now come to the definition of a straight segment. If two abstractive sets are prime in reference to the same two-fold condition of covering a given set of two points and of being equivalent to some ovate abstractive set, then two sets are equivalent and belong to one geometrical element : this geometrical element is called a straight segment. As can be readily seen, this definition itself shows the uniqueness of a straight segment. A similar definition is given of a flat geometrical element : instead of having two, we now have more than two, points. Whitehead observes that straight segments are also included under the designation of flat geometrical elements.

Realizing that it may so happen that the same geometrical element is difinable by some sub–set as is defined by a given set of points, Whitehead offers a definition and a postulate to meet this difficulty. A set of points which defines a flat geometrical element is said to be in its lowest terms when it contains no sub–set defining the same flat geometrical element; and, no two sets of a finite number or points, both in their lowest terms, define the same geometrical element.

Whitehead defines a straight line between two given points as the locus of points incident in a straight segment between those points. (A straight segment between two given points was defined as a certain geometrical element. Now, a straight line between two points is being defined as a certain locus of points.) Similarly a flat locus is defined as the locus of points in a flat geometrical element. He relates a given flat locus with a section thereof through the assumption that if any sub–set of points lies in a flat locus, that sub–set too defines a flat locus contained within the given locus. Now a complete straight line is defined as a locus of points such that (i) the straight line joining any two members of the locus lies wholly within the locus, (ii) every sub–set in the locus, which is in its lowest terms, consists of a pair of points, and (iii) no points can be added to the locus without loss of one, or both, of the characteristics (i) and (ii).

Whitehead defines a triangle as the flat locus defined by three non–collinear points; these points are the angular points of the triangle. A plane is defined as a locus of non–collinear points such that (i) the triangle defined by any three non–collinear members of the locus lies wholly within the locus, (ii) any finite number of points in the locus lies in some triangle wholly contained in the locus, and (iii) no set of points can be added to the lacus without loss of one, or both, of the characteristics (i) and (ii). Similarly, a tetrahedron is the flat locus defined by four non-coplanar points which are the corners of the tetrahedron. We now come to th the definition of a three dimensional flat space. It is a locus of non-coplanar points such that (i) the tetrahedron defined by any four non-coplanar points of the locus lies wholly within the locus, (ii) any finite number of points in the locus lies in some tetrahedron wholly contained in the locus, and (iii) no set of points can be added to the locus without the loss of one, or both, of the characteristics (i) and (ii).

II

Does the method succeed in derving the point from the region?

Professor Adolf Grünbaum answers this question in the negative on two grounds. According to Professor Grünbaum, (i) the convergence of the abstractive sets or classes is fatally ambiguous,[3] and (ii) Whitehead's method is vitiated by one of Zeno's arguments [4]

(1) Professor Grünbaum's first ground is valid in so far as Whitehead's earlier works, the *Enquiry* and the *Concept* are concerned. In those works, he had taken the expression 'A extends over B' to mean that B was a proper part of A. Now, if we take smaller and smaller (proper) parts of A as the members of an abstractive 'class', then, without appealing to the notions of a point, line, surface or volume, it cannot be determined as to what kind of an entity it is to which, e. g., to a point or a line, does a given abstractive 'class' converge. For example, if we take an event E and wish to take out parts of E to converge to a line, but take out parts E_1, E_2, E_3 ... such that the 'surfaces' of E_1, E_2, E_3 ... have one and only one point in common, then the abstractive 'class' so abtained connot con-

verge to a line. Thus, to what an abstractive class converges was not determinable. Whitehead had done nothing to forestall this ambiguity of convergence. And this ambiguity was fatal to his Method, since it entirely depended on the notion of sameness of convergence. In his *Process and Reality*, Whitehead removed this ambiguity by distinguishing between tangential and non-tangential inclusion and basing the notion of an abstractive set on that of non-tangential inclusion or non-tangential whole-part relationship. No two members of an abstractive set can now have a common outer surface or a common line-segment or point on their outer surfaces.

However, Professor Grünbaum holds that the Method even as presented in *PR* is beset by ambiguity of couvergence. He asks us to take two distinct but neighbouring points such as $x=0$ and $x=10^{-1000}$. It is clear that there is a non-denumerable infinity of other points between the two chosen points. Now, Professor Grünbaum asks Whitehead to tell us (i) whether we know from sense perception that there exist two *different* abstractive classes defining those two points, and, if the answer be yes, to tell us (ii) as to precisely how their particular difference is certifiable by sense perception. Professor Grünbaum, it is submitted, does not see that a circularlity is involved in his rhetorical question, and that he is raising an irrelevant issue. He first asks us to assume that *there are* two points and then demands that their difference should be certifiable by sense experience. To be able to demand that the difference between two points should be demonstrable in sense experience, he would have to point out two perceptible things which can be represented by $x=0$ and $x=10^{-1000}$. If he would have succeeded in doing that, then Whitehed too would have succeeded in pointing to the perceptible difference between those two things. However, the point is that empiricism does not demand that everything we talk about should be perceptible. It would suffice if what we talk about can be brought into some intelligible relation with what is observable in sense perception. Hence, it is not required that we should be able to distinguish between two such points in sense perception so long as some rational principle can be laid down for the purpose of distinguishing the one from the other. If it would have been the case that we are unable to distinguish between rwo abstractive sets of regions, A and B, converging respectively to points $x=0$ and $x=10^{-1000}$, then indeed Whitehead's method would have been fatally ambiguous and would have been a total failure on that account. But we see that **B** would have members (in fact, infinitely many members) which do not contain point $x=0$ (that is, some members of **B** would not include any region which is a member of some set of regions that would ordinarily be said to converge to point $x=0$).

