LOGICAL SPACE IN THE TRACTATUS

Like so much else in the *Tractatus*, Wittgenstein's remarks about logical space have been variously interpreted by the commentators. Alternative and sometimes conflicting accounts in the literature are suggestive of a certain obscurity or abstruseness in Wittgenstein's idea. In this paper, we criticise a number of these accounts, and argue that, so far as the conception of logical space is concerned, the difficulties are rather to be found in the commentaries than in the *Tractatus* itself. We believe that there is a simple interpretation which suffices to unify and explain all of Wittgenstein's remarks about logical space. Our account has much in common with what Stenius says in chapter four, parts eight and nine, of his *Wittgenstein's Tractatus* (Blackwell, 1960). We know of no discussion of logical space which adds anything of substance to Stenius' account, and, as we shall show, other commentaries have introduced confusions or interpretations which are not supported by the text.[1]

The line of interpratation which we believe to be the most seriously mistaken is that which sees Wittgenstein's notion as suggesting a parallel between an elementary sentence and a point in a co-ordinate system and comparing the names which it contains with single co-ordinate numbers. An interpretation along these lines is offered by Griffin,[2] and is also to be found among those offered by Black.[3] Although there are passges in the *Tractatus*, and in the *Notebooks*, which might superficially suggest that single co-ordinate numbers are to be compared to names, we believe this interpretation to be too facile, and will attempt to show that Griffin and Black have failed to make out a convincing case for it.

Here is what Griffin says about logical space :

> The essence of the metaphor, is I think, comparison of a sentence with a point in a co-ordinate system and so names with single co-ordinate numbers. In a given co-ordinate system putting two numbers together defines a point; in a given language putting two names together makes a statement. In this way, languages are a kind of logical

co-ordinate system. And as there are different co-ordinate systems as a result of choosing different point of origin, different scales, and so on, there are different representational forms in language. Now, from these roots the rest of the metaphor grows. If two names are of forms permitting combination, their combination is logically all right; it would be as impossible for a conjunction of names to contradict logic as for a pair of co-ordinates to contradict geometry. A conjunction will always determine a 'logical place' ('logischer Ort'). Further more, as in geometry to specify one set of co-ordinates involves the whole apparatus of grid, point of origin, and so on, so the determination of one logical place brings along with it a whole symbolism with all its rules and operations—or, as Wittgenstein puts it, the logical scaffolding will already be given by it. There is a logical place corresponding to every state of affairs.

In footnotes to this passage, Griffin refers to sections of both the *Notebooks* and the *Tractatus*. But even in the case of the *Notebooks*, Griffin's argument is unconvincing. For instance, Griffin doesn't explain what corresponds to the "different points of origin, different scales, and so forth" and gives rise to the "different representational forms in language", and it isn't obvious what they would be on his account, since names are not ordered or systematically interrelated as numbers are in a co-ordinate system. Now this comment about "different respresentational forms" is supported by reference to *Notebooks* 30.10.14. But what is the problem that Wittgenstein is actually discussing at this point in the *Notebooks*? Wittgenstein begins the entry by considering whether we could say that "$\phi(x)$" in "$\sim\dot{\phi}(x)$" images how things are *not* i.e., whether the tilde indicates that what follows represents not how things are but how things are not. He is considering whether the tilde introduces a different method of representation, and he goes on to ask what it is that is "really characteristic of the relation of *representing*" given that there are these different ways of giving a representation. He wonders whether we might not say: "It's just that there are different logical co-ordinate systems!", and conclude that what is characteristic is that every representation "can be right or wrong, true or false". If the co-ordinates are the names why should Wittgenstein have essayed an answer to his question here in terms of logical co-ordination ?

In the *Tractatus*, Wittgenstein is quite explicit that a proposition and its negation determine different logical places (4.0641), and therefore they must have different co-ordinates. Yet on Griffin's interpretation, this would lead to inconsistency since the negation will contain exactly those names which occur in the proposition negated and thus on this view ought to have the *same* logical co-ordinates.

Indeed, it appears that when Griffin says that there is a logical place corresponding to every state of affairs, he has it in mind that the correspondence is one-one as in the geometrical case. Yet while it is true that every state of affairs (*Sachverhalt*) corresponds to a place in logical space, it cannot be the case that every place in logical space corresponds to a *Sachverhalt*. For an elementary proposition, on Wittgenstein's view, asserts the existence of a *Sachverhalt* and its negation asserts the non-existence of the very same *Sachverhalt*. An elementary proposition and its negation can be said to refer to the one *Sachverhalt*. So if there is one logical place to every *Sachverhalt*, it could not be the case that a negative proposition determines a logical place different from that determined by the proposition it negates which is precisely what Wittgenstein asserts to be the case at 4.0641.

