THE LAWS OF THE TRADITIONAL LOGIC AND THE INTERPRETATION – PROBLEM*

The Antilogism Theorem as a test for the validity of the syllogistic moods of the traditional logic worked as a stimulus to analyze the interpretation-problem. It has become a fashion among the formal logicians to maintain that a consistent interpretation of the traditional system cannot be given. To this it has been replied by the opposite camp that a consistent interpretation can be given. I have, in this paper, elucidated and analyzed two soludions offered by Strawson – the Formalistic Solution and the Realistic Solution.

For extraneous purpose of my own, I have divided the paper into four parts. Part I gives the first interpretation and introduces Lord Franklin's Antilogism Theorem, and further shows that this interpretation not only fails to cover all syllogistic moods but even the trditional square of opposition as well as a few eductive inferences are left out. Part II tries to improve upon the first interpretation by giving the second one. And there although some progress is made, the interpretation fails to cover the whole system. In part III, I say that let us not give up hopes. There I give three more interpretations in order to reach the goal. I also examine there, Strawson's Formalistic Solution, and upon analysis, it is found not to be a solution. And lastly, Part IV explains Strawsons Theory of Presupposition which is conceived by him as a Realistic Solution.

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I

I will first interpret the traditional four-fold scheme as follows, and call this interpretation 'I_1'.

$f\ a\ g \rightarrow \sim(\exists x)\ (fx \cdot \sim gx)$ – It reads as : " It is not the case that there is an 'x' which is bath 'f' and 'not-g ."

f *e* g → $\sim(\exists x)(fx \,.\, gx)$ – "It is not the case that there is an 'x' which is both 'f' and 'g'."

f *i* g → $(\exists x)(fx \,.\, gx)$ – "There is an 'x' which is both 'f' and 'g'."

f *o* g → $(\exists x)(fx \,.\, \sim gx)$ – "There is an 'x' which is both 'f' and 'not-g'."

In I_1, the traditional square of opposition will lose its status, and three of the four types of the traditional relations will vanish. The contradictory relation is the only one which will stand in tact in I_1.

Contrary relation does not hold good if 'f' is assumed to be a null class. Traditionally, we say that the truth of 'A' implies the falsity of 'E'. In I_1, it reads as ;

" $\sim(\exists x)(fx \cdot \sim gx) \supset \sim\sim(\exists x)(fx \,.\, gx)$ "

Now if 'f' is a null class, then intuitively it is very clear that the truth of 'A' does not imply the falsity of 'E' inasmuch as the whole of antecedent is true while the consequent carries as its truth-value 'falsity'. Even its demonstration is indubitable. In I_1,

" $\sim(\exists x)\, fx \supset \sim\{\sim(\exists x)(fx \,.\, \sim gx) \supset \sim\sim(\exists x)(fx \,.\, gx)\}$ "

reads as : If 'f' is a null class, then it is not the case that the truth of "All f's are g's" implies the falsity of "No f is g"."

Subcontrary relation also vanishes in I_1 if 'f' is assumed to be a null class. Traditionally, we say that the falsity of 'I' implies the truth of '0'. In I_1 it reads as :

" $\sim(\exists x)(fx \,.\, gx) \supset (\exists x)(fx \,.\, \sim gx)$ "

Now if 'f' is a null class, then intuitively it is very clear that the falsity of 'I' does not imply the truth of '0'. This is because the antecedent gets 'true' truth value while the consequent becomes false. Its demonstration justifies the point.

" $\sim(\exists x)\, fx \supset \sim\{\sim(\exists x)(fx \,.\, gx) \supset (\exists x)(fx \,.\, \sim gx)\}$ "

which reads as : " If 'f' is a null class, then it is not the case that the falsity of " Some f's are g's " implies the truth of " Some f's are not g's " ", can be demonatrated.