However, it seems to me that the convergence of the abstractive sets *is* ambiguous in one case, namely, in the case of a set that is supposed to converge to a region but which *may* only converge to a surface. That is to say, Whitehead does not provide a criterion to distinguish between those abstractive sets that would ordinarily be said to converge to a solid and those that would ordinarily be said to converge to a surface.

Let there be an abstractive set that would ordinarily be said to converge to a sphere s. Let point p be the centre of sphere s. Now take a large sphere R concentric with and containing s. Spherical parts of R having p as their centre and larger than *s* would then constitute an abstractive set converging to s. Let us call this abstractive set A. It is clear that s is not a member of A : if we construct a set having as members R, R_1, R_2, R_3,.... such that p is the centre of each of these spheres and R_1 is contained in R, R_2 in R_1, R_3 in R_2, and so on, and such that each member is larger than s, then we have an infinite convergent series whose first member is R and s is in the nature of the *omega–plus–one th* term, i.e., s is the 'limit' of this series. We now take another abstractive set of regions B such that every member of B contains some member of A and similarly every member of A contains some member of B. It follows that B must also converge to s, for, otherwise, some member of A would fail to contain any member of B or some member of B would fail to contain any member of A. *Enquivalence* of two abstractive sets (in Whitehead's sense) ensures *sameness of convergence*. Now, our objection is that having chosen the abstractive set A (and, consequently, set B as well as the 'complete' group of abstractive sets equivalent to A and to one another and equivalent to no other abstractive set outside the given group), if we were to assume that sphere s does not exist—that is, if we assume that R is a hollow region—then Whitehead's method partially fails, for, now abstractive set A can only be said to converge to a surface, the surface of sphere s, but, Whitehead's method does not ensure that region R must not be hollow, Whitehead simply assumes that a region is not hollow. In other words, Whitehead should have made sure that something hollow cannot be taken to be a region but he failed to do so. However, this is not a crucial failure. The defect can be remedied by defining a gap and postulating that there are no gaps in any region. For example, Whitehead could have added two propositions at the end of section II (p. 420), as follows :

Definition 9 A. A region A is said to have no 'gap' in it when there are two regions B and C such that A and B are a dissection of C, and C includes B non–tangentially.

Assumption 18 A. By 'region' we shall henceforth mean a region that has no gap in it. This assumption is merely a convenient arrangement of nomenclature.

It may moreover be pointed out that this was not a very important matter for Whitehead. For, his method was to jump from a (four-dimensional) region to a 'point' and build up a line, surface and solid from 'points'. A group of abstractive sets that is a 'point' can be unerringly distinguished from any other group that is another 'point' or is a line, surface or solid, which is what alone matters.

(2) According to Professor Grünbaum, Whitehead's method is vitiated by Zeno's methematical paradox of plurality. The argument, in its details, is somewhat as follows :

Part of the edifice of contemporary mathematics rests on the conception that a spatial interval is literally composed of unextended point-elements. But, obviously, no finite set of point-elements can add up to a positive interval, and as argued by Zeno (and demonstrated by Professor Grünbaum), not even a denumerably infinite set of point-elements can constitute a positive interval. A positive interval can only be constituted by a non-denumerable infinite set of point-elements. For Whitehead, a point is a (complete) group of abstractive sets of regions. Hence, metrical consistency demands that there should be a non-denumerable infinity of (groups of) abstractive sets of regions. Now, Whitehead's programme of epistemological re-construction of geometry is that of beginning with something perceptible and by a process of abstraction arriving at things which are the termini of sense awareness. Hence, Whitehead's programme, in conjunction with the demand of metrical consistency, involves that there should be a non-denumerable infinity of abstractive sets and that these sets should be among the termini of sense awareness. Empiricists deny the existence of something actually infinite. Even if it is assumed that the existence of somthing actually, but only denumerably, infinite is certifiable by sense awareness, it is evident that the notion of actually infinite sets having a cardinality exceeding *aleph-null,* i.e., the notion of non-denumerbly infinite sets, would inexorably defy encompassment by the sensory imagination. Hence, Whitehead's eimpirical programme is seen to be at variance with the demand of metrical consistency.

Professor Grünbaum expects this argument to demolish both Whitehead's method in particular, and the empiricist's aspiration to reduce non-empirical notions to empirical ones in general. Insofar as the latter expectation is concerned, it is quite unjustified. In the first place, an epistemological reconstruction of geometry along empiricist lines would begin by removing from geometry the conception that supports part of the edifice of contemporary mathematics, viz., that an interval is constituted of magnitudeless elements. As such, no question of certifying the existence of a non-denumerable infinity of anything in sense experience or in anything else at all arises. In that case, the empiricists have of course to evolve points and instants, mass-points and particles, from phenomena that are perceptible, and would have to demonstrate that no illogicality was involved in such evolution. We believe that the empiricists' programme can be executed even though Whitehead may not have succeeded in evolving points from regions. (The notion of a point, we believe, is a rational notion. Hence, there must be a non-circular process through which human intellect arrived at the notion of a point. We have only to rediscover it consciously.) We are thus only left with the question of this argument's particular application to Whitehead.