Griffin's difficulties here are a symptom of a more general problem for him over non-elementary propositions. Griffin writes as though Wittgenstein's discussion of logical space related to elementary propositions alone. But this is evidently false. We have just considered the evidence of 4.0641 : the negation of an elementary proposition is not itself an elementary proposition, and yet it does determine a logical place. Then, there is the concern, in 4.463, with the question how tautology and contradiction relate to logical space. The manner in which 4.463 relates the truth-conditions of a proposition to the notion of logical space shows clearly that the discussion of this notion is not concerned solely, or even primarily, with the elementary proposition. (Of course, it doesn't exclude the elementary proposition : compare Wittgenstein's parenthetical remark in *Notebooks* 30.10.14 that what he says about the relation of representing naturally holds for the elementary proposition too !)

Is there any reason to suppose that in the 3.4's (or in *Notebooks* 1.11.14r-t) Wittgenstein's references to propositions are implicitly restricted to elementary propositions ? The only apparent reason is that Wittgenstein here speaks of the existence of the logical place as " guaranteed by the mere existence of the constituents ". But this is not a reason for restricting his use of " proposition ", for the possibilities represented by compound propositions no less than the possibilities represented by elementary propositions are guaranteed by the existence of their constituents. Wittgenstein's point is not that the constituents uniquely determine the proposition, (as Black claims[4]) but rather that the possible logical combinations, which will include their " immediate combinations ", but also truth functional combinations of these, are already " given ", are already possibilities, once the constituents are given.

A related difficulty for Griffin's account would be how to explain what Wittgenstein means when he says, at 4.463, that " a tautology leaves open to reality the whole.... of logical space: a contradiction fills the whole of logical space.... ". If a place in logical space corresponds to a *Sachverhalt*, how can it be said that the conjunction of a proposition which asserts that Sachverhalt's existence and one which asserts its non-existence (i.e., a contradition) fills *the whole* of logical space ? Nothing in Griffin's model is suggestive of Wittgenstein's distinction of positive and negative senses of the proposition; nor is it clear how a proposition can be the analogue of a point and yet be thought of as leaving open some part of space or being like a body which partly ' fills ' a space.

Griffin does attempt to give some explanation of Wittgenstein's remarks in 3.42, but the explanation offered is quite inadequate. It is true that the determination of one logical place will bring along with it " a whole symbolism with all its rules and operations ", but Wittgenstein means something more than this when he says that a proposition " reaches through " the whole of logical space. And it is the whole of logical space which " will already be given by it ", not the " logical scaffolding " as Griffin has it. These are arguably distinct notions, since one is said to determine the other; and we shall suggest later that what Wittgenstein has in mind is that whereas we might think of the construction of a proposition as involving only a certain amount of actual " scaffolding ", it always involves the whole of logical space.

If we turn to Black's *Companion*, we find a view similar to Griffin's stated in the " General Introduction "[5] and developed in the Commentary, particularly in section 23 : " The Proposition as a Point in Logical Space ", which is the commentary on the 3.4's.[6] Alongside this, however, we find another account of logical space very different from the first and bearing some relation to that of Stenius, from whom Black quotes on page 157, and to whom he refers the reader on pages 158, 234 and elsewhere. This second account comes to the forefront in the commentary on those passages which we saw presented special difficulty for Griffin, specifically 4.0641 and 4.463. Yet Black makes no attempt to reconcile the two accounts, and apart from a suggestion on page 157 that they are " alternative " ways of using the analogy, writes as though he were presenting a single coherent interpretation.

Thus, on page 155 (discussing the 43.4's) the analogue of a point is an elementary proposition, on page 233 (discussing 4.463) it is " a conjunction in which each elementary proposition or its negation occurs "—i.e. what Wittgenstein had called a set of " truth-possibilities of elementary propositions ". Or again, on page 184, commenting on 4.0641, Black writes : " If the proposition considered is not elementary, it would ' determine ' a region in logical space (rather than a ' place ' i.e., a point). The positive and the corresponding negative proposition might then be thought of as drawing a boundary around the *same* region of logical space." But 4.0641 is not restricted to the case where the negated proposition is itself compound. (Of course this holds for the elementary proposition too !) Indeed, 4.0641(4) goes on to say that the negated proposition can be negated again, and as Black points out in his comment on this, " negation always works in the same way." In his commentary on 4.0641 (1) Black adds that in the *Notebooks* (9.11.14) " Wittgenstein compares p and ~p, respectively, to a picture and the infinite plane outside the picture." If we turn to the *Notebooks* we find that the entry continues " I can construct the infinite space outside only by using the picture to bound that space." I couldn't do that, though, if the picture was a *point*. Black says that an elementary proposition determines a 'place', which is a point, but that a compound proposition determines a *region* of space; but this view cannot be worked out consistently with the text, not merely because it cannot deal with the relationship between the negated proposition and the negating proposition when the former is elementary, but for the very simple reason that 4.0641 says quite explicitly of one kind of compound proposition, viz. a negation, that *it*, like the proposition negated, determines a logical place.

