

**SAVITRIBAI PHULE PUNE UNIVERSITY**  
**DEPARTMENT OF MATHEMATICS**  
**SYLLABUS**  
**MA/M.Sc.(Mathematics)**  
**(w.e.f. A.Y. :2014-15)**

**Duration: Min. 2 Years (4 Semesters)**

**Credits: Min 100**

**FIRST YEAR**

**SEMESTER I**

(All are compulsory and each course is of 5 credit points)

MT 101 Linear Algebra  
MT 102 Topology  
MT 103 Measure and Integration  
MT 104 Algebra  
MT 105 Numerical Analysis.

**Total credits: 25 points**

**SEMESTER II**

(All are compulsory and each course is of 5 credit points)

MT 201 Functional Analysis  
MT 202 Complex Analysis  
MT 203 Field Theory  
MT 204 Advanced Calculus I  
MT 205 Differential Equations

**Total credits: 25 points**

## SECOND YEAR

List of the Elective Courses for semester III and semester IV. Each course is of 5 credit points.

- MT 01. Operations Research
- MT 02. Integral Equations and Transforms
- MT 03. Number Theory I
- MT 04. Coding Theory
- MT 05. Graph Theory I
- MT 06. Lattice Theory I
- MT 07. Computational Geometry
- MT 08. Cryptography
- MT 09. Financial Mathematics
- MT 10. Modeling and Simulation
- MT 11. Artificial Intelligence
- MT 12. Symmetries
- MT 13. Wavelets
- MT 14. Combinatorics
- MT 15. Partial Differential Equations
- MT 16. Fuzzy Logic
- MT 17. Statistics and Probability
- MT 18. Fluid Dynamics
- MT 19. Banach Algebra
- MT 20. Boundary Value Problems
- MT 21. Baer \* Rings
- MT 22. Matroid Theory I
- MT 23. Sperner Theory
- MT 24. Differential Equation and Dynamical System
- MT 25. Mechanics
- MT 26. Complex Analysis II
- MT 27. Representation Theory of Groups
- MT 28. Fourier Analysis on Finite Groups
- MT 29. Differential Geometry
- MT 30. Non-Linear Dynamical System
- MT 31. Topics in Lie Groups.
- MT 32. Algebraic Topology
- MT 33. Advanced Calculus II
- MT 34. Projective Geometry
- MT 35. Algebraic Geometry
- MT 36. Algebraic Number Theory
- MT 37. Algebraic Curves
- MT 38. Commutative Algebra
- MT 39. Advanced Lattice Theory II
- MT 40. Graph Theory II
- MT 41. Matroid Theory II
- MT 42. Group Theory II
- MT 43. Ring Theory
- MT 44. Topics in Non Commutative Rings.

## SEMESTER I

### MIM 101 : Linear Algebra

1. Prerequisites: Vector Spaces: Definition and Examples, Subspaces, Bases and Dimensions, Linear Transformations, Quotient Spaces, Direct Sum, The matrix of Linear Transformation, Duality.
2. Canonical Forms: Eigenvalues and Eigenvectors, The minimal Polynomial, Diagonalisability, Triangularisable Operators, Jordan Forms, The Rational Forms.
3. Inner Product Spaces: Inner Product Spaces, Orthogonally, The Adjoint of Linear Transformation, Unitary operators, Self Adjoint and Normal Operators.
4. Bilinear Forms: Definition and Examples, The matrix of a Bilinear Form, Orthogonality, Classification of Bilinear Forms.
5. Modules: Definition and Examples, Further notions and Results.
6. Free Modules: Linear Independence, Bases of Free Modules, Matrices and Homeomorphisms.

#### Reference Books:

1. Luthar and Passi, Modules (Narosa Publishing House).
2. Vivek Sahai and Vikas Bist, Linear Algebra (Narosa Publishing House).

## MT 102 : Topology

1. Prerequisites: Cartesian Products, Finite Sets, Countable and Uncountable Sets, Infinite Sets and Axiom of Choice, Well Ordered Sets.
2. Topological Spaces : Basis for a topology, Order topology, Subspace Topology, Product topology, closed sets and limit points, Continuous functions, Metric Topology
3. Connected and Compact Spaces: Connected spaces, Connected Subspaces of Real Line, Components and Local Connectedness, Compact spaces, Compact Subspaces of the Real Line, Limit point compactness, Local Compactness.
4. Countability and Separation Axioms: Countability Axioms, Separation axioms Normal Spaces, Urysohn's Lemma(without proof), Tietz Extension Theorem (Without Proof), Metrization Theorem (without proof), Tychonoff's Theorem.