Now, in relation to Whitehead, let it be noted that the argument involves both his derivation of the 'point' from the region and his derivation of the 'line', 'surface' and 'volume' from 'points'. Insofar as his derivation of the point is concerned, this does not involve non–denumerable infinity, at least directly. However, if spatial intervals are constituted as modern mathematicians suppose it to be constituted, then the 'complete' group of equivalent sets that is a geometrical element must have a non–demumerable infinity of members. But this should present no insurmountable difficulties since the abstractive sets would be overlapping in the sense that the member regions of one set would overlap with the members of the other sets. An abstractive set is not itself non-denumerably infinite, and, in fact, Whitehead asks us to think of them as a series of discrete members even though every one of them non-tangentially contains 'all' members coming after itself.

Insofar as Whitehead's derivation of the 'line' etc., from the 'points' is concerned, it is true that he does not explicitly lay it down that only a non–denumerable infinity of points can constitute a line–segment, surface of a region, or a region. But he does not lay it down either that a positive interval is constituted only of a denumerable infinity of 'points'. Rather, since he uses the expression 'all points' he may be taken to have supposed a complete locus of points to be constituted of a non–denumerable infinity of points. Hence, if it be correct that Prof. Grunbaum's view succeeds in meeting Zeno's argument in question, then Whitehead too may be taken to have succeeded in meeting Zeno's argument. As for the claim that the existence of a non–denumerable infinity of abstractive sets in sense awarness is impossible, I think, poses no problem for Whitehead, for, in sense awarness only finite number of regions would suffice to give rise to the supposition of non–denumerable infinity of abstractive sets. Moreover, as Professor Mays points out, it is by no means clear that Whitehead intended epistemological recon-

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struction of geometry along empiricist lines, and, as Nicod suggests, the Method may be considered after the fashion of an abstract mathematical model.[5] Had Whitehead had any such reconstruction at heart, he could hardly have tried to define lines, surfaces and volumes in terms of points. However, it is clear that he did not like to take the point as (intuitively) given and that he endeavoured to bring it into a rational relation with something sensible. Even so, this does not commit Whitehead to having a non-denumerable infinity of abstractive sets in perception.

In any popular exposition of Whitehead's method, it is inevitable that the words "point", "line", and "surface" should occur before his definitions thereof occur, just as we had to do earlier. (Whitehead himself found it necessary, in an aside, to talk of convergence to a point before he had defined the point.[6]) This leads to the objection that a circularity is involved in the method. But, the fact is that the apparent circularity is involved only in the exposition of the method, not in the method itself. The definition of a point given by Whitehead does not presuppose the notion of a point : a point is a geometrical element in which no other geometrical element is incident, or, in other words, a complete group of equivalent abstractive sets in which no other complete group of equivalent abstractive sets is incident. And, as argued by Broad and Stebbing, there is no circularity in popular expositions either, since 'convergence to a point' is itself understood in terms of regions and their relations.[7] (I am not happy with the actual defence though. But, we shall not argue this point since it relates only to popular expositions and not to Whitehead's method itself.)

III

(i)

We submit however that Whitehead's method does not really succeed in deriving the point from the region for the following reasons.

(1) Whitehead *either* unwarrantably presumes that an abstractive set does converge to a point, line, surface or region *or* abstractive sets fail to converge to a point, line etc.

We see that a given abstractive set **A** (consisting of concentric spheres having point p as their common centre) converges to p, but only because we know (or suppose that we know) that there are points and that points are contained in regions and that any given member region of A contains p. If we assume, for example, that there are no points, then set A will still converge as it did before, but now it will not converge *to* point p. Whitehead's postulates and definitions do not ensure that there must be points (in the ordinary sense of the word), it is simply assumed by us all that there are points, lines and other geometrical entities. In other words, Whitehead had endeavoured to derive point p, not circularly from an abstractive set converging to point p, but quite logically from the *sameness of convergence* of two converging sets of regions; but, in doing so, he failed to ensure that either of the two converging sets should converge *to* point p, that is, he failed to ensure that there was something to which either of the two sets converged. The circularity reappears in that Whitehead simply takes it for granted that region R (a member of set A) cannot lack point p which will ordinarily be said to constitute its centre. He made no effort to ensure, without appealing to the notion of a point, that a point be contained in a region; he merely surrounded the intuitively apprehended point on all sides and assumed that the poor point could not run away from the surrounding region without seeing to it that his postulates and definitions which help surround the intuitively apprehended point ensure that it does not disappear.

