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ON THE RELATION BETWEEN SCIENCE AND LOGIC

Scientific theories reslut from observable phenomena: logical and mathematical theories from the relation and use of concepts. By science, I mean the physical and natural sciences. Many logicians and mathematicians refer to such things as the sciences of logic and of mathematics. And although I may be partial to such interpretations, I would suggest that the use of such expressions without first attempting to specify what is meant by a "science" would be either misleading or lead to confusion. To that extent, a definition of science will not be attempted because that would involve a different sort of analysis than the one intended here. Since my primary concerns here are with logic and mathematics, I do not think my conclusions will be endangered by this omission.

If these statements are to be regarded as true a perplexing question is raised viz. what is the relation between physical science, logic and pure mathematics? The question can be made much more explicit. What is the relation between observable phenomena and our concepts, that is, the concepts themselves and the holding and the use thereof?

The propositions of natural science are said to extend human knowledge; those of logic and mathematics are said to be reflective since they are not about anything (physical) in the world. There are no mathematical or logical entities in nature — and truth in logic and mathematics is a function of things other than a correspondence between propositions and matter-of-fact conditions. As such logical and mathematical propositions do not bring about an extension of our knowledge — they are simply based on introspection.

A concerted effort will be made to argue against this view of logic and mathematics in the hope of demonstrating two important points. The first is that logic is the means by which we understand reality; the second that mathematical ideas provide inspiration for new ideas (progress) in natural science. In the latter, the primary concern is with physics, chemistry and astronomy. It is quite possible, however, that biology and botany etc., are also affected by my thesis and conclusions.

I

Progress in scientific research may be seen as an attempt to achieve at least three things : classification, comparison and quantification.

Classification simply means the arrangement of natural objects and ideas; the assigning of a name to a thing, or more explicity the placing of an object within a class in accordance with certain laws or principles. Scientists are well aware of the difficulties involved in such an endeavour and the difficulties vary directly with the degree of differentiation desired. Difficulties result also from the kind of classification sought, that is, natural or artificial. One need only regard the classificatory systems of Linnaeus and Cuvier (15th century) to appreciate this point. But even here the expressions *natural'* and artificial are problematic. Linnaeus for example, by classifying plants according to their sexual characteristics, that is, according to the number and form of the pistils, petals, and stamens of flowers, arrived at a much more general systems than by classifying them according to say the shape of their leaves. Thus when one system is replaced by a more satisfactory system, the new one is referred to as natural because it expresses analogies between two characteristics. The abandoned classification is considered *artificial.* The concept of *natural* classification is therefore seen to be relative.

Currently, we mean by a natural classification a system, e.g., in Zoology, arranged according to the number of genes. Serum reactions i.e. chemical reactions, have demonstrated consanguity between different kinds of animals, which points to a genetic relationship.

The quantitative aspect of physical science is another form of classification. It is essentially the assigning of anumerical value to an object: and the assigning is effected through the use of rules. Perhaps the simplest application of quantitative concepts in science is counting. The principle of counting which was formulated by the mathematician Dedekind is as follows: To count the objects of an aggregate or collection K is to establish a one-to-one correspondence between the objects of K and a set N of numbers or numerals, such that:

1. N includes I;
2. there is at most one number in N whose immediate successor is not in N;
3. the number of objects in K is the number mentioned in (step 2) if it exists; otherwise the number of objects in K is infinite;
4. the one-to-one correspondence may be carried out (i) by actually attaching one number or numeral to each object; or
5. as when we count by two's or five's or hundreds; or
6. by specifying a rule for actually attaching numerals to as many objects as we please.

The above are formal principles which have been given to demonstrate that what is generally taken to be an empirical exercise really involves the use of logically set theoretical concepts and the ability to attach or assign these concepts to specific objects.

The number one may be defined as follows:
a class or set X contains only one element, or more precisely X is a member of the class of classes I, $(X \in I)$ if

(a) there exists an entity e.g., U, such that $(U \in X)$ and if
(b) for any two entities V and W, if $(V \in X)$ and $(W \in X)$ then $V = W$ (that is the elements are identical).

The number two is defined as:
Y is a member of the class of classes, 2, $(Y \in 2)$ if

(a) there exists an entity e.g., U_1 such that $(U_1 \in Y)$ and another entity U_2, such that $(U_2 \in Y)$ and if

(b) for any entity, say V, if $(V \in Y)$ then $(V = U_1) \vee (V = U_2)$.

We can thus express "1 + 1 = 2" in terms of the definition of 1 and 2, with the help of the logic of quantification, that is, the universal quantifier.