### Reference Book:

J.R. Munkres, Topology : A First Course. Second Edition.  
( Ch.1 : Sec 5,6,7,9,10; Ch.2 : Sec 12 to 21; Ch.3 : Sec 23 to 29; Ch.4 : Sec 30 to 35; Ch.5 : Sec 37).

### MT 103 Measure and Integration

1. Prerequisites: Cardinal Numbers and Countability, Properties of Open Sets, Cantor Like Sets.
2. Measure on Real Line : Lebesgue Outer Measure, Measurable Sets, Regularity, Measurable Functions, Borel and Lebesgue Measurability.
3. Integration of Functions on Real Variable : Integration of Non Negative Functions, General Integral, Integration of Series, Riemann and Lebesgue Integral.
4. Differentiation : Functions of Bounded Variation, Lebesgue Differentiation Theorem, Differentiation Theorem, Differentiation and Integration.
5. Inequalities and  $L^p$  spaces : The  $L^p$  Spaces, The Convex Functions, Jensen's Inequalities, Inequalities of Holder and Minkowski, Completion of  $L^p$ .
6. Convergence : Convergence in Measure, Almost Uniform Convergence, Convergence Diagrams, Counter Examples

#### Reference Book:

1. G. de Barra, Measure Theory and Integration, New Age International Ltd, Publishers.  
( Sec 1.5 to 1.7., 2.1 to 2.5., 3.1 to 3.4., 4.1 to 4.5., 5.1 to 5.6., 6.1 to 6.5, 7.1 to 7.4.).
2. H.L.Roydon, Real Analysis (Third Ed.), Prentice Hall 1995.

## MT 104 : Algebra

3. Prerequisites: Semigroups and groups, Homomorphisms, Subgroups and cosets. Rings, Examples of rings, types of rings, subrings and characteristic of a ring.
4. Cyclic groups, permutation groups, generators and relations.
5. Normal subgroups and quotient groups. Isomorphism theorems, automorphisms, conjugacy and G-sets.
6. Normal series, Solvable groups, Nilpotent groups.
7. Group Homomorphisms, First Isomorphism Theorem, Fundamental Theorem of Finite Abelian Groups.
8. Permutation Groups, Cyclic decomposition, Alternating group  $A_n$ , Simplicity of  $A_n$ .
9. Structure of groups, Direct products, Finitely Generated Abelian Groups, Invariants of a finite abelian group
10. Sylow Theorems, Groups of order  $p^2$ ,  $pq$ .
11. Ideals and homomorphisms, maximal and prime ideals, nilpotent and nil ideals, Zorn's lemma
12. Unique Factorisation Domains, Principal Ideal Domains, Euclidean Domains. Polynomials over UFD.

### Reference Books:

1. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra (Second Ed.), Cambridge Univ. Press (Indian Ed. 1995).
2. Joseph A. Gallian, Contemporary Abstract Algebra (Fourth Ed.), Narosa, 1999.
3. I. S. Luthar and I. B. S. Passi, Algebra-Vol. 1: Groups, Narosa, New Delhi, 1996.

## MT : 105 Numerical Analysis

1. Iterative solutions of nonlinear equation: bisection method. Fixed-point iteration, Newton's method, secant method, acceleration of convergence, Newton's method for two non linear equations, polynomial equation methods.
2. Polynomial interpolation: interpolation polynomial, divided difference interpolation, Aitken's formula, finite difference formulas, Hermite's interpolation, double interpolation.
3. Linear systems of Equations: Gauss Elimination, Gauss-Jordan method, LU decomposition, iterative methods, and Gauss-Seidel iteration.
4. Numerical Calculus : Numerical differentiation, Errors in numerical differentiation, Numerical Integration, Trapezoidal rule, Simpson's 1/3 - rule, Simpson's 3/8 rule, error estimates for Trapezoidal rule and Simpson's rule.
5. Numerical Solution of Ordinary differential Equations : Solution by Taylor series, Picard Method of successive approximations, Euler's Method, Modified Euler Method, Runge-Kutta Methods, Predictor-Corrector Methods.
6. Eigenvalue Problem: Power method, Jacobi method, Householder method.
7. Practicals with Scilab.