This, however, is not a very effective argument. Whitehead could have said that he did *not* postulate the entities ordinarily called points, lines, etc., and that he had no use for them and that it sufficed for his purpose that two abstractive sets had *sameness of convergence* even though neither converged *to* anything. If Professor Grünbaum were to insist that this would affect the continuity of the continuum, that if there were no surfaces, lines and points (as we understand these terms) then there would only be discrete regions, then Whitehead could say that he did not have to begin by assuming that spatio–temporal continua were continuous in the sense of there being boundaries between regions and that it would suffice for his purpose if regions were continuous in the sense that regions were contiguous and had no gaps in them. What is important for Whitehead is that the 'point' as defined by him does all the work that a point is required to do in geometry. However, it seems to me that (apart from the question whether Whitehead's point can do for our point) the fact that two abstractive sets have sameness of convergence but neither can be said to converge to anything (without already assuming that there are points, lines and surfaces and thus begging the question) presents at least an infelicity. (And this infelicity turns into perplexity when in popular expositions 'convergence to a point' etc., is glibly mentioned : convergence to a point *or* 'convergence' to a complete group of equivalent abstractive sets in which no such other group is incident, and if the latter then what does 'convergence to a certain group of abstractive sets' mean ?)

Whitehead also appeals to the fact that abstractive sets have different kinds of convergence : some converge to a region, some to a surface, some to a line, and only some to a point. But differences in the nature of convergence can be apprehended only when a point, line and surface has already been defined. If we do not know what a point, line or surface is, and we are not supposed to know it till points, lines and surfaces have been defined, then we only see that, as we proceed toward the regions included in the earlier regions, the members of the abstractive set are becoming smaller, we have no means of discovering that some abstractive sets are converging to a region while others are converging to a surface, line or point. Thus, in appealing to the fact that abstractive sets have different kinds of convergence, Whitehead's method involves a circularity.

(2) Whitehead's definition of a point as a *complete* group of equivalent abstractive sets is necessary but impossible.

The qualification of completeness is necessary because otherwise it would have been possible that a given group of equivalent abstractive sets is point p and another group of equivalent abstractive sets is point q but the members of p and q are equivalent and, hence, either a distinction would have to be drawn between the two groups, which seems impossible, or a rule would have to be laid down that p and q are the same, which in effect would amount to the completeness of the group.

The qualification of completeness is impossible because no group of equivalent abstractive sets can be complete. It is evident that no finite collection of equivalent abstractive sets can be complete, since no matter how many such sets have been taken, there will still be some other set which is equivalent to each member of the collection but is not itself a member of this collection. The reason is that space is *ex hypothesi* infinitely divisible and hence given any two equivalent abstractive sets there is a third which is equivalent to both and in a sense lies between them. (Suppose we take set $S_1=R_1, R_2, R_3 \ldots, R_n, \ldots$, and set $S_2=E_1, E_2, E_3, \ldots E_n \ldots$, such that R_1 contains E_1 and E_1 contains R_2, and so on. Then there is an abstractive set $S_3=F_1, F_2, F_3, \ldots$ such that R_1 contains F_1, F_1 contains E_1 and

E_1 contoins R_2, and so on.) This means that given any abstractive set S, there are infinitely many abstractive sets that are equivalent to S. But there can be no infinite group or collection of anything, i. e., no determinate collection or group of anything can be infinite. (This point will be elaborated later in connection with the question whether 'an infinite set of points' has any meaning; please see sub-section ii.)

(3) Whitehead's 'point' does not answer to what we call a point.

We may not be able to state what we mean by the word "point" beyond what has been said by Euclid, but, I believe, we all mean the same thing (otherwise there would have been no geometry), and certainly what we mean by this word is not a complete group of equivalent abstractive sets of regions in which no other such group is incident (and whose member sets would ordinarily be said to converge to a point). C. D. Broad says that we must not be aghast at finding that the point had turned out to be different from what we had expected it to be.[8] Indeed, if we had supposed a ball to be made of iron and on analysis found out that it was made of silver, or we supposed the ball to be spherical and found out that it was oblong, then we ought not to be aghast at our finding. But, here we do not begin by assuming that the point is given and on analysis is discovered to be different from what we had expected it to be. Here, we believe we know what a point is and if we find that we are being presented with something different then we can at least say, "Well, your 'point' is different from ours". The crucial test here, as Broad rightly observes, is to see if Whitehead's 'point' can do for our point, and, we see that we cannot replace the *definiendum* (the ordinary word "point") in geometrical sentences by the *definiens* of Whitehead's definition of a point ('a geometrical element in which no other element is incident').

This point may be seen in connection with Whitehead's definition of being situated in a region. It is for us a truism that a point is situated in a region. But we do not comprehend what is meant when we are told that a certain group of equivalent abstractive sets of regions is said to be 'situated' in a region when that region is a member of one of the abstractive sets which compose that group of equivalent abstractive sets. Shorn of its technicalities, the definition tells us that a group of abstractive sets of region is *situated* in any region which is a member of any of the abstractive sets of regions included in the group in question. We feel that 'to be situated in a region' as used by Whitehead does not mean what we mean when we say that a point is situated in a region. The gulf between the two usages appears to widen when a complete group of equivalent abstractive sets of regions is said by Whitehead to be situated in the *surface* of a region which is a member of one of the given abstractive sets of regions.

In short, a group of abstractive sets of regions is *not* a point (as ordinarly conceived) but merely a *route* or *pointer* to a point. It is unquestionably a better route or pointer than any that we have hitherto had, for example, better than the attempt to arrive at a point by dividing and subdividing a region. All the same, a group of abstractive sets is only a pointer or route to a point, not a point in itself. This, Whitehead had himself conceded in an earlier work, when he said :

> There is no one event which the series [of events forming an abstractive class] marks out, but the series itself is a route of approximation towards an ideal simplicity of content. [9]

A route of approximation towards an ideal simplicity of 'content' it is submitted, is not itself an ideal simplicity of content.

