ANALYSIS OF GENERAL PROPOSITIONS*

The importance of being clear about the meaning, or correct analysis, of general propositions cannot be exaggered. This has been recognised since antiquity by formal logicians. But, as I argued at length in my paper "Approaches to the Paradox of Confirmation" (*Ajatus* 1972) and shall discuss briefly in what follows, the need for being clear about the semantics of general propositions is no less urgent in the theory of confirmation. It requires very little reflection to see that we cannot tell what kind of logical consequence a general proposition will have, or what kind of evidence will confirm it, if we do not know what it means.

It should be pointed out first that in my discussion today I shall confine myself to only those qualified or restricted general propositions which were examined in details by the traditional logicians and which include particular as well as universal propositions. These propositions are, in short, the A, E, I, and O propositions of traditional logic.

Let us consider, to begin with, the view of the traditional logicians themselves. According to them, as we all know, general propositions, both universal and particular, are categorical. In fact, some of the things which are said by these logicians may even suggest that they are the only categorical proposition, that are there. There are, the dogma goes, four kinds of categorical proposition : Universal Affirmative, Universal Negative, Particular Affirmative, and Particular Negative. But what is a *categorical* proposition ? No definition in the strict sense seems available; though the idea of a categorical proposition appears to be that of

* This is a slightly revised version of the Presidential Address for Logic and Scientific Method Section at the 48th Session of Indian Philosophical Congress held in Allahabad in December, 1974.

a proposition which involves unconditional assertion, a proposition which is amenable to a subject-predicate analysis, and in which a certain predicate is directly and unconditionally, ascribed to a certain subject. Thus what is typically contrasted with a categorical proposition is a hypothetical proposition which involves a conditional predication only.

An important feature of a categorical proposition is that it has existential import with respect to its subject term. Ignoring for the time being the ambivalent attitude which is taken by the traditional logicians towards it, we can take a ***singular*** subject-predicate proposition to bring out this point about categorical propositions. The proposition " Socrates is mortal " has existential import with respect to its subject term " Socrates " in the sense that it cannot be true if Socrates does not exist. The proposition has existential import in this sense because when we assert the proposition we refer, by use of the term " Socrates ", to a certain individual in order to say something about it, by attaching a predicate to the term. Now, if Socrates does not exist, I cannot refer to any individual by use of the term " Socrates ", and, if I fail to refer to an individual, I cannot succeed in saying anything true about the individual either. A slightly different and more succinct way of putting the matter is as follows : When I assert " Socrates is mortal ", I want to say something true about Socrates, but, if Socrates does not exist, I cannot say anything true ***about*** him. Now, what is true of " Sccrates is mortal " will be true of the general propositions also, if the latter are construed as categorical. If general propositions are categorical, then, if we assert any of the propositions,

(A) All men are mortal,

(E) No men are mortal,

(I) Some men are mortal,

and (O) Some men are not mortal,

We refer to human beings by use of the term " man ", and say about them, that each one of them is mortal in the first proposition, that none of them is mortal in the second, that some of them are, and, that some of them are not, mortal in the third and fourth respectively, Consequently, if there are no human beings, there is nothing which any of these proposition can be true about, and, therefore, none of them is true.

It is important to note that there are (at least) two different interpretations of the doctrine of existential import. According to the orthodox interpreation, a categorical proposition has existential import with respect to the subject term in the sense that it ***implies*** existence of objects answering to it. According to the other interpretation, which is associated with the name of Strawson, but perhaps goes back to some of Frege's ideas, a categorical proposition has existential import in the sense that it ***presupposes*** this existence. Strawson explains the difference between implication and presupposition of existence by saying that, while in the first case existence of objects answering to the subject term is a necessary condition of the ***truth only***, in the second case, this existence is a necessary condition of ***both truth and falsity*** of the proposition in question. One consequence of this difference is that a nonfulfilment of the existence--condition would render a categorical proposition ***false*** under the first interpretation, but would render it ***neither true nor false*** under the second. (See P. F. Strawson, ***Introduction to Logical Theory***, Chapter VI., Part III., particularly Sections 7 and 8.) Another consequence — the importance of this consequence we shall see later—is that we can ***infer*** the existance of objects answering to the subject term from a categorical proposition if the first interpretation, but ***not*** if the second interpretation, is correct.