Thus "1 + 1 = 2" is defined by

$$(x)\,(y)\,[\,(X \in I) \cdot (Y \in I)\,] = [\,(X \vee Y) \in 2)\,].$$

If we apply this generalisation to a particular case, e.g., the analysis of one orange and another orange make two oranges, then if a and b are two classes of oranges using the above formula we have:

$$(a \in 1) \cdot (b \in 1) \equiv [\,(a \vee b) \in 2\,].$$

That is "$1 + 1 = 2$" is a statement of logic about classes of classes in general, whereas "1 orange and 1 orange make 2 oranges" is a statement of logic about classes of classes in particular — not an empirical statement about the world in which they happen to be physical oranges. From this, it follows that what is logically true of classes of classes in general is logically true of classes of classes in particular, that is, classes of oranges, cows, men, or numbers, etc.

II

We mentioned earlier on that quantification in science involved the assigning of numerical values to subjects through the use of rules. We have provided as an example of this the method of counting. It must be stressed that we are not about to suggest that every individual on each occasion that he counts thinks of the rules so as to guide his activity or indeed that each and every individual is even aware of the rules. What is intended, and has been attempted is to provide an analysis (conceptual analysis) of what is involved in counting. One advantage we have all had was that during our childhood we were made to mimic others who know how to count and simply memorised the procedure as we did when learning sums, the multiplication table etc. Perhaps other examples involving the application of quantitative concepts and their rules will aid in the clarification of this point, viz.

a. Determination of temperature (of a biological specimen, of the surface of a distant planet, etc.).
b. Chromosome counting.
c. Determination of specific heat.
d. Determination of density.
e. Determination of the velocity of an object (of light, radio-waves, atomic particles, etc.).

All these involve what can be regarded as standard procedures, i.e., the use of rules. The inescapable conclusion that must be drawn as regards the use of quantitative concepts or measurement is that such concepts are meaningless without rules, i.e., without procedures for measurement. The quantitative concept actually developed out of the process of measuring. The concept of temperature, e.g., exists only because there are thermometers.

Experience or "factual knowledge" is necessary in order to decide which kinds of conventions can be carried out without coming into conflict with the facts of nature and as such various logical structures must be accepted in order to avoid logical inconsistencies."[1] Or to assert the case for logic more forcefully, the use of numerical concepts, i.e., numbers, as values in science implies a structure of logical relations that are not conventional because we cannot abandon them (the relations) without becoming involved in logical contradictions.

Comparative concepts in science are concepts like *lighter* and *heavier*, *warmer* and *cooler*, *lighter* and *darker*, etc. Whereas one might think on first hearing that these concepts play either no role or a very minor role in physical science their application may be used to demonstrate the importance of logical relations in scientific investigation, and the distinction between these concepts and convention. If, for example, we have a pair of objects and wish to determine how they compare in weight—assuming that we are unable to assign numerical values to either, we may appeal to the following rules:

1. If, when placed in separate pans on a balance, the two objects balance each other, then they are of equal weight.
2. If the objects do not balance, the object on the pan that goes down is heavier than the object on the pan that goes up.

Since we are assuming that the quantitative concept of weight has not yet been introduced we may not speak in terms of one object having *greater weight*. We can formalize the above two rules, by which we define the comparative concept of equally heavy, heavier than, and lighter than.

Let E stand for equality of weight, L stand for less than—in weight. We can then define the relations between E and L as follows : If (1) above obtains, the relation E holds between the two objects. If (2) above obtains, the relation L holds between the two objects. From these, the consequences of our two relations would be thus :

a. The two relations apply to all objects that have weight.
b. The reation E must be symmetric, i.e., if it holds between any two objects a and b, it must also hold between b and a. Thus if a has the same weight as b, b also nas the same weight as a.

c. The relation E must be transitive, i.e., if the relation holds between a and b, and b and c, then it holds between a and c.

$$a \rightarrow b$$
$$b \rightarrow c \,/\, \therefore\, a \rightarrow c$$

d. If E is both symmetric and transitive it must be reflexive, i.e., a has the same weight as itself (as a).

e. The relation L is a symmetric, i.e., if a is lighter than b, b cannot be lighter than a.

f. The relation L is transitive, i.e., if a is lighter than b, and b is lighter than c, then a is lighter than c.

What follows from (a) through (f) is this: by means of relation E we can classify all objects into equivalent classes; and by means of relation L we can order the class.

III

The scheme demonstrated here has its roots in the works of Hemple and Carnap[2]. What is being shown is that in defining a class, i.e., in classifying, we can specify any conditions at all, provided that we remain consistent. However, comparative concepts imply a complex structure of logical relations. Once introduced we are not free to reject or modify the structure—we are bound by the logical structure of relations. Comparative concepts in physical science must therefore :

a. correspond with matter of fact conditions; and

b. must conform to a logical structure of relation.