The shortcomings of Black's account are already evident in his discussion of the 3.4's. In his commentary on 3.41 he says that "the word 'co-ordinate' can be viewed as synonym of *Zuordnung*—in 2.1514, 2.1515". Accordingly, though no argument has been given for the identification, he takes the logical co-ordinates to be the links between the elements of the sign and the objects for which they stand, and gives a gloss on 3.41: "The propositional sign, together with the meanings assigned to its constituents—that is what we mean by a 'logical place'". Yet on the very next page Black proceeds to raise the idea of the co-ordinates of a proposition being given by its representation as a truth-function of the elementary propositions; and the reading which takes names to be the co-ordinates becomes "an alternative way of using the analogy."

The commentary on 3.411 consists of a quotation from Stenius, a remark to the effect that "something can exist in it" means that some state of affairs can exist in a place; and a literal rendering of the sentence. No attempt is make to relate the quotation from Stenius to the commentary on 3.41. With regard to 3.42, logical places are again identified with possible states of affairs, and the account of what Wittgenstein means by saying that a proposition gives the whole of logical space is consequently inadequate (for compound propositions containing p as a truth-functional component must determine places in logical space no less than p does if p's giving the whole of logical space is to be relevant to the logical relations of these compounds and p). For the effect of negation on the "logical place" we are referred to 4.0641, Black's comment on which we have already remarked on above. On "logical scaffolding" Black refers us to its two other occurrences, and his explication of this notion is confined to a remark that the scaffolding of the world is the same as the logical form of the world.[7] For comment of " reaches through the whole of logical space ", we are referred to Anscombe's *Introduction*. So let us turn to that.

Anscombe's account of logical space proceeds via a discussion of her own spatial illustration, which purportedly educidates many of Wittgenstein's remarks, though its precise relationship to his own analogy is never made clear. Anscombe constructs her spatial illustration in the following passages:

> If you consider an island marked on the surface of a sphere, it is clear that it defines not merely its own shape, but the shape of the rest of the surface. A proposition is to be compared to such an island, its negation to the rest of the surface.
>
> Let us say that you illustrate the concept of truth by painting the island white and the rest of the surface black, to correspond to calling a proposition true and its negation false; if on the other hand, it is the negation that is true, the island is black and the rest of the surface white. Obviously you could do this with a real globe; and any map, real or imaginary, would divide the globe. Only.... the divisions would not necessarily correspond to any actual coastlines. But the division made by the two senses of any proposition is a division of truth from falsehood; each coastline partitions the whole earth's surface, so each proposition ' reaches through the whole of the logical space. '. But it *is* a proposition precisely by making a division of true from false. Now let us represent the proposition saying that *either* this *or* that is ture, by a new globe with both the corresponding areas white; what corresponds to saying that either a proposition or its negation is true is painting the *whole* surface of the globe white—in which case you have no map. And similarly for painting the whole surface black, which would correspond to ' not (p or not p) '. But it is clear that an all-white or all-black globe is not a map.[8]

Anscombe's account has the appearance of illustrating Wittgenstein's ideas, but closer analysis reveals that it is superficial and unilluminating. Anscombe admits that her own analogy "goes lame", and she implies that this is true too of Wittgenstein's analogy of logical space; but the lameness to which she refers (that "one *is* saying something about the globe if one says that either this or that representation of it is true", since a globe with an island will retain the shape of a coastline if land and sea replace one another), is due to the peculiar features of her own illustration, and in fact provides a reason for not thinking of the way in which a proposition determines reality in terms of coastlines or actual divisions between the kinds of things which exist in space. One can say that Anscombe's analogy doesn't actually go lame : it is rather a congenital cripple.

In Anscombe's illustration, one area—the island—represents "p", and the remainder of the surface represents "$\sim$p". "A proposition is to be compared to such an island, its negation to the rest of the surface." But Wittgenstein's claim that a proposition determines a place in logical space suggests the possibility of a more illuminating spatial representation of propositions than this. The notion of logical space is intended to show something about the relations between propositions, and not merely about the relation between a proposition and its negation. But does Anscombe's discussion even illuminate the matter of negation ?