We may put this argument as follows. If we knew what the word 'point' meant and were looking for point p then Whitehead's method would unerringly take us to point p and to no other point. That is, if we were in search of a route to p then nothing I know of could provide a better route to p than this method, for, it is by taking p as the point of departure that the group of abstractive sets has been arrived at.

However, if we are innocent of the notion of a point then despite guiding us towards point p by making sure that we do not chance wander on to any other point or to anything else of a different nature such as a line, Whitehead's method completely fails in yielding a point. What we have is a set of overlapping regions which become smaller and smaller indefinitely, and beckon a person wise to the situation towards P and leave an ignoramus like myself greatly baffled.

To sum it up, if we had to *represent* a point by something so that we could retain the distinction between points p_1 and p_2 then groups of equivalent abstractive sets could be used for this purpose : the distinction would be retained in as much as group g_2 cannot lead to p_1 nor can group g_1 lead to p_2, g_1 being a route of approximation to p_1 and g_2 being a similar route to p_2. But if we desired to have something *equivalent* to what we call a point, or, what is the same, if we desired to learn what the word 'point' *means*, then the expression 'a complete group of equivalent abstractive sets of regions in which no other group of equivalent abstractive sets of regions is incident' is *not* equivalent to the word 'point', it does not tell us what a point really is. If so, Whitehead fails to *define* a point, and, *a fortiori*, fails to derive the point from the region.

(4) Whitehead jumps from the (four demensional) region to the point directly instead of deriving the surface from a region, a line from a surface, and a point from a line.

If Whitehead would have succeeded in deriving the point from the region then this objection would have been pointless, although, even in that case, it would have pointed out an aesthetic infelicity.

If fine, we see that inspite of taking advantage of the infelicity of jumping directly from the region to the point and of having sameness of convergence without there being a convergence *to*, Whitehead fails to find a non-circular method for defining the point.

(ii)

In addition, it may be pointed out, Whitehead's method fails to derive the line from the point.

Insofar as the derivation of the line, surface and volume is concerned, there is no difference between Whitehead and modern mathematicians—both derive the line, surface and volume from the point—and the arguments which can be urged against the one can be urged against the other.

(1) First of all, it seems strange that a magnitudinous whole should consist of magnitudeless parts. This difficulty is overcome by distinguishing between 'components' and 'constituents'.[10] Even so, it seems strange that a set of things each one of which is of zero magnitude should give rise to something that has positive magnitude.

Strange though it seems, this is what the mathematicians, Dedekind and Cantor in particular, are supposed to have succeeded in doing. If S be a set of points such that for any value of x, if x is a point on line segment l then x is a member of S and if there is no x such that x is a member of S but does not lie in l, then the members of S ordered in the manner they occur in l would be equivalent to l. Thus, all we need to do to dissolve the line-segment into a set of points is to find a set which has the property of set S. Now suppose that the line segment l is of the length of one centimetre. Let p_0 be the first point of l, and p_1 be the last point of l. Now, any point p_n on l can be defined in terms of its distance from p_0; e.g., if p_n is at a distance of 0.4 centimetres then we represent it by p. 4 However, this is not sifficient to derive the line. We have to determine the relations that cubsist between the paints when they form a line. Dedekind and Cantor, therefore, endeavoured to determine what characteristics the supposed set of points S has. Now, the first characteristic of points is that no two points are consecutive. So, no two members of S may be consecutive if set out in the order of increasing (or decreasing) magnitude of their subscripts. Secondly, every point is an end-point of some sub-segment of l, and every sub-segment of l is such that an omega-sequence of points can be obtained having as its 'limit' the end-point of that sub-segment. So, every sub-set of S must contain a progression of members and the limits of such progression of must be members of the set S. Thirdly, if p_m and p_n be any two members of S and if m and n be rational numbers, then there must be a member of S, say p_r such that r is an irrational number greater than m and smaller than n, and conversely, if m and n are irrational numbers, then there must be a p_r such that r is a rational number greater than m and smaller than n. Given these conditions, to run through the members of S in the ascending order of magnitudes would be tantamount to running through l from p_o to p_1." [11]

Thus, the objection seems to have been overcome : we see how a line having a positive magnitude can be dissolved into points, or if you like, how can magnitudeless points give rise to a line.

However, it seems to me that the line is not done away with completely. Of course, the obvious objection that each point was defined in terms of its distance from a given point and that no definition of 'distance' in terms of points alone had been given, would be based on a mistake. In order to show that a line can be analysed in terms of points, the points were initially defined in terms of distances, but once we see that an equivalence can be established between set S and line 1, we can take the points independently of distances and in themselves : if the members of S have three characteristics given above they give rise to a continuum of points. However, no rule appears to have been given to distinguish between the lengths of two continua of points. That is, since any continuum of points has a non-denumerable infinity of points, their magnitudes cannot be differentiated by the number of points. Indeed, in some cases, magnitudes of continna can be differentiated, e. g. where one is a part of the other, but, even in such cases, the ratios between the two can be worked out only by taking some continuum as the unit of comparison, which in effect means that some line-segment, in itself and quite independently of the points supposedly constituting it, would be adopted as the unit of measurement.