Before attempting an analysis of logical concepts one more example which will demonstrate the three aspects—classificatory comparative and quantitative—of scientific research will be examined. The example selected concerns the periodic table of chemical elements, i.e., the Periodic System. The Periodic System is a theoretical construct formulated by Mendeleev resulting from chemical research beginning from approximately the end of the 17th century. The designation of the end of the 17th century or the beginning of 18th century is by no means arbitrary since before that time scientific attidues, at least, in chemistry, were shrouded with a metaphysical veil. Before then the recognized elements were metaphysical viz., fire, air, earth and sometimes, sulphur, mercury and salt. After this time an empirical attitude prevailed.

The Periodic System is a classificatory system of the chemical elements. It is both conceptual and physical and involves the application of numerical concepts. Because it is a conceptual instrument it can be and has been employed to predict new elements, to predict unrecognised relationships, and serves as a corrective device. It is flexible and extendable, and can yield a variety of interpretations, i.e., the table can be reproduced in many forms.

Very briefly the historical background[3] is as follows : Eighteenth century scientists (chemists) having adopted empirical procedures, endeavoured to isolate *simple substances.* Boyle provided a definition of element but was unable to demonstrate an elementary substance. Lavoisier succeeded. Some of the difficulties facing Boyle and his contemporaries were the adherence to the Phlogiston Theory and the fact that very little was known about chemical reactions (their nature was in fact misunderstood). Some degree of success was achieved, since by the end of the 18th century there were at least 100 simple substances, i.e., substances which could not be further decomposed. Furthermore the collection was by no means an arbitrary one as chemists were to distinguish families of elements having analogous properties. During the early 19th century numerical relations between the 100 elements were introduced; and the use of comparative concepts such as the concept of atomic weight was also introduced (atomic weight is the weight of an element in respect of Hydrogen). This new classification based on comparative and quantitative concepts was clearly a scientific achievement.

Early 18th century chemists were busy classifying simple substances while possessing no definition of element and little or no understanding of chemical reactions. Their classification was primarily based on physical properties. One reason for this was the positivistic attitude, that is to say, empirical evidence became primarily important. It can therefore be suggested that Positivism delayed the *discovery* (invention) of the Periodic System since it delayed Atomic Theory before Dalton. The Periodic System could not be formulated or discovered until specific quantitative relations between the elements had been found. Thus before periodicity could be found with respect to the elements, mathematical terms had to be constructed (or discovered), and mathematical formulations had to be effected.

This historical account can be terminated with the development of atomic theory or more specifically with the Bohr formulation of atomic structure. This last example has been selected to show that the Periodic System is a logical construct based on the three concepts mentioned earlier. The classification at first appeared to be a matter of convention, however, once comparative rules or principles were employed and once numerical concepts were assigned the theory was bound by the logic of relations. The Periodic System shows the relationship between theoretical laws and empirical events as follows :

a. The system is structured such that every element has two attributes, (1) a numerical attribute, i.e., Atomic weight, and (2) a qualitative attribute, i.e., chemical properties—chemical properties are theoretically defined, e.g., valence, exidation, reduction etc;
b. Numerical quantities change in the same direction, very discontinuously, but repetition of property occurs periodically—groups have analogous properties.

IV

Attempts will now be made to show how the relations of logic figure in scientific investigation. We may define logic as follows: Logic is the systematic study of the structure of propositions and of the general conditions of valid inference by a method which abstracts from their content or matter of the propositions and deals only with their logical *form*. This distinction between form and matter is made whenever we distinguish between the logical soundness or validity of a piece of reasoning and the truth of the premises from which it proceeds, and in this sense is familiar in every day usage. However, a precise statement of the distinction must be made with reference to a particular language or systems of notation, a *formalized language*, which shall avoid the inexactness and systematically misleading irregularities of structure and expression that are found in ordinary (colloquial or literary) English and in other natural languages, and shall follow or reproduce the logical form—at the expense, where necessary, of brevity and facility of communication. To adopt a particular formalized language is thus to adopt a particular system or theory of logical analysis. And the formal method may then be characterised by saying that it deals with the objective form of sentences which express propositions, and provides in these concrete terms criteria of meaningfulness, of valid inference, and of other notions closely associated with these[4].

Logic is concerned with the forms of arguments and as such with the principles of valid inference (or simply validity). Validity—the soundness of an argument is associated not so much with the content of an argument or with its conclusion being true,rather it is concerned with the relations of the consequences that hold between the premises and the conclusion. If a conclusion follows from or is a consequence of its premises it (the argument) is said to be valid or correct.