Subaltern relations too do not find place in I_1 if ' f ' is an empty class. The traditional expressions are that the universal proposition ' A ' implies the particular proposition ' I ', and similarly, the truth of ' E ' implies that ' O ' is true. In I_1, these expressions read respectively as :

" $\sim (\exists x)(fx . \sim gx) \supset (\exists x)(fx . gx)$ " and

" $\sim (\exists x)(fx . gx) \supset (\exists x)(fx . \sim gx)$ "

Here even if the class ' f ' has no members, then these implications do not hold good. If ' f ' is a null class then it is not the case that the truth of " All f's are g's " implies the truth of " Some f's are g's ", and surely this can be demonstrated.

" $\sim (\exists x) fx \supset \sim \{ \sim (\exists x)(fx . \sim gx) \supset (\exists x)(fx . gx) \}$ "
is a logical truth. So is :

" $\sim (\exists x) fx \supset \sim \{ \sim (\exists x)(fx . gx) \supset (\exists x)(fx . \sim gx \}$ "
And taking the same trend of thought, we can demonstrate that converese and obverted converse of ' A ', partial and full contrapositions of ' E ', partial and full inverses of ' A ', partial and full inverses of ' E ' and the six subaltern eductive inferences do not find a place in I_1.

After having made this clear, I at once pass on to elucidating the Antilogism Theorem. It reads as follows : A syllogistic mood in order that it be valid under I_1, should have its antilogism with the below-mentioned three caaracteristics :

(a) The antilogism should contain two propositions beginning with the denial of existential quantifier and one proposition beginning with the existential quantifier.

(b) In the two propositions beginning with the denial of existential quantifier, there should be a common variable, and if ϕ is that variable, then ϕ should stand negated in one of these two propositions and should remain unnegated in the other.

(c) If ψ and Ω are the remaining variables in the propositions beginning with the denial of existential quantifier, then their occurrences there should be as they are in the proposition beginning with the existential quantifier.

Consider, for instance, Celarent of Figure I, Festino of Figure II, Disamis of Figure III and Camenes of Figure IV, whose antilogisms respectively read as follows :

$$\sim (\exists x)(fx \cdot gx) \cdot \sim (\exists x)(hx \cdot \sim fx) \cdot (\exists x)(hx \cdot gx)$$

$$\sim (\exists x)(fx \cdot gx) \cdot (\exists x)(hx \cdot gx) \cdot \sim (\exists x)(hx \cdot \sim fx)$$

$$(\exists x)(fx \cdot gx) \cdot \sim (\exists x)(fx \cdot \sim hx) \cdot \sim (\exists x)(hx \cdot gx)$$

$$\sim (\exists x)(fx \cdot \sim gx) \cdot \sim (\exists x)(gx \cdot hx) \cdot (\exists x)(hx \cdot fx)$$

It will be noticed that the above antilogisms, selected at random, satisfy the conditions put forth in the Theorem, and thus the corresponding moods are proved valid. Nevertheless, following observations need to be made : (1) Out of the 256 possibilities, 15 moods are proved valid, and the rest proved invalid. (2) Darapti, Felapton, Bramantip, Fesapo and the five subaltern moods which, from the traditional theory, are valid, are proved in I_1 as invalid.

Thus, to recapitulate, in I_1, the square of opposition fares very badly, not all eductive inferences find place, and the above-mentioned syllogistic moods fail to fit themselves.

II

From what has been said, we need not jump to the conclusion that we are left in a chaotic condition. A slight reflection on to the seeming chaos will show that the fragments of the traditional system which are left out, are not arbitrary fragments, but parts which have some common characteristics. It will be noticed that those valid inferences of the traditional scheme in which a proposition which begins with the existential quantifier is inferred from one or two premises which begin with the denial of existential quantifier, turn out to be invalid in I_1. This is very much explicit in the outcasted moods, eductive inferences and the subaltern relations of the square of opposition. But even in Contrary and Sub-contrary relations of the square of opposition, where the abovesaid characteristics do not leap to the eye, upon analysis we find that the conclusion begins with the existential quantifier while the premise begins with the denial of existential quantifier. In Contrary, we say : If " All f's are g's " is true, then it is not the case that " No f is g " is true; (f *a* g) ⊃ ∼ (f *e* g), and the consequent is equivalent to (f *i* g). Again, in Subcontrary relation, we say : Denial of the truth of " Some f's are g's " implies the truth of " Some f's are not g's "; ∼ (f *i* g) ⊃ (f *o* g), and the antecedent is equivalent to (f *e* g). Here too, therefore, we infer a proposition beginning with the existential quantifier from a proposition with the denial of existential quantifier.