### Reference Book:

1. S. S. Sastry, Introduction Methods of Numerical Analysis ( 4th Edition)(Prentice-Hall).
2. K.E. Atkinson,; An Introduction to Numerical Analysis.
3. J. I. Buchaman and P. R. Turner, Numerical Methods and Analysis..

## SEMESTER II

### MT 201 : Functional Analysis

1. Normed spaces, continuity of linear maps, Hahn - Banach theorems, Banach spaces.
2. Uniform bounded principle, Application - Divergence of Fourier Series of Continuous Functions, closed graph theorem, Open mapping theorem, bounded inverse theorem, spectrum of Bounded Operator.
3. Duals and transposes, duals of  $L^p[a, b]$  and  $C[a, b]$ .
4. Inner product spaces, orthonormal sets, approximation and optimization, projections, Riesz representation theorem.
5. Bounded operators and adjoints on a Hilbert space, normal, unitary and self adjoint operators.
6. Fourier Series and Integrals.

### Reference Book:

- B.V. Limaye, Functional Analysis (Second Edition) - New Age International Limited.  
(Ch. 1: ; Ch. 2: Sec 5 to 8; Ch. 3: Sec 9 to 12; Ch. 4: Sec 13, 14;  
Ch. 6: Sec 21 to 24; Ch. 7: Sec 25, 26).



## MT 202 : Complex Analysis

### 1. Pre-requisites:

- (a) Topological and Analytical Preliminaries: Point sets in the plane, sequences, compactness, stereographic projection, continuity.
  - (b) Elementary Functions: Exponential functions, mapping properties, logarithmic function, complex exponents.
2. Analytic Functions: Cauchy-Riemann Equations, analyticity, harmonic functions.
  3. Power Series: Sequences, uniform convergence, Maclaurin and Taylor series, operations on power series.
  4. Complex Integration and Cauchy's Theorem: Curves, parameterizations, line integral, Cauchy's Theorem.
  5. Applications of Cauchy's Theorem: Cauchy's integral formula, Cauchy's inequality and applications, maximum modulus theorem.
  6. Laurent Series and Residue Theorem: Laurent series, classification of singularities, evaluation of real integrals, argument principle.
  7. Bilinear Transformations and Mappings: Basic mappings, linear fractional transformations, other mappings.

### Reference Books:

1. S. Ponnuswamy and Herb Silverman, *Complex Variables with Applications*, Birkhäuser.
2. J. B. Conway, *Functions of one complex variables*, Narosa Publishing House.

### MT 203 : Field Theory

1. Prerequisites: Definitions and basic properties Rings and fields, Ideals and homomorphisms, Characteristic of fields, Euclidean domains, Unique factorization, Polynomials.
2. Field Extensions: The degree of an extension, Extensions and polynomials, Polynomials and extensions.
3. Applications to Geometry: Ruler and compasses construction, An algebraic approach.
4. Splitting Fields.
5. Finite Fields.
6. The Galois Group: Monomorphisms between fields, Automorphisms, Groups and subfields, Normal extensions, Separable extensions, The Galois correspondence, The fundamental theorem, An example.
7. Equations and Groups: Solution by radicals of quadratics, cubics and quartics. Cyclotomic polynomials, cyclic extensions.
8. Groups and Equations: Insoluble quintics, General polynomials.

#### Reference Books:

1. J. M. Howie, Fields and Galois Theory, Springer Undergraduate Mathematics Series, 2006.(Chapters 1 to 8 and Chapter 10).
2. M. Artin, Algebra, Prentice-Hall, Englewood Cliffs, N.J., 1991.
3. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, Second Ed., Cambridge University Press, Cambridge, 1995.

## MT 204 : Advanced Calculus I

1. Compact and Connected Subsets of  $\mathbb{R}^n$ .
2. Differentiation : Derivative, Continuously Differentiable functions, Chain rule, Inverse function theorem, Implicit function theorem.
3. Integration: integral over a rectangle, Existence of the Integral, evaluation of the integral, integral over a bounded set and rectifiable sets, improper integrals
4. Change of Variable Theorem (Proof of one variable) and Statement of n-variables (with Illustrations)
5. Line Integrals with Applications

### Reference Book:

1. J.R. Munkres, Analysis on Manifolds. ( Sections 4 to 15 and Section 17).
2. T.M. Apostol, Calculus (Volume II). (Chapter 10 : Sections 10.1 to 10.9).