It ought to be possible, on Wittgenstein's theory, to use a spatial diagram in place of, or rather *as*, a proposition. And if the coastline in Anscombe's illustration is marked, not by say a black line on a white background, but by one region being painted black and the other white, then we have two possible diagrams with a coastline, one of which can be assigned to "p", the other to "$\sim$p". This seems to be Anscombe's intention, since a globe which consisted of a black line representing the coastline of an island marked on an otherwise uniformly white surface would not be a "perfect and absolute blank", and therefore couldn't be what Anscombe has in mind when she speaks of painting both areas white. However, Anscombe's exposition is far from clear, and she writes of areas of the globe's being white variously as representing a proposition ("....let us represent the proposition saying that *either* this *or* that is true...."), as corresponding to *calling* a proposition true, and even (in one apparently careless formulation) as corresponding to a proposition's *being* true (" ....if on the other hand it is the negation that is true,.... ").

Given two contiguous areas, each of which can be painted either black or white, we have of course four possibilities, only two of which as we said have a " coastline "—i.e., two distinct areas. On what seems to us to be the most plausible reading, Anscombe's illustration involves identifying one black and white globe with " p ", the other black and white globe with " ~p ", the all-white globe with the tautology, and all-black globe with the tautology, and all-black globe with the contradiction. (We say that this is the most plausible reading, partly for the reason that the concept of logical space has to do with the logical relations between propositions, whether or not they are affirmed (cf. 4.064) and whether or not they are true (cf. 4.023); though it clearly doesn't explain why Anscombe should say that her analogy illustrates the concept of truth.)

Now it would seem that *any* such division of a space could serve equally well to " illustrate " some of the things Wittgenstein says about negation—e.g. that " p " and " ~p " could be used to say what the other says (4.0621). Does Anscombe's model offer anything more than this ? If it does it must permit the representation of more than just a proposition and its negation. However, if we wish to consider two elementary propositions, it will not do to have a sphere with two islands on its surface. For then what would correspond to calling one of the propositions true and its negation false would be painting one of the propositions true and its negation false would be painting one island white and rest of the surface black, which would mean that if one affirms the truth of one proposition one affirms the falsity of every other proposition. It might seem that we could have two spheres, and compare one proposition with an island on one sphere, another proposition with an island on the other sphere. But either that leaves us with *two* logical spaces, or we must still be able to mark both islands on the surface of *one* sphere. Alternatively, if we take the sphere itself with the island painted white and the rest of the surface black, as standing in place of the proposition, with the painted areas thought of as corresponding to its positive and negative senses, then of course we would have distinct " maps " for distinct propositions " p " and " q ", but again they must have a framework in common, and relate to further " maps " corresponding to " p and q " " p or q ", etc. Since a globe with two islands marked on it contains only three distinct areas, it is logically impossible to produce black and white " Maps " of this kind for all the possible propositions. But in that case, if conventions are used to represent negation and either conjunction or disjunction, they must break down when systematically applied. So the use of such conventions could only *appear* to elucidate the concepts of tautology and contradiction.

Even with regard to the passages in the *Tractatus* that Anscombe discusses then, her claims exaggerate the illustrative power of her analogy and fail to spell out its departures from Wittgenstein's own " similar but double analogy ". Again in relation to the discussion of tautology and contradiction, it isn't at all clear why in Anscombe's illustration the all-white globe should leave open to reality the whole of logical space, whereas the all-black globe fills the whole of logical space. At 4.463 Wittgenstein says that a proposition in the negative sense is like a solid body that excludes other bodies from the space that it occupies, and that in the positive sense it is " like a space bounded by solid substances in which there is room for a body ". Anscombe argues from that this that " since any proposition p divides the whole space, then the positive proposition ' p or not p ' leaves the whole space empty, both the island indicated by p and the rest of the space; and its negative ' not (p or not p) ' blocks the whole space."

Since we could have represented the contradition and the autology by " p and not p " and its negative " not (p and not p) " respectively it is clear that Wittgenstein's claims about the positive character of tautologies and the negative character of contradictions have nothing particularly to do with the fact that a tautology might actually be represented by an " affirmative proposition " (a disjunction) and a contradiction by an explicitly " negative proposition ". What Wittgenstein means by his claim that propositions have both a positive and a negative sense is explicated by the spatial analogy, which helps us to see why it is that tautology and contradiction are limiting cases or not really propositions : that tautology " leaves the whole space empty " and does not have a negative sense, while contradiction " blocks the whole space " and does not have a positive sense. It would be a misunderstanding of Wittgenstein's thought here to take him to be presenting an argument to the conclusion that tautology leaves the whole of the space empty and contradiction fills the whole of the space, from a set of premises which included the statement that a tautology is a positive proposition and a contradiction is a negative proposition.