An advantage of the view that all general propositions are categorical is that it enabled the traditional logicians to set up some neat relationship between the four kinds of general proposition in their doctrine of Opposition of Propositions. If both the universal and particular propositions are categorical, they both have existential import, and, in that case, the universal affirmative entails the corresponding particular affirmative and the universal negative, the corresponding particular negative, and the relation of sub-alternation is secured. Again, if " all " is taken to mean " every " and " some ", to mean at " at least one ", the relation of contradiction would obtain between the universal affirmative and the corresponding particular negative on the one hand, and between the universal negative and the corresponding particular affirmative on the other. And, if the relations of sub-alternation and contradiction obtain, then the two other relations, viz. the relation of contrariety between the universal affirmative and the corresponding universal negative, and that of sub--contrariety between the particular affirmative and the corresponding particular negative, must also obtain.

We can consider next the view of (the overwhelming majority of) modern symbolic logicians. According to this view, a universal proposition is a universal material conditional. So the universal affirmative " All men are mortal " can be formulated as

$$(x)\ (x \text{ is a man} \supset x \text{ is mortal}),$$

and the universal negative " No men are mortal " as

$$(x)\ (x \text{ is a man} \supset \sim x \text{ is mortal}).$$

Since a particular affirmative is the contradictory of the corresponding universal negative, the proposition " Some men are mortal " can be formulated as

$$\sim (x)\ (x \text{ is a man} \supset \sim x \text{ is mortal});$$

but this negative formula is logically equivalent with

$$(Ex)\ (x \text{ is a man} \ \& \ x \text{ is mortal}).$$

And, since a particular negative is the contradictory of the corresponding universal affirmative, the propostion " Some men are not mortal " can be formulated as

$$\sim (x)\ (x \text{ is a man} \supset x \text{ is mortal});$$

but *this* negative formula, in its turn, is logically equivalent with

$$(Ex)\ (x \text{ is a man} \ \&\ \sim x \text{ is mortal}).$$

This view of the nature of general propositions has been developed in more or less conscious opposition to the traditional view, and some of the circumstances which have led to this are as follows :

(a) J. S. Mill recommended that both the subject and the predicate terms be taken in ***connotation*** in a general proposition.

(b) F. H. Bradley argued forcefully against the traditional view that universal propositions were categorical and in support of the view that they were hypothetical. He also formulated the basic objection to the categorical interpretation which is that a universal proposition, not to be confused with such concealed collections of ***singular*** propositions as " All planets of the sun move in an ellipse ", does ***not*** have any existential import because it is ***not*** falsified by non-existence of objects answering to the subject term. (One of Bradley's favourite examples is " All tresspassers are prosecuted ", but a better example would be " All bodies free from bacteria are free from diseases ".)

(c) Bertrand Russell picked up from Bradley the idea that universal propositions were conditional, but saw clearly that they must be ***universal*** conditionals, and explicitly identified the conditional with Frege's ***truth-functional*** conditional, calling it " material implication ", and derived the consequences which this interpretation of universal propositions inevitably has for the particular.

(d) Gottlob Frege himself did the same thing with general propositions in his logic, though without any explicit reference to the traditional theory which he was clearly undermining.

(e) Franz Brentano gave an explicitly ***existential*** analysis o general propositions. He maintained that a particular proposition always ***asserts*** the existence of objects of a certain kind, while a universal proposition ***denies*** the existence of objects of a certain kind. To take an example, when we assert " Some men are mortal " we assert the existence of individuals which are both human and mortal, and when we assert " Some men are not mortal " we assert the existence of individuals which are human but not mortal "; but when we assert " No men are mortal " we deny the existence of individuals which are both human and mortal, and when we assert " All men are mortal " we deny the existence of individuals which are humon but not martal.