Now all these are left out for quite an obvious reason. In I_1, " All f ' s are g ' s " and " No f is g " have been interpreted non-existentially. In " All f ' s are g ' s ", we have denied the existence of an ' x ' having ' f ' and ' not-g '. In " No f is g ", we have denied the existence of an ' x ' having ' f ' and ' g ', but not affirmed or asserted the existence of an ' x ' having ' f '. But so far as " Some f ' s are g ' s " and " Some f ' s are not g ' s " are concerned, they have been interpreted in I_1 existentially. In " Some f's are g's ", we have asserted the existence of an ' x ' having ' f ' and ' g '. In " Some f's are not g's ", we have asserted the existence of an ' x ' having ' f ' and ' not-g '. The reason demands, therefore, another interpretation of the traditional scheme. While retaining the interpretation of ' I ' and ' O ' as given in I_1, we now modify the interpretations of ' A ' and ' E ', interpret them in explicitly existential terms, and call the whole unit I_2.

f *a* g → (∃x) fx · ∼ (∃x) (fx · ∼ gx)— There are a few individuals in the universe who are ' f ' and it is not the case that those individuals fail to be ' g '.

f *e* g → (∃x) fx · ∼ (∃x) (fx . gx)— There are a few individuals in the universe who are ' f ' but it is not the case that those individuals are ' g '.

f i g → (Ex) (fx . gx) – (Reading as in I_1)

f *o* g → (∃x) (fx . ∼ gx) – (Reading as in I_1)

But construing I_2 in this way should not lead any one to believe that the game is over. While trying to gloss over the gaps in I_1, we have succeeded a little bit; nevertheless, we have failed to cover all, and moreover, dug a few different graves. Let us have a bird's eye view :

I *Square of Opposition* : again fares badly. Subcontrary and the two contradictories do not hold good.

II *Eductive Inferences* : The following inferences fail to fit in Converse and Obverted converse of 'E',

Partial and full Contrapositions of 'A', Partial and Full Inverses of 'A', Partial and Full Inverses of 'E', Subaltern of Converse of 'E', Subaltern of Obverted Gonverse of 'E', and Subaltern of Partial and Full Contrapositions of 'A'.

III *Syllogistic Moods* : Camenes of Figure IV and its subaltern mood AEO do not find place in I_2.

Have we made in I_2 any progress ? In I_1, 27 items were left out. In I_2, 17 items are left out. The goal is not yet near.

III

But let us not give up hopes. Let us see why we have failed where we have failed. Consider the Square in I_2. The Contradictories have vanished. But isn't it obvious ? The contradictory of 'A' in I_2 (∃x) fx . ∼ (∃x) (fx . ∼ gx)
is not 'O' of I_2 (∃x) (fx . ∼ gx) but rather
∼ (∃x) fx v (∃x) (fx . ∼ gx). Similarly, the contradictory of 'E' in I_2 (∃x) fx . ∼ (∃x) (fx . gx) is not 'I' of I_2
(∃x) (fx . gx) but rather ∼ (∃x) fx v (∃x) (fx , gx.)
Thus retaining 'A' and 'E' of I_2, let us give this new interpretation to 'I' and 'O', and call the whole unit I_3.

f *a* g → (∃x) fx . ∼ (∃x) (fx . ∼ gx)	(Reading same as in I_2)
f *e* g → (∃x) fx . ∼ (∃x) (fx . gx)	(Reading same as in I_2)
f *i* g → ∼ (∃x) fx v (∃x) (fx . gx)	(Either nothing is 'f' or there is an 'x' which is both 'f' and 'g'.)

f *o* g → ∼ (∃x) fx v (∃x) (fx . ∼ gx) (Either nothing is ‘ f ’ or there is an ‘ x ’ which is both ‘ f ’ and ‘ not–g ’.)