An attempt to overcome some of the difficulties we have found in Anscombe's discussion, by providing for the representation of any finite number of states of affairs, can be found in David Favrholdt's *Interpretation and Critique of Wittgenstein's Tractatus*. But Favrholdt's attempt fails, and introduces additional confusions. He begins his discussion of logical space as follows :

A model of the logical space may look like this :

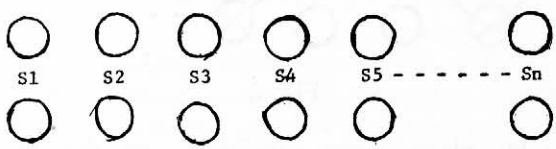

Fig. 1

Here S1, S2, S3 .... etc. each indicate a state of affairs and the series S1, S2, S3 .... Sn consequently indicates the totality of all states of affairs. The two circles that are placed above and below each state of affair mark the possibilities of existence and non-existence of the states of affairs. Thus, the existence of a state of affairs can be indicated by :

and the non-existence correspondingly :

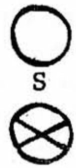

Fig. 1 can now be viewed as a ' logical space that gives place for all possible realities. For instance, we may imagine that S1, S3, and S4 exist while S2 and S5 do not exist, and correspondingly we can imagine that the further states of affairs in the series either exist or do not exist up to Sn which in this case does exist. The ' reality ' which is hereby ' created ', can then be represented by a line :

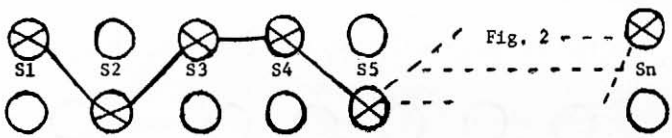

Fig. 2

Favrholdt places three stipulations on this model : All states of affairs must be marked as either existing or not existing; no marking of the existence or non-existence of one state of affairs determines the value of any other, and no state of affairs may be marked as both existing and not existing.

Favrholdt confidently asserts that, with these stipulations his model " meets all the demands of the *Tractatus* concerning " logical space ".[9] He proceeds to examine certain passages of the *Tractatus*, which by implication contain all the demands concerning logical space. He begins with 2.11 and 2.202, which tell us only that a picture presents a situation in logical space, this possible situation being " a possibility of existence and non-existence of states of affairs " (2.201). This may be consistent with Favrholdt s model, though he appears implausibly to take Wittgenstein's " existence and non-existence of states of affairs " as meaning the " existence of a state of affairs or its non-existence ". The next " demands " considered are those in the 3.4's and immediately Favrholdt has to admit that on his account 3.42 " remains obscure ". 3.42 consists of several remarks, the first of which is that a proposition can determine only one place in logical space. On this remark, Favrholdt comments : " It is clear, however, that if, in our model, we mean by a logical place, a vertical column, a proposition always determines only one place."[10] The error of equating logical places with *Sachverhalten* is one that we have already exposed in our discussion of Griffin, where we pointed out that this equation contradicts 4.0641.

Curiously enough, while Favrholdt ignores 4.0641 at this point, he does refer to it a few pages further on in the course of a discussion on the need to distinguish " p is false " from " ~p "—a distinction which in Favrholdt's view, requires a distinction between two kinds of falsity. Now, although Wittegenstein " never succeeded in solving " the problem of how we are able to distinguish between the two expressions ' p is false ' and ' ~p ',[11] he " seems to have noticed that he ought to speak of two kinds of falsity, since he writes in 4.0641 : ' The negating proposition determines a logical place *different* from that of the negated proposition. The negating proposition determines a logical place with the help of the logical place of the negated proposition. For it describes it as lying outside the latter's logical place '. But it remains a problem how he could think that this passage could be reconciled with his remarks on ' logical space '."[12]

Now a certain perversity is apparent here. By implication Favrholdt is aware that 4.0641 makes a " demand " concerning logical space. And he is aware that it contradicts the account of logical space he has given. Yet 4.0641 is not mentioned when Favrholdt is attempting to reconcile the *Tractatus* demands on the notion of logical space with his own account of it. But the inference he draws here is that Wittgenstein's notion is (or "seems" ?) inconsistent, not that his own account of it may be in error.

Having said that a proposition determines only one place in logical space, Wittgenstein adds in 3.42 that " nevertheless the whole of logical space must already be given by it ". Favrholdt's comment on this is that " it is evident that any place must be fixed in relation to other places, i.e. to the logical space as a whole, as the word 'place' otherwise loses its meaning. Therefore, a proposition establishes the whole of logical space in establishing a place in it."[13]