(f) Geroge Boole formulated general propositions in his class-theoretical terminology which represented a particular proposition as asserting the non-emptiness of the product of one class with another class or its complementary, and a universial proposition as asserting the emptiness of the same. (This formulation enabled John Venn to give a diagrammatic representation of the general propcsitions with overlapping circles.)

There is much to recommend the theory that a universal proposition is a universal material conditional and that a particular propcsition is the negation of such a conditional. The greatest attraction of this theory, particularly for those who care for a systematic development of logic, is that it enables us to construct the logic of general propositions on the foundation of truth-functional logic which happens to be one of the finest and most solid logical systems. But the theory also raises a large number o questions. I shall discuss only a few of them.

In the first place, the theory invalidates a large part of traditional logic. Since, according to this theory, the universal proposition dces not have any existent:al import (it rather denies existence) but the particular proposition does have it (as it ***asserts*** the same), the universal proposition does not entail the particular, and hence sub–alternation dces not hold. But if sub-alternation fails, so do contrariety and sub-contrariety; and, with them, conversion *per accidents* and the so–called weakened syllogisms. This dces seem to an unbiased mind to be a deficiency of the theory, because it does seem that traditional logic codifies well at least some uses of propositions like " All men are mortal ", " Some men are mortal ", etc.

In the second place, the theory of general propositions which we are considering gives some counter–intuitive results. I shall mention two of them which are not unconnected with one another. The first of these counter- intuitive results is that the non-existence of objects answering to the subject term is *sufficient for the truth* of a universal proposition. Thus, to take a random example " All dragons can climb trees " is true simply because there are no dragons. (Note that it was *not* necessary to maintain this in order to be able to give a non–categorical, or conditional, analysis of a universal propcsition. What is demanded by such an analysis is only that the non–existence of objects answering to the subject term should not be sufficient for making the universal proposition untrue (i. e. either false or neither true nor false). There is indeed a difference between saying that something is not sufficient to make a proposition untrue and saying that it is sufficient to make the proposition true.)

The second counter-intuitive consequence of theory, that a general proposition is either pcsitively or negatively existential, which I want to discuss, shows itself in confirmation theory. Consider the following argument :

1. The existence of a white handkerchief confirms the hypothesis " All non-black things are non-ravens. "
2. For any evidence e and any pair of hypotheses h_1 and h_2, if e confirms h_1, and h_1 is logically equivalent with h_2, then e also confirms h_2.

3. But the hypothesis " All non-black things are non-ravens " is logically equivalent with the hypothesis " All ravens are black ".
4. Therefore, the existence of a white handkerchief confirms the hypothtsis " All ravens are black ".

(This argument generates what is known as the ' ravan paradox. ') The conclusion of this argument is counter-intuitive, for the existence of a white handkerchief does seem to be entirely irrelevant to the hypothesis " All ravens are black ". But how could we arrive at such a counter-intuitive and paradoxical conclusion when the argument itself is obviously perfectly valid? There must be something wrong about one or more of the premises of the paradox generating argument even although each of them seems to be true. The first of the premises restes on what is known as the Instance Theory of Confirmation, the second simply formulates Hempel's Equivalence Condition, and the third is just an example of the Law of Contraposition. As I argued at length elsewhere (*op. cit.*), the Instance Theory of Confirmation seems to be entirely unexceptionable, and so the paradoxical conclusion must be due to something wrong either with Hempel's Equivalence Condition or with the Law of Contraposition. I think that it is due to both. But there cannot be anything wrong with the contraposition in the third premise of the paradox generating argument if the universal hypotheses mentioned in them are both universal material conditionals, for the very simple reason that the Law of Contraposition *is* valid for material conditionals.

One may say at this point that it would not do to give up the existential interpretation if one wants to avoid accepting the Law of Contraposition, for the law is valid in the traditional logic of general propositions as well. The answer to this is two-fold. First, it is ***not*** necessary to take the traditional interpretation of general propositions when one gives up the existential because there are other alternatives as well. Second, it is in fact extremely difficult to preserve the validity of contraposition on the traditional interpretation, although this is not usually recognised, I shall discuss the second point first.