While trying to save the contradictories, we have, in I_3 not only succeeeed in doing so, but also have been able to cover the whole Square. All the six relations hold good in I_3 But so far as the eductive infernnces are concerned, seven of them fail to hold good in I_3. They are Converses of ‘ E ’ and ‘ I ’, Obverted Converse of ‘ E ’, Partial and Full Contrapositions of ‘ A ’, and Partial and Full Contrapositions of ‘ O ’. And we fail to cover Camenes, Dimaris and Fresison also.

Let us check the progress : In I_7, 27 items were left out, in I_2 17 items did not find place, and now in I_3, 10 i ems are thrown out.

It will be interesting to note that I_3 is not the only interpretation which saves the whole of the square. If we bring in a few changes in I_3, then not only we will be able to save the whole Square but also feel elevated for covering all the 24 val d moods. While retaining the interpretations of ‘ A ’ and ‘ O ’ of I_3, we may bring in the interpretations of ‘ E ’ and ‘ I ’ of I_1. Let us call this unit I_4.

f *a* g →(∃x) fx . ∼(∃x) (fx . ∼ gx)

f *e* g → ∼ (∃x) (fx . gx)

f *i* g → (∃x) (fx . gx)

f *o* g → ∼ (∃x) fx v (∃x) (fx . ∼ gx)

Thus so far as the Square and the Moocs are concerned, we have made a good progress. Nevertheless, we have regressed in so far as the total number of items kicked out are concnrned. The frollowing thirteen educative inferences are not covered by I_4 : Obverses of ‘ A ’ and ‘ O ’, Obverted Converse of ‘ E ’, Full contra positions of ‘ A ’ and ‘ E ’, Partial and Full Contrapositions of ‘ O ’, Full Inverse of ‘ A ’, Partial and Full Ieverses of ‘ E ’, Subaltern of Obverse of ‘ E ′, Subaltern of Obverse of ‘ E ’ and Subaltern of Full Contraposition of ‘ A ’.

For a final break-through, let us go back to I_3 and see why the converse of 'E' failed. In I_3, 'E' reads as : $(\exists x)\ fx \,.\, \sim(\exists x)(fx \,.\, gx)$. And this leaves open the possibility of 'g' being a null class In other words, the denial of existence of an 'x' belonging to 'g' is consistent with this scheme. And it should be consistent with the simple converse of this scheme if we are to say that "No f is g" implies "No g is f". But in fact, it is not consistent. $\sim(\exists x)\ gx$ is not consistent with $(\exists x)\ gx \,.\, \sim(\exists x)(gx \,.\, fx)$. Hence Converse of 'E' in I_3 fails. But let us repair the damage. 'E' may be interpreted as $(\exists x)\ fx \,.\, (\exists x)\ gx \,.\, \sim(\exists x)(fx \,.\, gx)$. And this will imply its simple converse :

$(\exists x)\ gx \,.\, (\exists x)\ fx \,.\, \sim(\exists x)(gx \,.\, fx)$.

Now 'I', the contradictory of 'E' will have to be obviously modified. The negation of 'E' as now interpreted will yield the following :

$\sim(\exists x)\ fx \vee \sim(\exists x)\ gx \vee (\exists x)(fx \,.\, gx)$.

Following this trend of thought, we interpret 'A' as follows :

$(\exists x)\ fx \,.\, (\exists x) \sim gx \,.\, \sim(\exists x)(fx \,.\, \sim gx)$.

And when we negate this, the contradictory will yield '0' which will read as :

$\sim(\exists x)\ fx \vee \sim(\exists x) \sim gx \vee (\exists x)(fx \,.\, \sim gx)$.