## MT 205 : Differential Equations

1. Prerequisites: Linear equations of the first order.
2. Linear equations with constant coefficients : Second order homogeneous equations, Initial value problems, Linear dependence and independence, Nonhomogeneous equations of n-th order, Algebra of constant coefficients.
3. Linear equations with variable coefficients : Initial value problems, Solutions of the homogeneous equation, Wronskian and linear independence, Reduction of order, Nonhomogeneous equations, Legendre equation.
4. Linear Equations with regular singular points : Euler equation, Second order equation with regular singular points, Exceptional cases, Bessel equation.
5. Existence and uniqueness of solutions to first order equations: Separation of variables, exact equations, Method of successive approximations, Lipschitz condition, Approximation to and uniqueness of solutions.
6. Existence and uniqueness of solutions to systems and n-th order equations: Complex n-dimensional space, Systems as vector equations, Existence and uniqueness of solutions to systems, Existence, Uniqueness for linear systems and equations of order n.

### Reference Book:

1. E. A. Coddington, An Introduction to Ordinary Differential Equations (Prentice-Hall).
2. G. F. Simmons and S. G. Krantz, Differential Equations (Tata McGraw-Hill).

## MT 01 - OPERATIONS RESEARCH

Unit I - Kuhn – Tucker conditions of Optimality – Quadratic Programming (Sections 19.2.2B, 20.2.2)

Unit II - Inventory Models  
(Sections 14.1 to 14.3)

Unit III - Queuing Models (Section  
15.1, 15.2, 15.4, 15.5)

Unit IV - Project Scheduling By PERT – CPM (Sections  
13.1 to 13.4)

Unit V - Simulation Modeling with SIMNET – II (Sections  
17.1 to 17.10)

Reference Book :

Hamdy A.Taha, Operations Research, Fifth Edition, Prentice Hall of India.

## MT 02- INTEGRAL EQUATIONS AND TRANSFORMS

1. Classification of Linear Integral Equations : Fredholm, Volterra, Integro-Differential Equations, Singular Integral Equations, Converting Volterra Equation to ODE, Conversion of IVP to Volterra equation Conversion of BVP to Fredholm equation
2. Fredholm Integral Equations - Decomposition method, Direct Computation method, successive approximation method, method of successive substitutions, Homogeneous Fredholm Equations, Comparison between alternative methods.
3. Volterra Integral Equation - Adomian Decomposition method, Series solution method, converting Volterra equation to IVP, Successive Approximation method, successive substitution method, comparison between alternative methods.
4. Integro-Differential Equations - Introduction, Direct Computation method, Adomian Decomposition Method. Conversion to Fredholm integral Equation. Volterra Integro-Differential equations Series Solution, Decomposition Method, Conversion to IVP.
5. Singular Integral Equations - Abel problem, Generalized Abel Integral Equation, Weakly-singular Volterra Equations.
6. Non Linear Integral Equations - Non linear Fredholm Integral equations, Direct Computation, decomposition method, Non linear Volterra Integral Equation, Series solution, Decomposition method.
7. Existence and uniqueness of solutions using fixed-point theorems in case of Linear and nonlinear Volterra and Fredholm integral equations.
8. Fourier Transforms: [FT] Definition Properties evaluation of Fourier and inverse Fourier transforms of functions, Convolution theorem for FT. Sine and Cosine Fourier transforms. Solving differential equations and integral equations using FT.
9. Laplace Transform: Definition Properties, evaluation of Laplace and Inverse Laplace transforms of functions. Convolution theorem for Laplace Transforms. Solving initial value problem using Laplace Transforms. Solving integral equation using Laplace Transforms

### Reference Books:

- 1) A First course in integral equations –A.M. Wazwaz (1997) (World Scientific)
- 2) Introduction to Integral Equation with Applications –A.J. Jerri (1999) Second edition Wiley Interscience.

### MT 03 - NUMBER THEORY

1. Congruences : Solutions of congruences. Chinese Remainder Theorem, Techniques of numerical calculation Public-Key Cryptography.
2. Prime power moduli. Prime modulus. Primitive roots and power residues, Congruences of degree two.
3. Quadratic Residues, Quadratic Reciprocity.
4. Greatest integer function, Arithmetic functions, Multiplicative functions, Dirichlet multiplication. Mobius Inversion Formula.
5. Diophantine equations. The equation  $ax + by = c$ , Pythagorean triangles, Assorted examples. Rational points on curves.