On the traditional view, " All ravens are black " is committed to the existence of ravens, but does not seem to be committed to the existence of non-black things at all; and, on the other hand, " All non-black things are non-ravens " is committed to the existence of non-black things, but does not seem to be committed to the existence of ravens (i. e. non-ravens) at all. So the two propositions do not seem to be equivalent, and contraposition seems to fail on the traditional view. One way in which we may try to save contraposition is as follows : We disallow the occurrence of universal terms, i. e., terms which are true of everything, as we have already disallowed empty terms, i. e., terms which are false of everything, in order to save sub-alternation etc. [This latter is done whenever the existential import of the general proposition is made secure.] This strategy may also receive support from some unexpected quarters. Many recent philosophers have argued in some such way as that if every coin is counterfeit then it does not make any sense to say about any particular coin that it is counterfeit, if every perception is illusory, and if everything is uncertain then it does not make any sense to say about any particular thing that it is uncertain. So if there is to be any significant predication with " counterfeit ", " illusory " and " uncertain " there must, respectively, be things which are genuine, veridical and certain. According to this principle, which underlies the well-known Argument from Contrast, there must be non-ravens and non-black things if the terms " raven " and " black " are to be used in significant predication. And if that is so, contraposition remains valid. I may argue the point at some other place, but for the present I only dogmatically assert that this defence of the Law of Contraposition is not ultimately tenable.

Let me turn now to the question whether it is necessary to accept the traditional view of general propositions if the existential interpretation fails. My answer to this question is that it is not necessary to do so, for there is at least one other interpretation which is possible. I shall now try to explain what that other interpretation is.

Let us begin with the universal proposition. According to traditional logic, a universal proposition is a kind of categorical proposition, and, according to the modern standard predicate logic, it is a universal material conditional. The third view which I want to present falls just in between the two. According to this, a universal proposition is a universal proposition is a universal ***non-material*** conditional. Let me explain, by taking a cue from W. V. Quine, what this non-material conditional is like. In describing our " usual attitude " towards a conditional affirmation of the form " If p then q " Quine says :

> If, after we have made such an affirmation, the antecedent turns out true, then we consider ourselves committed to the consequent, and are ready to acknowledge error if it proves false. If on the other hand the the antecedent turns out to have been false our conditional affirmation is *as if it had **never been** made.* (*Methods of Logic,* 1st edn., p. 12; italics mine)

But if an affirmation, if it is fit, had never been made then the question of truth or falsity cannot arise with regard to such an affirmation. So we get the result that a conditional, in one of the possible attitudes-may be the ***usual*** attitude as Quine considers it to be, is true or false only if its antecedent is true, true and false according as the consequent is true, true and false according as the consequent is true and false, and neither true nor false-we shall call it " indeterminate "-if the antecedent is false, or itself indeterminate. Using the letters " T ", " F ", and " I " as abbreviations of " True ", " False ", and " Indeterminate " respectively, and " * " as the sign of the conditional, we can present the semantics of the non-material conditional of which Quine speaks, and which Quine speaks, and which I have in mind, in the following table :

p	q	p*q
T	T	T
T	I	I
T	F	F
I	T	I
I	I	I
I	F	I
F	T	I
F	I	I
F	F	I

It is important to note the following points about this table :

(a) " I " does not stond for a ***truth***-value; it only stands for a ***failure*** of truth-value.

(b) Consequently, " p*q " is *not* a truth-function.

(c) And, for the same reason, the table is not an ordinary truth-table.

(d) Although, in a very important sense, the truth of the antecedent, as well as of the consequent, is a necessary condition of the truth of the conditional, it cannot be said that the conditional implies, i. e. ***entails***, either its antecedent or its consequent.

For, one truth about entailment is that if the entailed proposition is false then the entailing proposition is also false, but a look at the table reveals that there are circumstances under which " p " or " q " is false but " p * q " is not false, but only ***indeterminate***. This fact really gives edge to Strawson's dictum (***op. cit.***) that is a necessary condition both for the truth and for the falsity of a proposition cannot be said to be implied, i. e. entailed, by it.