Thus our I_5 is ready :

$f\, a\, g \rightarrow (\exists x)\ fx \,.\, (\exists x) \sim gx \,.\, \sim(\exists x)(fx \,.\, \sim gx)$

$f\, e\, g \rightarrow (\exists x)\ fx \,.\, (\exists x)\ gx \,.\, \sim(\exists x)(fx \,.\, gx)$

$f\, i\, g \rightarrow \sim(\exists x)\ fx \vee \sim(\exists x)\ gx \vee (\exists x)(fx \,.\, gx)$

$f\, o\, g \rightarrow \sim(\exists x)\ fx \vee \sim(\exists x) \sim gx \vee (\exists x)(fx \,.\, \sim gx)$

I_5 covers all the six relations of the Square of Opposition, all the twenty-six eductive inferences, and all the twenty-four syllogistic moods. Seeing this, Strawson says : "For this interpretation, all the laws of the traditional logic hold good together." This he regards as the formalistic solution which according to him, overthrows the orthodox criticism hurled by formal logicians at the traditional system.

It would have been better if before making this remark, Strawson had realized that the traditional Law of Identity is very much in existence and is hostile to I_5. Perhaps someone argues : " But 'A' in 'A is A' surely is a variable standing for an individual and not for a term and hence cannot be within the domain of syllogistic formulation ! " To this it has been replied that even if the scheme " All f 's are f 's " is rejected as a formulation of the Law af Identity, the fact remains that it is well formed. Being a well-formed formula, it should be covered by the formalistic solution. But there is a difference between 'should' and 'is'. The formalistic solution should cover it but in fact, does not; and hence I_5 ceases to be a solution. " All f's are f 's " will read in I_5 as :

" $(\exists x)\, fx \,.\, (\exists x) \sim fx. \sim (\exists x)\,(fx. \sim fx)$ "

Something is 'f' and something is not 'f' but nothing is both 'f' and 'not-f'. This sounds pretty good and intelligible. We say, something in this universe is blue and something is not blue, but nothing is both blue and not blue. But now let 'f' stand for " barren woman's son. " Surely the first conjunct is false ! '$(\exists x)\, fx$' is not true. And now let 'f' stand for " identical with itself ". Here the second, $(\exists x) \sim fx$', is false.

The assertion " There is a barren woman's son " makes " All barren woman's sons are barren woman's sons " hostile to I_5. And so does the assertion " There is something which is not identical with itself " in the case of " All things which are identical with themselves are identical with themselves ". Thus, we say, " 'x' is blue " has an instance and a counter instance; " Barren woman's son " has a counter instance but does not have an instance; " Identical with itself " has an instance but does not have a counter instance. " All f's are f's " will work in I_5 only for values of 'f' having an instance as well as a counter instance. I_5 fails.

IV

Suppose we overlook the fact that the Law of Identity is hostile to I_5, and then ask the question : " Have we achieved something valuable ? " So far as the playing with symbols and polishing the mind by giving it some exercise is concerned, we will say : " well done ". But is it valuable in the given context ? Strawson himself admits that the interpretation is " a kind of adhoc patching up of the old system in order to represent it, in its entirety, as a fragment of the new." But further we ask: "Has this 'patching up' paid us anything or have we paid a price for it?" The answer lies in affirming the second alternant. While trying to bring in consistency, we have done a great injustice to Aristotle and his followers. "Either nothing is a flower or nothing is white or something is both a flower and white." Surely Aristotle would scratch his head if he hears that we are made to believe that this is the meaning of his 'I' form of proposition—"Some flowers are white"! But this is what I_5 does! The cases of f *a* g, f *e* g, and f *o* g are no better. What I_5 does is that it enables an individual to convert a good, simple English into a bad, complicated one.

Let us see if we can trace the trouble-shooter. We began to weave the web when I_1 failed. And the reason which we gave for its failure was that some propositions are interpreted existentially and some are not done so. Hence to repair the damage, we said that time, the interpretation should be "in explicitly existential terms". And this is the root cause of the present problem. If we stick to those words, all that we finally get is a 'patch-up' and that too not fully pervasive. But, on the other hand, if we free ourselves from these words, then a new light might show us the right path.