(2) Moreover, there is a more fundamental objection to the mathematicians' position, viz., that there is no set of terms which could be the set S, or in other words, that 'set S' is not a 'set of terms' but a formula for generating terms, and a formula which is in principle incapable of yielding any given set of terms. We have argued this point elsewhere;[12] here we shall content ourselves with presenting a summary of our arguments.

(i) Since lines and periods of time, on this view, are nothing but sets or series of points and moments, whatever be the state of affairs, the interval must be capable of being given in terms of points and moments. But we find an insurmountable difficulty in doing that. Even in relation to points and moments themselves we are beset with the difficulty of accounting for the end-points (and every point or moment is an end–point). If we divide a set (or rather series) into two halves, do we take a certain point or moment in both the sub–sets or in neither of the sub–sets or in one but not in the other sub–set ? If we take it (a moment) in both the sub-sets, then we may have to concede that, e. g., a body is both green and is not green at that moment. If we take it as being a member of neither sub–set, then there will be no last moment of being green and no first moment of being non-green, and, at the moment in question the body would neither be green nor not be green. If we take this moment as a member of one of the sub–sets, then, firstly, either there will be no last moment of being green or no first moment of not being green; and secondly, it will become a matter of arbitrary choice as to which of the two alternatives is chosen in a given instance.

Perhaps, the only way of meeting this objection is to claim that we start with wrong data—the body cannot be said to be green during a sub–period unless we know during what set of moments it is green; if it is green at moments m_1 to m_n then it is green during period t_1 such that $t_1 = \{ m_1 \ldots m_n \}$, but if it is green at moment m_1 and every moment before m_n but not at m_n then we would say that the body is green at moments m such that $m_1 \leqslant m < m_n$ ('$<$' meaning 'is before' and '$\leqslant$' meaning 'is the same as or before').

But the problem will not get resolved. In the first place, there will still be cases in which there will be no last moment or no first moment of being in a given state, for, two consecutive periods cannot have a common moment and no two moments are consecutive. Moreover, in the case of motion, if a body must be motionless at any and every moment (since it cannot traverse any distance in a moment) then it would seem a little strange that the body is not invariably at rest during a set of moments. In the second place, it seems unreasonable to suppose that a body can be in any state in a durationless moment.

(ii) That what we have called 'set S' cannot be a *collection* of terms is quite clear, since an 'infinite collection' is a contradiction in terms.

But 'set S' cannot be a *class* of terms either. It is true that the word 'class' is ordinarily used quite ambiguously so that we have both a defining property and the terms which have that property. And it is this practice which has given rise to the problem of universals. We are here using the word differently. We are so using the word that a given aggregation of terms each of the same sort or kind constitutes a collection and not a class, so that a class can stand in relation only to other classes and cannot be said to have an n number of members no matter what n may be and no matter how many entities be known to have the defining property of the given class. Moreover, the property that defines a class must be general and must not in any manner be restricted. That is, restriction on a class must come only from an additional qualification being imposed which must itself be general. Thus, there can be a class of animals and a class of points, but there cannot be a class of animals living in Pakistan or a class of animals existing in the 19th century; similarly there can be no class of points lying in this solid or that line–segment. For, 'living in Pakistan' or 'lying in that line–segment' are not general attributes or properties. If so, there can be no such *class* as the class of points between points p_m and p_n.

Let us suppose that 'set' means something different from a collection and a class. What will the expression 'all the members of S' now mean? If S would have been a collection, it would have meant '$x_1, x_2, x_3, \ldots, x_n$' but S is not a collection. If S would have been a class it would have meant the whole class to the exclusion of on sub–class, but S is not a class. What then can the expression in question signify ? To me, it signifies nothing except the obstinate desire to do the impossible—to derive the line from the point.

(iii) Finally, it appears to me that mathematicians took the wrong course in relating the line and the point : it is the point which is to be derived from the line and not the line from the point. Mathematicians thus not only reify the point, they completely fail to understand the nature of a point. A point is a potential division of a line just as a line is a potential division of a surface, and a surface that of a solid. To talk of all the points of l is thus to talk of all the division of l, and to equate a set of points with l is to equate a set of division of l with l and to hold that line–segment l is nothing but all the divisions of l. In a sense, the equation is true. If there is such a thing as 'all the divisions of l' then no matter how disparate the category of 'divisions' and 'line–segments' may prima facie appear to be, nothing would be left in l if all its possible divisions were obtaimed. However, 'all the divisions of l', though it very much looks like 'all the boys in this room'' has at best the same status as 'all men' and any attribute predicated of it must be analytic, i.e. the predicate must be a component of the complex of defining properties. But when we claim that 'all the divisions of l are given' then 'being given' does not at all seem to be a property of 'the class of divisions of l' (even assuming it to be a class).

(iii)

Furthermore, Whitehead fails to define a straight segment. He defines a straight segment in terms of, inter alia, an 'ovate' abstractive set which he has not been able to define.