Favrholdt's diagram is, inevitably, a spatial one, so each "place" on it stands in a fixed spatial relation to every other place. But how does it follow from this that each proposition establishes the whole of logical space in establishing a place in it ? The spatial feature in question is one which belongs to *every* spatial diagram, but it isn't any diagram that will illustrate the logical feature that Wittgenstein is pointing to in 3.42. Indeed, Favrholdt has debarred himself from giving any more meaningful explanation by his having taken too narrow a view of the purpose of Wittgenstein's analogy. "The only reason", he says, for introducing logical space, which "can be conceived as a sort of ontological duplicate of the logical theory of the independence and true-false-poles of elementary propositions", is "that it enables us to deal with false propositions in the same way as with true or to make only one theory of meaning comprising both categories of propositions."[14] On the remainder of 3.42 Favrholdt admits that he is stumped, but we are expected to share his confidence that none of it contradicts the model. However, in one of these remarks Wittgenstein attempts to provide a *reason* why a proposition "gives" or "reaches through" the whole of logical space, and in the other he introduces the notion of "logical scaffolding". Favrholdt gives no account of this, or of the related notion of "logical co-ordinates" introduced in 3.41; and there seems to be nothing in his model which could correspond to them.

Favrholdt next discusses the "demand" expressed by 4.463, which deals with tautology and contradiction. We would contend that it is a condition of adequacy for a model of logical space that it should *show* that as Wittgenstein says here, a tautology leaves the whole space empty, and that a contradiction fills the whole space "leaving no point of it for reality". Can Favrholdt's model show this ? Clearly, it can do so only if the tautology and contradiction can be represented in it. But they cannot, at any rate without a good deal of outside assistance from Favrholdt.

Thus Favrholdt writes : "Since any proposition, according to the extensional view of logic, is implied by a contradiction, it is clear that if it is possible to assert a contradiction at all, then one can maintain that any circle in fig. 1 can contain an X. In this way the whole of logical space would be filled and no place would be left for reality. On the other hand, the assertion of the tautologies is the assertion of all the possibilities indicated by fig. 1."

Here we have Wittgenstein turned completely on his head, with " the extensional view of logic " elucidating the model that was supposed to elucidate it. But the logical contrivance cannot really engage with the model, which is equipped to represent nothing more than the existence or alternatively the non-existence of each state of affairs. It is still incapable of representing the tautology and contradiction, because it has no machinery to display the operations of disjunction and conjunction. So when Favrholdt speaks of putting a cross in every circle of his figure, this doesn't correspond to a conjunction, but rather to each state of affair's both existing and not existing, which is why Favrholdt had stipulated on p. 44 that it mustn't be done, i.e., that no vertical lines should occur in the figure. Perhaps we have the explanation here of why Favrholdt hedges on the possibility of asserting a contradiction, though he seems not to be worried about the assertion of tautologies.

In Favrholdt's model, a line joining crossed circles was to represent a " reality " so that even if we do allow that we could fill the whole space by putting a cross in every circle, there would still be a difficulty with Wittgenstein's remark that a contradiction leaves no place for reality. And even if the assertion of the tautologies is " the assertion of all the possibilities indicated in fig. 1 ", how does this correspond to leaving all the circles, and thus presumably the whole space, empty ? In Wittgenstein's discussion in 4.463, these remarks about tautology and contradiction are linked to, indeed they emanate from, a thesis about positive and negative senses of the proposition, which Wittgenstein explains in terms of the logical space metaphor. The irrelevance of Favrholdt's model is evident from the fact that it completely loses this connection. We conclude that the demands of the *Tractatus* are already too much for it.

In yet another mistaken account of Wittgenstein's concept of logical space, James Morrison's *Meaning and Truth in Wittgenstein's Tractatus* (Mouton, 1968), identifies Wittgenstein's logical places with possible worlds. Any logically possible combination of the positive and negative values (existence and non-existence) of all *Sachverhalten* is a possible world : we might say that a possible world is an ontological analogue of a single truth possibility of (ξ), the complete set of elementary propositions. For a world of three *Sachverhalten* P_1, P_2, P_3 there would be 2^3 or 8 possible worlds. Morrison says that for such a world there would be 2^3 or 8 logical places. This is unacceptable since it is not the case that for every proposition there is a possible world to which the proposition bears a relation it bears to no other possible world, and which may be identified with that of determining a logical place. In fact, Morrison evidently arrives at his view by misinterpreting Stenius. On page 98 Morrison says :

> ...." .... the individual ' places ' of the logical space are places of possible worlds (or facts) ". For a world of three states of affairs there would be 2^3, or 8 logical places.

The sentence in inverted commas comes from Stenius, p. 54, and Morrison appends a reference to Stenius, p. 55 to the second sentence. Morrison thus writes as if his own account was the same as that given by Stenius. But it is not. Stenius does not hold that for a world of three *Sachverhalten* there would be 8 logical places. The sentence Morrison quotes says not that logical places are possible worlds, but that they are ' places of possible worlds '. Later Stenius defines the logical place of a proposition p, as the class of possible worlds consistent with p : only in special cases will this be a singleton class. Logical space Stenius characterizes as a ' space of possible worlds '. Briefly, logical space consists of possible worlds each distinct set of which consitutes a distinct logical place.