So far the expression logic or logic of relations had been extended to include mathematical relations. Thus we have spoken of numbers as sets of sets (or classes of classes) and characterised the method of counting and addition in terms of set theoretic functions. This has been done with the full knowledge of the view that all of arithmetic (or mathematics) is not reducible to a logical system, i.e., based on Godel's Theorem about consistency in arithmetic. The thesis that mathematics is not logic, however, does not affect the position so far adopted here, since it has been proved that various branches of mathematics can be axiomatised-formulated in a deductive system (such branches as cardinal and ordinal arithmetic can be logically formulated). Thus on every occasion of the mathematical expression, we can substitute the logically parsed expression.

The question now is : How is logic (logic and mathematics) related to reality ? The question may also be formulated as follows :

a. What is the relationship between theory and observation ?
b. What is the relationship between theoretical (?) and applied (?). For example, the relationship between theoretical physics and applied physics; theoretical chemistry and applied chemistry, etc.
c. How can thinking be a source of knowledge ?

The source of difficulty in an attempt to answer these questions is this—a classical empiricist would suggest that all knowledge results from the observation of phenomena. He might also be prone to provide a distinction between appearance and reality because he has come to learn that the senses are sometimes deceptive, for example, a straight stick in water, dreams, eliptical kobo (penny) etc. The strict empiricist furthermore is unable to explain why the laws of logic and mathematics are universally valid—"All Ravens are black—

$(x)\,(Rx \longrightarrow Bx)$

Ra

Therefore Ba

Or, 'Heated bodies expand'.

But the empiricist says that the propositions of logic and mathematics are tautologies (have universal validity) and as such cannot be derived from experience. Since such propositions comprise an important part of our knowledge, the classical empiricist's position seems untenable.

The rationalist on the other hand seeing the problems the empiricist has as regards deception of the senses, claims that we cannot be deceived in our thinking, thus formulates the view that thinking (or reflection) is the source of knowledge. Unfortunately not only have the thoughts and reflections of the rationalist been devious as regards what actually happens in the world but there is often disagreement between individual exponents of the rationalist epistemology.

It seems therefore that either we must develop a new epistemological tradition (phenomenology ?) substantially different from either empiricism and rationalism or we must attempt to explain logical and mathematical propositions using elements of both traditions viz., logical empiricism. A purely epistemological attempt to explain logic and mathematics leads to dualism.[5] An empiricist would argue that laws of science (general and universal propositions) can in fact be refuted given a large number of experiments, but if that were true, what would prevent some one from *discovering* on occasion that $2 + 2 = 5$, or a square having 5 sides ? To avoid this the empiricist must resort to dualism.

The position of the logical empiricist as regards thought and observation in terms asserted by one of its exponents Hans Hahn is : "Thought grasps the most general laws of all being as formulated perhaps in logic and mathematics; observation provides the detailed filling of this framework."[6] A number of problems result from Hahn's point of view :

a. Is geometry a priori or a posteriori ?

b. Is the law of inertia, for example, a priori or a posteriori ?

Likewise the laws of conservation of mass and energy, universal law of attraction of masses, etc. In order to avoid these problems, i.e., that scientific laws must be confirmed empirically, scientists and philosophers have qualified the above view (thought/observation) as follows : The experimental physicist, e.g. provides knowledge of laws of nature by direct observation. The theoretical physicist thereafter changes this knowledge tremendously by thinking, in such a way that we are in a position also to assert propositions about processes that are far from us in space and time and about processes which, on account of their magnitude or minuteness, are not directly observable but which are connected with what is directly observed by the most general laws of being, grasped by thought, the laws of logic and mathematics.[7]

This view cannot withstand criticism since in it is the claim that thinking about propositions which are not about the world adds to our knowledge about the world. How then do we find our way out of the difficulty ? I shall conclude with an attempt to answer this question. The above dilemma appears to stem from a faulty view of logic and mathematics. The view of logic it presupposes is that logic is the account of the most universal properties of things; the account of those properties which are common to all things; as such logic is the science of all things. The view of logic that has been presented and argued for throughout (in this paper) is that logic is not about objects at all but about relations. Logic is concerned with the way we speak about objects. Or to put it another way, laws of logic are *performance rules*.

V

When we operate in strict accordance with performance rules we go through the following stages, among others :

a. Consideration and/or acceptance of an open-rule formula;

b. Consideration and/or acceptance of that specification of this rule-formula which is appropriate;

c. Construction in theory of a plan of operation conforming to the rule and to its specification;

d. Putting the plan into operation.