Whitehead begins by mentioning what he calls an 'oval' region and contrusts it with a non–oval region in a very vague and ambiguou fashion. He claims that it is *evident* that two (as yet undefined) oval regions *can* only overlap with unique intersection. I do not profess to undrstand what he means. In the literal sense of the word, a region would be called oval if it had the shape of an egg, and a region which did not have this shape for example, a sphere, an obelisque or a pyramid, would be called a non–oval region. If so, why two oval and not two non–oval regions should overlap with unique intersection is by no means evident to me. Whitehead further say that anys non oval region overlaps some oval regions with multiple intersection, from which it appears as if some oval regions may not overlap any non–oval region with multiple intersection. Even so, we fail to have any definite idea of an oval region or of the distinction between an oval and a non–oval region.

Whitehead holds that a class of ovals can be defined although a single oval cannot be defined. It is submitted that this expression is logically inappropriate. An individual can be described, possibly, exhaustively described, but cannot be defined. A class of things can be defined but if a class is defined then every individual which belongs to that class can be distinguished from any other individual not belonging to that class. The cat called Pussey cannot be defined, it can only be described; the class of cats can be defined, which only means that cat–ness or the properties which a thing must posses in order to qualify to be called a cat can be exhaustively enumerated. Thus, if it were possible to define a class of ovals, then it would be possible to say what an oval was. But, Whitehead, in saying that a single oval cannot be defined, meant to say that it was not possible to state what characteristics a region must possess to be called an oval. If so, in a logically proper sense, it was not possible to define the class of ovals. Thus, we may take it that in claiming that the class of ovals was definable, what Whitehead really meant to say was that without defining the terms 'oval' and 'non-oval' a set of protocol propositions could be laid down stating relations between these terms which could lead us to divine in what senses the two terms might have been used.

Whitehead further confuses the issue by saying, "...we cannot define a single oval, but we can define a class of ovals. Such a class will be called 'ovate'",. At first sight, this decision seems to be senseless : why not persevere with the term 'oval', why bring in yet another undefined term ? But, on reflection, we see that Whitehead is not using the word "class" to mean things of the same kind in general, i e , things having common characteristics whether or not there actually be a thing having the characteristics in question—in short, in a sense in which the notion of a null class is not a contradiction in terms. Hence, it would seem that by "ovate" he means that group of ovals which can be defined. This makes sense, but makes the notion of an oval even more confusing and out of our reach.

Coming to the ovate 'class', what Whitehead does is to tell us what relations two ovate regions must bear to one another, what relations an ovate region must bear to some non-ovate region, what relations a non-ovate region must bear to some ovate region, and that there are ovate abstractive sets. This is indeed no way of defining what an ovate region is. But, let us try to see what picture of an ovate region emerges from the protocol propositions.

First of all, an ovate region is not necessarily oval in shape. For, a sphere satisfies both the abstractive and non-abstractive conditions laid down by Whitehead. Going over the conditions of the two groups, we came to the conclusion that what Whitehead may have had in mind is what we may call a 'regular' region, i.e., a region bounded by a 'regular' surface and comprehending all that lies within that surface. In other words, a region having a surface free from all protuberances and depressions and whose interior is free from all gaps or hollowness. We arrive at this conclusion from the fact that two regular regions; neither of the two having any protuberance or depression, can overlap only in a single, continuous stretch, whereas a regular region with some non–regular region and a non–regular region with some regular region must overlap with multiple intersection. And the surfaces of any two regular regions must meet either in a point or in a continuous set of points, that is, in a line or a surface, whereas a regular surface and some irregular surface, and, similarly, an irregular surface and some regular surface, must meet in a non–continuous set of points, i.e., in a group of points which do not by themselves from a line or a surface.

Although we cannot be definite that this is what Whitehead must have meant by an ovate region, I feel that we cannot be far wrong in our belief, for, for purposes of extensive abstraction the notion of a regular region is indispensable. Hence, we may at least tentatively assume that by an ovate region Whitehead must have meant a regular four–dimensional region.

Now, if Whitehead did really mean by an ovate region what we have designated a regular region, then it is all too clear that, instead of endeavouring to determine the essential properties of a regular region and defining a regular region in terms of those properties, Whitehead only seized upon two characteristics of pairs of regular/irregular regions/surfaces, namely, those of unique/multiple intersection and of intersecting in a group of points forming/not forming a line or surface, and tried to 'define' the regular region in terms of these two non–essential characteristics of pairs of regular/irregular regions/surfaces, i. e., characteristics which cannot be used to define the term 'a regular region', for, these properties characterize *relations* between *two* regions/surfaces, and, consequently, an ovate, region.

Thus, even though Whitehead's definition of a straight segment is such that the uniqueness of a straight segment is immediately deducible from the definition itself, which is clearly an improvement on the traditional treatment, this definition does not succeed in defining a straight segment since Whitehead had not succeeded in defining an ovate region even if he is regarded as having succeeded in telling us what he meant by an 'ovate' region. (It is to be noted that although our description of a regular region as 'a region whose surface is free from all protuberances and depressions and whose interior is free from all gaps or hollowness' seems quite clear and intelligible, if the notion of a point has not already been defined, means nothing. To become meaningful, the words 'protuberance', 'depression', 'gap' or 'hollowness' will have to be defined without resorting to the notion of a point. When we try to do so, we find it very difficult even to distinguish between the surface and the interior of a region !)

(iv)

Since, in our opinion, Whitehead has failed to derive the line-segment from the point and to define a straight segment, it follows that he has failed to derive the surface and volume from the point and has failed to define a plane.