We are in basic agreement with Stenius' characterizations of logical space and logical place. However, in our own account we add an explanation of Wittgenstein's notion of " logical co-ordinates ", and suggest what we think is a more perspicuous diagrammatic representation of Wittgenstein's ideas. On page 54 Stenius presents the following diagram of logical space :

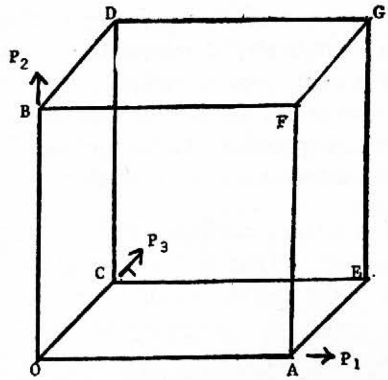

Fig. 3

The lines $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, we are told, correspond to states of affairs and the points O, A, B, C, D, E, F, G correspond to possible worlds. Denoting the non-existence of p_i by $\mathbf{p}_i$ the possible worlds corresponding to the points O – G are : O ($\mathbf{p}_1$ $\mathbf{p}_2$ $\mathbf{p}_3$); A (p_1 $\mathbf{p}_2$ $\mathbf{p}_3$); B ($\mathbf{p}_1$ p_2 $\mathbf{p}_3$); C ($\mathbf{p}_1$ $\mathbf{p}_2$ p_3); D ($\mathbf{p}_1$ p_2 p_3); E (p_1 $\mathbf{p}_2$ p_3); F (p_1 p_2 $\mathbf{p}_3$); G (p_1 p_2 p_3). Stenius calls the expressions in the parentheses ' logical coordinates of a possible world " p. 54).

In our own diagram, each of the lines P_1, P_2, P_3, corresponds to a *Sachverhalt*, but we have areas rather than points corresponding to possible worlds :

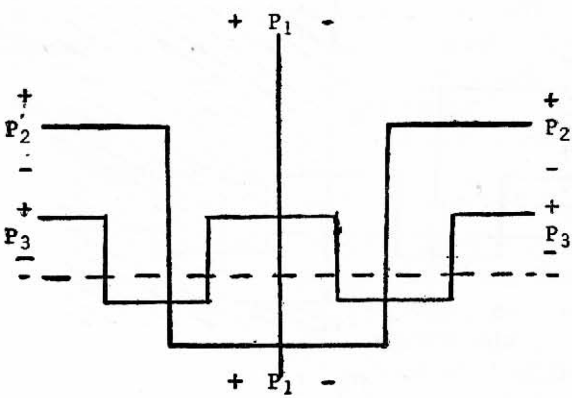

Fig. 4

The intersecting network of lines represents the " logical scaffolding " which presents the possibilities in logical space. Here we have drawn the diagram for three *Sachverhalten* but the principle of extension to any number n of *Sachverhalten* is clear.[15] Where n is actually the total number of *Sachverhalten*, the framework of the world would then have been completely constructed. Each of the mutually exclusive areas on the diagram corresponds to a possible world, and which possible worlds contain P_i and which do not contain P_i is indicated in an obvious way.

A proposition, if it is true confines reality to some subset of the set of possible worlds. That is the logical place which it determines. In terms of the spatial analogy, a proposition " holds open " this space for reality by occupying and so " closing off " to reality the remainder of the space. For convenience, we have given a two-dimensional model, and will use hatching to indicate that part of the space which a proposition " closes off " to reality. But the model can be conceived three-dimensionally, and it then provides a quite precise illustration of Wittgenstein's remarks at 4.463 about a proposition being, in the positive sense, " like a space bounded by solid substance in which there is room for a body " and, in the negative sense, " like a solid body that restricts the freedom of movement of others ". As an example, we show the place determined by " $p \vee (q \wedge \sim r)$ ", which consists of the areas without hatching in the following diagram :