We can now formulate the universal propositions in terms of this non-material conditional. We can formulate the universal affirmative proposition " All men are mortal " as

(x) (x is a man * x is mortal),

and the universal negative proposition " No men are mortal " as

(x) (x is a man * ∼ x is mortal).

After having formulated the universal propositions we can proceed to formulate the particulars. We shall naturally be guided by the idea that a particular affirmative is the contradictory of the corresponding universal negative, and a particular negative is the contradictory of the corresponding universal affirmative. So we can formulate " Some men are mortal " as

∼ (x) (x is a man * ∼ is mortal).

But this is equivalent with

(Ex) ∼ (x is a man * ∼ x is mortal),

and this latter, again, with

(Ex) (x is a man * x is mortal).

To explain this last equivalence, we have to introduce first the table for negation, and, then, by help of this and the table for the conditional, establish the equivalence between " ∼ (p * q) " and " p * ∼ q ". It is, I think, intuitively clear that the table for negation will be as follows :

p	∼ p	∼ ∼ p
T	F	T
I	I	I
F	T	F

It is now a matter of simple calculation to see that " ∼ (p * q) " and " p * ∼ q " are equivalent with one another, as well as that " ∼ (x is a man ∗ ∼ x is mortal) " and " x is a man ∗ x is mortal " are so, for the law of double negation continues to hold in this logic as is obvious from a comparison of the respective columns for " p " and " ∼ ∼ p " in the table for negation.

It is not difficult at all to see now that " Some men are not mortal " can be formulated as

(Ex) (x is a man ∗ ∼ x is mortal).

Abbreviating " is a man " and " is mortal " to " H " and " M " respectively, and using these predicate letters in the usual way, we can set out in a compact form our formulations of the four general propositions as follows :

(A) (x) (Hx ∗ Mx)
(E) (x) (Hx ∗ ∼ Mx)
(I) (Ex) (Hx ∗ Mx)
(O) (Ex) (Hx ∗ ∼ Mx)

I can now explain why I am interested in this interpretation of general propositions. One of the reasons–there may be many–is that this interpretation invalidates contraposition, and the simple reason why it does so is that contraposition is not valid for the kind of conditionals which this interpretation involves. This can be easily brought out by the following tables :

p q	p∗q	∼q∗∼p
T T	T	F I F
T I	I	I I F
T F	F	T F F
I T	I	F I I
I I	I	I I I
I F	I	T I I
F T	I	F I T
F I	I	I I T
F F	I	T T T

Since the tables for " p*q " and " ∼q*∼p " are not the same, they are not equivalent. Now, if contraposition is not valid for this interpretation of general propositions, we can now solve the raven paradox by maintaining that the general propositions involved in the paradox generating argument are open only to this kind of interpretation and, hence, not subject to contraposition.

This new interpreation not only invalidates the Law of Contraposition but also many other laws which hold in both traditional and modern standard predicate logic. The most important of them are perhaps Modus Tollens and the Laws of Conversion. But, on the other hand, it restores the whole of the traditional Square of Opposition. It is obvious that a particular proposition, in this interpretation, can be deduced from the corresponding universal by simple applications of the rules of Universal Instantiation and Existential Generalisation, which must be valid in any logic which can codify the logic of " all " and " some " at all. This secures sub-alternation; and we have taken care, in our very formulation of the propositions, to make universal affirmative and particular negative contradictories of one another, and the remaining pair or general propositions to be the same. How, if these two relations hold, the other relations in the square of Opposition must also do.

I want to conclude by pointing out what happens to existential import. What happens is simply fascinating. If we take existential import in the sense of ***implication***, then ***neither*** the universal ***nor*** the particular has any existential import; but if we take existential import in the sense of ***presupposition***, then ***both*** the universal ***and*** the particular have existential import. (Note that sub-alternation broke down because there was ***disparity*** between the universal and particular in respect of existential import.) And this is so ultimately for the very simple reason that the truth of " p*q " presupposes but does not entail the truth of " p ".

I have discussed here three different analyses of general propositions. I do not want to maintain that any of them is correct while the other two are not. It is quite possible that each of them is correct in its own sphere of application, and that there are others which are correct in theirs.

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