A NOTE ON TRUTH—POSSIBILITIES

A black and white (silent) television film is made up of a number of separate pictures, each picture of a number of lines, and each line of a number of light and dark spots – light where an electron hits the screen, dark where none does. Patterns of black and white spots make shades of grey.

Multiplying these three numbers – number of pictures x lines per picture x spots per line – gives the total number of spots, white or black, in the film – say ' n '. We can give a name or number to each spot position and each picture, and now, by saying of each spot during the film whether it is white or not, we can *describe the film completely.*

This would be only one of 2^n possible arrangements of white and black spots or possible ' films ' of the same duration (most of which would look like nothing on earth).

Now imagine for the sake of argument that the whole history of the universe is like such a television film, with one time and three space dimensions. So its size and duration, though huge, would be finite; and it would not be infinitesimal but consist of a finite though enormous number (call it ' n ' again) of space–time positions (perhaps millions in the space of an electron).

And suppose each such position – each place at any one moment – is in one of two possible states : let us call these states ' positive ' and ' negative ' (whatever that may mean); and whatever happens in the universe – eveny phenomenon or event (movements of stars or atoms, colours, sounds, . .) – corresponds to some space–time pattern of positive and negative positions.

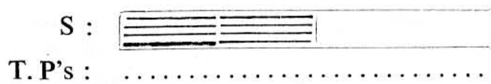
Such a universe could, in theory, be described *completely* throughout all time, by saying of each space–time position if it was positive or not. There would be 2^n possible histories of the universe – ' possible worlds ' or *Truth Possibilities.* Only one of these would be the true description of the actual universe.

(Finite complete description could also be given, in special cases, for an infinite or infinitesimal universe, where the boundaries of positive and negative regions are given by mathematical formulae and general rules.)

What statements could be made ?

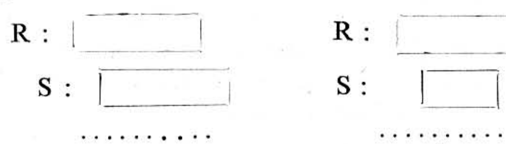
Let us represent the whole set of Truth Possibilities by a dotted line : each point on the line indicates one T.P. and *one* of these pointed represents the whole truth.

The oblong, ' s ', shows a completely defined statement which is *compatible* with the possibilities under its *white* part, *incompatible* with those under its *black* part.



(For simplicity the T.P's compatible with S are here put the right of all the incompatible ones. Otherwise S might show a complicated alternating pattern of black and white.)

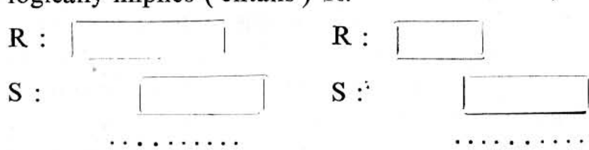
Possible logical relations of statements can be shown with such a diagram. Assume black where white is not outlined.)



(*a*) (*b*)

(*a*) R and S are logically independent. They may both be true or both false, or either true and the other false.

(*b*) S logically implies (entails) R.



(*c*) (*d*)

(*c*) R is the contradictory of S.

(*d*) R and S are incompatibls : " $\overline{RS}$ " is logically true.

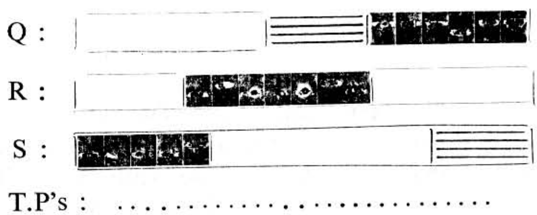
(*e*) (*f*)

(*e*) S is logically true : a ' tautology '.

(*f*) S is logically false : a ' contradiction '.

A complete description—a truth possibility—is expressed by an oblong white at one point, the rest black.

For simplicity's sake I took all statements above as completely defined, but there can be possibilities for which the truth value of some statements is determined and of other not. Uncertain areas can be shown in lined & Quares.



Here R is completely defined while Q and S, having doubtful areas are not.

On the line of T.P.'s the postition of the one that is actully true lies under the white areas of all true statements and the black areas of all false statements. A statement shown grey at this point connot be said to be true or false—it has no meaning here.

But can we know anything of such ' absolute ' possibilities ? Are there some, not dreamt of in our philosophy, for which the truth value of *no* statement of our language is determined ? Perhaps it is safe to ignore them : certainly they are *indiscribable*. If one of these were actual, actual our whole language would be useless. We should need to build a new language from scratch, whoever ' we ' might be in such a situation.

We can evade these problems by abandoning our model of the universe and approaching truth possibilities from the statements possible in *our* language. Two possiblites are distinguished if and only if we can make statements true of one false of the other. A *truth possibility* is what entails *all* statements logically compatible with it within a given language or set of statements (with all their truth functions). Life will be still simpler if we ignore, for now, ill-defined statements (with ' grey ' area)).

We cannot now claim that a truth possibility gives a complete description of the universe, which would almost certainly not be possible in, say. English–even if the universe were finite.

We can still use the same kind of diagrams to show logical relations of statements. And each statement can be defined by listing all the compatible truth possibilies (or all the incompatible ones), provided their number is finite.

One point of all this was to introduce the notion of *synonymy*.

R :

S :

T.P'S : ....

Here R and S are the same statement. Two sentences are *synonymous*, however different their wording or the symbols used in them, if they are compatible with the same truth possibilities and incompatible with the same truth possibilities; that is if they express the same statement. Synonymous sentences are formally equivalent; each entails the other; in no conceivable circumstances could one be true and the other false.