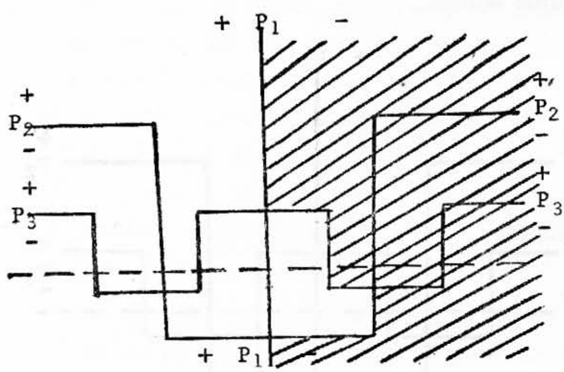

Fig. 5

The logical place determined by a proposition is made evident if the propositional sign explicity gives the truth conditions of the proposition. As Wittgenstein · points out, in place of a propositional sign like " p $\wedge$ (q ~ γ r) " we could use its truth table or—given the appropriate combinatotial rule for ordering the truth possibilities—the propositional sign (TFTFTTTF) (pqr) " (Cf. 4.44's). Then the " propositional sign with logical co-ordinates ", which Wittgenstein associates with the logical place in 3.41, would in our view be manifest. The ordered n–tuple of the values T and F which a proposition assigns to each possible world (or ordinate) expresses the logical co-ordinates of the proposition. The " logical co-ordinates " are thus distinct from the co-ordinations or what Wittgenstein calls the " correlations " (*Zuordnungen*) of name and object. Of course, in Wittgenstein's view, the two are interdependent, and a sign does not *have* logical co-ordinates, it is not a proposition, without the correlation of name and object; but recognition of this interdependency should not lead us to confuse the two.

When we think of a logical co-ordinate system as a system with 2^n ordinates so that " (FT) (p) " for instance, with co-ordinates F along the T-ordinate for p and T along the F-ordinate for p might be used to represent what " ~p " represents in Russell's system, we see the relevance of the notion of logical co-ordination to Wittgenstein's speculations in the *Notebook* entry discussed earlier in connection with Griffin. Moreover, on this interpretation it is explicable that in 29.10.14 Wittgenstein should say of the system of co-ordinates that it is an *internal* relation which " projects the situation into the proposition "; and it is possible too on this interpretation to make sense of Wittgenstein's remark about the proposition corresponding to the " fundamental " co-ordinates, since we could not fix the logical place of a proposition without fixing its values along the ordinates created by a certain set of elementary propositions, generally those explicitly represented in the propositional sign.

It is propositions and logical places that have logical co-ordinates, not possible worlds. Stenius' notion of logical co-ordinates is not Wittgenstein's. Stenius admits this himself : " Wittgenstein seems . . to use the notion of ' logical co-ordinates ' in a sense which is related to, though not identical with the notion of 'logical co-ordinates' as used above" (p. 55). But he does not give an explanation of this "related sense".

A proposition determines only a place in logical space, but the system of co-ordinates which gives it that place implicitly relates it to every other place. Specification of a proposition in terms of the full set of ordinates fixes its position relative to every other proposition, and specification in terms of the ordinates generated by its component propositions fixes its position relative to their positions. Thus, given the co-ordinate system, and knowing the places of 'p' and 'q', we have all that is required to determine the position 'p $\vee$ q'. If this were not so, if, roughly, the logical co-ordinates of 'p' and 'q' did not suffice to fix the position of 'p $\vee$ q', then something else would be required: this could only be a new element introduced by the logical constant, giving rise to additional logical co-ordinates. This is our explanation of Wittgenstein's remarks at 3.42 about the sense in which the proposition gives "the whole of logical space". So a model of logical space should show, as ours does, that each proposition divides the whole space into two, that each additional elementary proposition doubles the number of 'possible worlds', and that one has only to construct the logical scaffolding to be in a position to see how a proposition relates to every possibility in the world so constructed. (Cf. 4.023).

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NOTES

1. The other works we discuss all appeared later than Stenius' work with the exception of Anscombe's *Introduction*, which appeared in 1959. However, Anscombe mentions in the Acknowledgments that she had "had the advantage of reading through Professor Erik Stenius' highly interesting book on the *Tractatus* before its publication". We are ourselves grateful to Professor Stenius for comments on an earlier draft of the present paper. Professor Stenius' comments have led us to modify our views in one or two particulars, though we have also persisted with some views that he would regard as errors.

2. James Griffin: *Wittgenstein's Logical Atomism* (Oxford University Press, 1964), pp. 103–104.

3. Max Black: *A Companion to Wittgenstein's Tractatus* (Cambridge University Press, 1964) p. 156.

4. Max Black, op. cit. p. 155.

5. Op. cit. p. 9.

6. See esp. pp. 155–156.

7. Ib. p. 329.

8. G. E. M. Anscombe : *An Introduction to Wittgenstein's Tractatus*. (Hutchinson, 1959), pp. 75–76.

9. David Favrholdt : *An Interpretation and Critique of Wittgenstein's Tractatus* (Munksgaard, 1967) p. 44.

10. Ib.

11. Ib. p. 49.

12. Ib. p. 47.

13. Ib. p. 44.

14. Ib. p. 51.

15. This method of diagramming logical space for n *Sachverhalten* was suggested by an article "Venn-type Diagrams for Arguments of N terms" by D. E. Anderson and F. L. Cleaver in 30 *JSL* 1965, to which John Fox drew our attention.