#### Reference Books:

1. Niven and Zuckerman, An introduction to the Theory of Numbers, Wiley Publishers.
2. David Burton, Elementary Number Theory

## MT 04 - CODING THEORY

1. Error detection: correction and decoding: Communication channels, Maximum likelihood decoding, Hamming distance, Nearest neighbor / minimum distance decoding, Distance of a code.

2. Linear codes: Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes, Generator matrix and parity check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes, Cosets, Nearest neighbor decoding for linear codes, Syndrome decoding.

3. Cyclic codes: Definitions, Generator polynomials, Generator and parity check matrices, Decoding of cyclic codes, Burst-error-correcting codes.

4. Some special cyclic codes: BCH codes, Definitions, Parameters of BCH codes, Decoding of BCH codes.

Reference: 1. San Ling and Chaoing xing, Coding Theory- A First Course

2. Applied Abstract Algebra - Lid and Pilz 2nd Edition



## MT 05 - GRAPH THEORY

1. Fundamental concepts : Definitions and examples, graphs as models, matrices and isomorphism, paths, connected graphs, bipartite graphs, external vertex degree, the Pigeonhole principle, Turan's theorem, degree sequences, graphic sequences, degree and digraphs.
2. Tree and Distances : Properties of tree, distance in graphs, stronger results, disjoint spanning trees, shortest paths, trees in computer science, Eulerian circuits.
3. Matching and Factors : Matching in bipartite graphs, maximum matchings, Hall's matching conditions, Min-Max in bipartite graphs, sets, applications and algorithms, maximum bipartite matching, weighted bipartite matching, in general graphs, Tutte's 1-factor theorem, factors of graphs.
4. Connectivity and Paths : Cuts, connectivity, edge-connectivity, blocks, 2-connected graphs, connectivity of digraphs, k connected and k-edge connected graphs, applications of Menger's theorem, Network flow problems maximum network flow, integral flows.
5. Edges and cycles : Line graph and edge-coloring, Hamiltonian cycles: necessary conditions, Sufficient conditions.

### Reference Book :

1. Douglas B. West, Introduction to Graph Theory Prentice- Hall, New Delhi (1999)
2. John Clarke and D.A. Holton, A First Look at Graph Theory, Allied Publisher (1991)
3. Nora Harsfield and Gerhard Ringel , Pearls Theory, Academic Press (1990)
4. Harary, Graph Theory, Narosa Publishers, New Delhi (1989)

## MT 06 - LATTICE THEORY

1. Two definitions of lattices, Hasse diagrams, homomorphism, isotone maps, ideals, congruence relations, congruence lattices, the homomorphism theorem, product of lattices, complete lattice, ideal lattice, distributive –modular inequalities and identifies, complements, pseudocomplements, Boolean lattice of pseudocomplements, join and meet-irreducible elements.
2. Characterization theorems and representation theorems-Dedekind`s modularity criterion Birkhoff`s distributivity criterion, hereditary subsets, rings of sets, Stone theorems, Nachbin theorem, statements of Hashimoto`s theorem.
3. Modular lattices, isomorphism theorem, Upper and lower covering conditions, Kurosh- Ore theorem, independent sets (Drops results involving projectivity and sublattice generated by sets / elements )
4. Semimodular lattices Jordan-Holder chain condition, Modular pair, M-symmetric lattices.

Reference Book : General

Lattice Theory

Author- G. Grätzer (Birkhauser, IInd Edition 1998)

Chap. 1 Section 1,2,3,4,6, Cha. 2 Section-1, Chap.3. Section –1,2.

## MT 07 - COMPUTATIONAL GEOMETRY

1. Geometric primitives [Chap. 1]
2. Line intersection [Chaps. 2] plus randomized incremental
3. Triangulation and visibility and [Chaps. 3,15]
4. Linear programming in two and three dimensions [Chap. 4]
5. Orthogonal range searching [Chaps. 5,10]
6. Point location and Binary Space Partitions [Chaps. 6,12]
7. Voronoi diagrams and Delaunay triangulation [Chaps. 7,9]
8. Convex hulls [Chap. 11]
9. Non-orthogonal range searching [Chap. 16]
10. Curved Elements (Bezier, B-Splines)
11. Curve Reconstruction (reconstruction a curve(surface) from sample points)
12. 3-Dimensional Geometry

Reference Book :

Computational Geometry Algorithms and Applications, 2nd ed., by de Berg, van Kreveld, Overmars, and Schwarzkopf (Springer-Verlag, 2000).