Many sentences are *variable* : I mean that different utterances (usually spoken or written) of the same sentence, in different contexts may express different statements. So we can speak of synonymous *utterances* of sentences. And, in a slightly modified sense, we can say synonymous sentences can express the same statement as each other in *any* context.

(I define a *sentence* by the words symbls in it and their arrangement; a *statement* by what makes it true. So a sentence may vary its truth-value with context; a statement, in my sense, does not.)

I symbolize " is a synonymous with " by the sign " $\approx$ ". This sign is used to show a relation between *expressions*—symbols or sets of symbols. For instance : " Somebody doesn't like cheese $\approx$ Not everybody likes cheese " means the same as (or : is synonymous with) " Somebody doesn't like cheese " is synonymous with " Not everybody likes cheese ". So quotation marks are to be understood round the expressions on each side of the sign " $\approx$ ".

Any two expressions are synonymous if (and only if) in any sentence containing one of these expressions this can be replaced by the other expression producting (in the same context) a sentence synonymous with the first.

Thus : bachelor $\approx$ unmarried man.

Since, e. g. : My only son is a bachelor $\approx$ My only son is an unmarried man. The qualification about context is important; here the parent of the son must be the same, and roughbly the same time meant–since a bachelor may marry in time.

Are some ideas *logically simpler* than others ? Are axioms different from other logical truths or a definition from what is defined ? If so, how ?

In the model universe, if *a* is one of the space–time positions, " *a* is positive " is an elementary proposition in the Tractatus sense. It is logically independent of the other elementary propositions: it can be true or false while all the rest remain thesame. And all statements are truth functions of the elementary propositions.

Suppose there are four ' elementary propositions ' or independent statements, *p*, *q*, *r* and *s*.

		q		~q	
		p	~p	p	~p
s	r	1	2	3	4
	~ r	5	6	7	8
~ s	r	9	10	11	12
	~ r	13	14	15	16

		u		~u	
		t	~t	t	~t
w	v	11	1	8	13
	~ v	2	16	10	6
~ w	v	9	5	3	7
	~v	15	4	12	14

fig. (i) fig. (ii)

In fig, (i) the small numbered squares represent the Truth Possibilities. Truth Possibility No. 1 = D6, *p, q, r, s.* T. P. No. 6 = D6. *p, q, r, s,* and so on. Now each argument (*p. q.* etc.) denies half of the total number (here 16) of T.P.'s – e. g. *p* denies the eight even–numbered ones. And *each denies half of the T.P.'s denied by any other given argument* (end half those not so denied) – so *q* denies No. 4, 8, 12 and 16 (i. e. half those denied by *p*) besides Nos. 3, 7, 11 and 15 (half those not denied by *p*). *This is a necessary and sufficient* condition for the logical independence of the arguments.

Next, using the 16 Truth Possibilities, we can define a quite defferent set (in fact many such sets) of independent arguments, such as *t, u. v* and *w* in flg. (ii). So, e. g., *w* is defined here as; LV 2 V 6 V 8 V 10 V 11 V 13 V 16. Here again each of *t, u, v* and *w denies half of the truth possibilities and half of those denied by any other argument of the set* – i. e. they are *independent.* And any truth function of *p, q, r* and *s* (including *p, q, r* and *s* themselves) can be expressed as a truth function of *t, u, v* and *w.*

Is there any logical reason for regarding p, q, r and s as more ' elementary ' than t, u, v and w, or some other set ? (There might be good non–logical reasons – for instance nataral laws such as those of physics might be easier to state with one set than another. For instance a law : p ≡ q. V r V s: p V q V∼r could be expressed more simply as: v V w.)

Taking a simpler case, suppose that p≡,q≢r is a logical truth (that is : p ≈ q ≢ r). Here any two of the three, p, q, r, can be taken as ' elementary ' and used to define the third. There are four truth possibilities.

P	q	r		
				P ≈ 1 V 2
T	T	F	1	
T	F	T	2	q ≈ 1 V 3
T	T	T	3	
F	F	F	4	r ≈ 2 V 3

Logically, the three are on exactly the same level and the choice of any two as ' elementary ' is quite arbitrary.

In general a set of independent arguments has a *unique* set of 2^n (exhaustive and naturally exclusive) truth possibilities, just one being actually time. But corresponding to 2^n truth possibilities there are *many* ' elementary ' sets of independent arguments.

There are in fact 2^n ! ways of distributing the truth possibilities through the squares (Cf. figs. (i) and (ii). But simply interchanging arguments say p with q – does not give new sets : it is just changing the names. This can be done in n ! ways :

This is the number of different sets of n independent statements, each set with the same lot of 2^n truth possibilities.

$2^n ! \div n !$ sets are left. And if we also disregard sets formed by switching arguments with their negations – P with *vp* and so on. We can further divide by 2^n, giving the formula

$$2^n ! \, (n ! \times 2^n) \text{ or } \frac{(2^n - 1) !}{n !}$$

For this reason it seems natural to regard the unique set of truth possibilities as the hypothetical foundation of a language at any rate one with a finite number of different statements– other than some arbitrary set of ' elementary ' independent statements. But in practice it is, of course, much more convenient to give your definition in terms of a set of n statements than a set of 2^n.

If it is truth possibilities that are fundamental, why should we suppose that the number of them in the (finite) universe or language is an exact power of 2 ? If, say, there are more than 8 but fewer than 16, some of the 16 squares in the diagrams would have to be left blank. Combinations of truth values of the arguments eorresponding to these blauk squares would be *logically impossible*.

The negations of these combinations would be logical truths (axioms or theorems) Generally, these are between 2^n and $2^n + 1$ truth possibilities then all statements can be given as truth functions of some set of $n + 1$ arguments, but because of these axioms the arguments would not now all be independent.

Richards