Let us consider some examples. Suppose I find in a botanical reference book a particular sort of plant classified in terms of colour, form of leaves, shape of blossom, shape of leaves, type of stem, root, etc. Say the plant is referred to scientifically as *helleborus niger* and more commonly as *white rose*. We can then formulate the following true expressions :

a. Every white rose is a helleborus niger. This proposition will always be true and cannot be refuted by observation. The statement is essentially about a convention, i.e., the way we talk about a particular plant. Suppose also we have an object—a ball which is red in colour. Employing two principles of logic, the principle of contradiction and the law of excluded middle, we may make further statements about the ball.

——that it is not the case that is not-red, and

——it is not blue or it is not green, etc.

An object cannot be red (all over), and blue, green, white at the same time. Again these logical propositions say nothing about the object itself (in itself). Logic therefore cannot inform us of the actual colour of the object. What it says, it says about the use of colour concepts, i.e., it stipulates the designation of such concepts. The a priori nature of logic results from the following—because of the two logical principles employed in the last example, we know an object is red all over—at a given time, and we can predict without observing it that :

——it will not be, red and blue or green.

——it will not be, not-red.

b. Iron when heated expands—if iron is heated then it will expand. Statement (b) is quite different from the two previous examples, it is an hypothetical. We can easily note the differences between them (the examples) by looking at their respective contradictions.

——this white rose is not a helleborus niger;

——this ball is red and not-red;

——I heated this piece of iron but it did not expand.

The first two statements are clearly and obviously contradictory and thus present no problems. The last statement, however, would prompt one to put several questions to the agent. Was the sample in fact a piece of iron ? Are you sure that you heated it sufficiently ? Or perhaps you simply did not notice the expansion.

VI

What has been attempted is to show by these seemingly superficial examples that logic, while not being *about* flowers, or red objects or pieces of iron, can provide us with information about such things. Information obtained from an inconsistency is not trivial because we cannot be inconsistent without being wrong about something substantive. We can at this point allow logic to recede into the background and focus more clearly on some aspects of mathematics.

We may often feel that mathematicians in their wild excursions soar to incredible heights of abstraction. However, scientific discovery has provided confirmation that mathematical epistemology is an epistemology that takes into account the objective nature of the physical world. Mathematicians do in fact make real discoveries about the world, and the truths of mathematics can, and must be *tested* in the light of that fact. It is not being suggested that all these truths are tested in the same way as scientific hypotheses. Euclidian geometry, set theory and calculus, parallel line axiom and the axiom of choice, all represent fields in which mathematical thinking had to be revised to accomodate inconsistencies between mathematics and reality. Quantum theory provides another instance in which revision has been deemed necessary, namely by posing special problems relating to the laws of logic — the distributive law and the law of the excluded middle. Mathematical rules, contrary to common belief appear therefore, to depend in some way on the physical world. This is evidenced by the fact that it is through experience that we interpret mathematical results as significant truths about the world.

In a review of Prof. Morris Kline's book *Mathematical Thought from Ancient to Modern Times,* New York 1973, the review rendered an apt description of a most illusive and perhaps most interesting aspect of mathematics:

We search for pattern in a chaotic universe. We look for patterns of a kind we can recognize. Mathematics is the study of those patterns which the human mind can recognize and understand. Any patterns we see in the universe will be one for which a mathematical treatment is possible.

Conversely whenever a new mathematical insight occurs, we are able to recognize new kinds of patterns. If any of these occur in nature, we have a totally unexpected application of the theory. And this is how mathematics gets its power; for a pattern which is hard to recognize in one area may be obvious in another. By taking inspiration from the second we discover the existence of the first[8].

Department of Philosophy, **NKEONYE OTAKPOR**
University of Benin,
Nigeria.

NOTES

1. Carnap, R. *The Philosophical Foundations of Physics*, Chicago, U. P. 1950 p. 68.
2. Ibid. See also Hempel, G. E. *Fundamentals of Concept Formation in Empirical Science*, Chicago, U. P.
3. Spronsen, J. *The Periodic System of Chemical Elements*, Amsterdam. 1969.
4. Alonso Church's Definition quoted in *Mathematical Logic and The Foundations of Mathematics* by Kneebone, G. T., London 1963, p. 6.
5. Dualism here refers to the acceptance of two categories of proposition.
6. Hahn, Hans. "Logic, Mathematics and Knowledge of Nature", in *Logical Positivism*, Ayer A. J., ed., New York 1959.
7. Ibid.
8. Times Literary Supplement, London June 15, 1973, p. 658.