Suppose Avinash says: "All Sucharita's philosophy books are stolen away". Immediately we say two things: Avinash would not have, under normal circumstances, made this remark if he knew that Sucharita did not have any philosophy books. If he knew that Sucharita did not have any philosophy bnoks, and yet made this remark, then the utterance, under normal circumstances, is improper. If he has a mistaken belief about Sucharita's possessing philosophy books (i. e. suppose in fact she did not possess any philosophy books) and if we ask whether what Avinash says is true or false, then a problem arises. The truth-value cannot be true. Yet, at the same time, to label it as false will also be very much misleading. But a slight reflection onto it makes us think that if Sucharita did not possess any philosophy books, then the question about her philosophy books having been stolen away does not arise! Let us check it with reference to I_1: $\sim (\exists x)(Sx . \sim Tx)$. It reads as : " It is not the case that there is an ' x ' which is Sucharita's philosophy book and which is not stolen away ". Now here if Sucharita did not in fact have any philosophy books, even then this non-existence will be sufficient to make the remark of Avinash true ! Let us check the same with reference to I_5 :

$(\exists x)\, Sx \,.\, (\exists x) \sim Tx \,.\, \sim (\exists x)\, (Sx \,.\sim Tx)$. Here the said non-existence will be sufficient enough to falsify what Avinash says ! But under these circumstances (viz., (1) Sucharita did not in fact possess any philosophy books and (2) Avinash remraked " All Sucharita's philosophy books are stolen away "), damad for a truth value for what Avinash says is not appealing to a reflective mind. And equally revolting will be the mind if someone attempts to assign a truth value. Reflection rather demands that we should say : If she did not possess any philosophy books and if Avinash made that remark, then the question of truth or falsity about what Avinash says does not arise. We should add : The existence of Sucharita's philosophy books is a " necessary precondition " for an intelligible assignment of a truth value to what Avinash says. Thus when we interpret the traditional four statements, the interpretation should follow this trend of thought viz., the statements, in order ta carry a truth value, should have members in the subject class. Or, in other words, the person (like Avinash) while making his statement, commits himself to the existence of members in the subject class (like Sucharita's philosophy books).

Strawson, at this stage, makes a distinction between ' Entailment ' and ' Presupposition '. And this follows from two sorts of logical absurdities. Schematically, Strawson's view is this : There is a statement ' X ' and there is a statement ' Y '. Now if ' Y ' is a necessary condition for the *truth* of ' X ' and if we assert ' X ' and ' not–Y ' in the same breath, then there will be one kind of logical absurdity. But now conceive another set of statements. There is a statement ' P ' and there is a statement ' Q '. Further suppose that ' Q ' is a necessary precondition for the *truth–value* of ' P '. That is, we cannot raise the question as to whether 'P' is true or false unless ' Q ' is true. We cannot intelligibly assign a truth value to ' P ' unless ' Q ' is true. If this be the case, and if we assert in the same breath ' P ' and ' not–Q ', then we will again have a logical absurdity. But the point which Strawson wants to draw home is that this second absurdity is different in kind from the first absurdity. The first is a "straight forward self-contradiction" but the second is not so. The relation between 'X' and 'Y' is that of Entailment. The relation between 'P' and 'Q' is that of Presupposition. 'X' entails 'Y' but 'P' presupposes 'Q'. "This is a pink flower" if conjoined with "This is not coloured" leads to a straightforward self-contradiction because "This is a pink flower" entails "This is coloured". "All Sucharita's philosophy books are stolen away" and "Sucharita did not possess any philosophy books" cannot be conjuncts of the same statement because "All Sucharita's philosophy books are stolen away" presupposes "Sucharita possessed philosophy books". Therefore, to interpret "f *a* g" as done in I_2 and then try to bring in consistency among the laws of the traditional logic will be improper, and will only lead us to a 'patch up' and that too, as earlier pointed out, not fully pervasive. If, on the other hand, we interpret the traditional propositions in keeping with the thesis of Presupposition, then, says Strawson, it will be a realistic solution.

Whether or not Strawson's theory of Presupposition works as a solution, has been for a long period, a live problem–the Russellian thinkers considering the relation between "All Sucharita's philosophy books are stolen away" and "Sucharita possessed philosophy books" as that of entailment while the camp of proponents trying io justify Strawson's stand, and it is this controversy that this paper now leads to.

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