IV

Now that we come to the conclusion that Whitehead's method of extensive abstraction did not succeed in deriving the point, and the line and the surface, from the region (the latter two via the point), or in defining a straight line or a flat surface, must we regard this method as a historical curiosity, as yet another instance of an aberration of the kind human mind affords ample evidence of being prone to ? I think that the answer is an emphatic " no ".

Solutions of most philosophical problems have only been possible by the trial-and-error method after many false leads had been thoroughly worked through. When, finally, a definitive solution is arrived at, all the earlier attempts at solution are seen to be complements of the actual solution without which such a solution could hardly have been possible. Even though a failure in the ultimate analysis, the very fact that such an attempt was made is in itself of immense value. In attempting to derive the point from the region, Whitehead's method is on the right track : we are certainly not born with the notion of a point, and, hence, it is obvious that we acquire it by some such sub-conscious process as Whitehead's method. The final solution of this problem will be arrived at by the same rigorous logical method of beginning with a few undefined notions embedded in sense perception and a few universally acceptable axioms.

It is clear that the notions of tangential and non-tangential inclusion will prove helpful in any attempt at extensive abstraction If the notions of point, line and surface are not given, then to be able to ensure that a given region is a plenum-i. e.; to ensure that a given outer surface encloses the entire region which would ordinarily be taken as enclosed within it—the notion of non-tatngential inclusion will be found to be of crucial importance.

The method of rigorous deduction, though not new with Whitehead, is of the greatest value and the only logical method for the derivation of the point from the interval. In relation to extensive abstraction, Whitehead's was the pioneering endeavour and will ever be a beacon to all those who might attempt extensive abstraction in the future.

Whitehead's procedure in defining a straight segment, that is, in offering a definition which shows the straight segment's uniqueness among the line-segments bounded by two given points was a wonderful attempt and one cannot but wish that it had succeeded. Whithead had taken the property of being the shortest distance as the crucial defining property without falling a prey to the circularity involved in other attempts to define the notion of a straight line. It is clear that if the concept of straightness is ever to be caught hold of in a non-circular definition, that definition will have to be such that either the property of being the shortest distance between two points can be immediately deduced from the definition or the concept of being the shortest distance between two points can be defined with the help of the defined notion of a straight segment.

In short, we owe a debt of gratitude to Whitehead for his having attempted to derive the point by extensive abstraction from a datum which was a deliverance of the only primary source of human knowledge. sense perception.

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NOTES

1. In his preface (written in 1914) to *Our Knowledge of the External World* (first published in 1914, revised in 1926) Russell says that he owed his definition of points and the treatment of instants to Whitehead and that what he had said on those topics in that book was in fact a rough preliminary account of the more precise results which Whitehead was giving in the fourth volume of their *Principia Mathematica.* I have not been able to consult this book. However, it is almost certain that

Whitehead's exposition of his Method in that book must have been about the same as he has given in the first two works listed here. (But at p. 119 of the 1961 reprint of the revised edition, Russell only mentions the *Enquiry* and the *Concept* in this connection. Moreover, Professor W. Mays, in his book, *The Philosophy of Whitehead*, 1959, reprint, New York, 1962, devotes a chapter to the Method of Extensive Abstraction, pp. 115-25, but makes no mention of the *Principia* in connection with the Method.)

2. At p. 432, however, he says that regions M and N intersect instead of saying that M and N overlap. (To overlap has been defined but not 'to intersect'. An 'intersect' has been defined. but from its definition one cannot go on to 'to intersect'.

3. "Whitehead's Method of Extensive Abstraction", *The British Journal for the Philosophy of Science*, IV, No. 15 (1953). pp. 215–26: See, pp. 219-26.

4. "Whitehead's Method", pp. 216–19 and 222–26.

5. *Philosophy of Whitehead*, pp. 118–19. (Prof. Mays only says that "Whitehead does not always make it clear whether his method is to be taken as an algorithm or as an exact description of some actual process of convergence". He futher says that "Nicod ... suggested that Whitehead's contribution could be taken as the construction of a pure geometry rather than as an analysis of the real World".

6. E. g., *Process and Reality*, p. 421.

7. C. D. Broad, *Scientific Thought*, reprint, London 1952, pp. 45–47; L. S. Stebbing, *A Modern Introduction to Logic*, reprint, London, 1958, pp. 446–52, esp. pp. 450–51.

8. *Scientific Thought*, p. 43.

9. *An Enquiry Concerning the Principles of Natural Knowledge*, reprint, Cambridge, 1955, p. 104. In the *Concept of Nature* (reprint, Cambridge, 1971), Whitehead says, 'Thus an abstractive element is the group of routes of approximation to a definite intrinsic character of ideal simplicity to be found as a limit among natural facts.'

10. C. D. Broad, *Scientific Thought*. p. 330.

11. R. Dedekind, *Essays on the Theory of Numbers* (tr. W. W. Beman), New York : Dover, n. d., esp. pp, 3–21; G. Cantor, *Contributions to the Founding of the Theory of Transfinite Numbers* (tr. P. E. B. Jourdain), New York, 1915.

12. "Infinzer–atomicity", *The Pakistan Philosophical Journal*, XIII, no. 3 (October 1975), pp. 47–84, and XIV, no. 2 (Jan.–June 1976), pp. 34-72.