## MT 08 - CRYPTOGRAPHY

1. Introduction : Overview of course, Classical cryptography [parts of Chapter 1].
2. Secret Key Encryption : Perfect Secrecy - One time pads [Chapter 2.1], Stream ciphers and the Data Encryption Standard (DES) [Chapter 3 (excluding 3.6)], The Advanced Encryption Standard (AES) - adopted September 2000.
3. Public Key Encryption : Factoring and the RSA encryption [Chapter 4.1 - 4.4], Discrete log. Diffie-Hellman Key Exchange [Chapter 8.4 (only pages 270-273)]. ElGamal encryption [Chapter 5 (only pages 162-164)] , Digital Signatures [Chapter 6 (excluding 6.5 - 6.6)], One-time signatures, Rabin and ElGamal signatures schemes, Digital Signature Standard (DSS).
4. Hashing : Motivation and applications. Cryptographically Secure Hashing. [Chapter 7.1-7.3,7.6], Message Authentication Codes (MAC). HMAC, Network Security , Secure Socket Layer (SSL), IPsec., Secret Sharing, Definition. Shamir's threshold scheme [Chapter 11.1], Visual secret sharing schemes.

### Reference Book :

D. R. Stinson. *CRYPTOGRAPHY: Theory and Practice*. CRC Press. 1995.

## MT 09 - FINANCIAL MATHEMATICS

1. Introduction to options and markets: types of options, interest rates and present values.
2. Black Sholes model : arbitrage, option values, pay offs and strategies, put call parity, Black Scholes equation, similarity solution and exact formulae for European options, American option, call and put options, free boundary problem.
3. Binomial methods : option valuation, dividend paying stock, general formulation and implementation.
4. Monte Carlo simulation : valuation by simulation
5. Finite difference methods : explicit and implicit methods with stability and convergence analysis methods for American options- constrained matrix problem, projected SOR, time stepping algorithms with convergence and numerical examples.
6. Lab component: implementation of the option pricing algorithms and evaluations for Indian companies.

### Reference Book :

1. D.G.Luenberger, Investment Science, Oxford University Press,1998.
2. J.C.Hull , Options, Futures and Other Derivatives, 4th ed., Prentice- Hall ,New York,2000.
3. J.C.Cox and M.Rubinstein, Option Market, Englewood Cliffs,N.J.: Prentice- Hall,1985.
4. C.P. Jones. Investments, Analysis and Measurement, 5th ed.,John Wiley and Sons,1996.

## MT 10 - MODELLING AND SIMULATION

1. Introduction to modeling and simulation. System analysis, classification of systems. System theory basics, its relation to simulation.
2. Model classification: conceptual, abstract, and simulation models. Heterogeneous models. Methodology of model building
3. Simulation systems and languages, means for model and experiment description. Principles of simulation system design
4. Parallel process modeling. Using Petri nets and finite automata in simulation. Models of queuing systems. Discrete simulation models. Model time, simulation experiment control.
5. Continuous systems modeling. Overview of numerical methods used for continuous simulation.
6. Combined simulation. The role of simulation in digital systems design
7. Special model classes, models of heterogeneous systems.
8. Checking model validity, verification of models. Analysis of simulation results
9. Simulation results visualization. Interactive simulation
10. Design and control of simulation experiments. Model optimization
11. Generating, transformation, and testing of pseudorandom numbers. Stochastic models, Monte Carlo method
12. Overview of commonly used simulation systems.

### Reference Book :

1. Fishwick P.: Simulation Model Design and Execution, PrenticeHall, 1995, ISBN 0-13-098609-7
2. Law A., Kelton D.: Simulation Modelling and Analysis, McGraw-Hill, 1991, ISBN 0-07-100803-9
3. Rábová Z. a kol: Modelování a simulace, VUT Brno, 1992, ISBN 80-214-0480-9
4. Ross, S.: Simulation, Academic Press, 2002, ISBN 0-12-598053-1.

## MT 11 - ARTIFICIAL INTELLIGENCE

1. Overview of history and goals of AI : Tentative definitions. Turing's test. Knowledge vs. Symbolic Level. Relations with other disciplines, from Philosophy, to Linguistics, to Engineering. Review of AI successes and failures.
2. State Spaces, Production Systems, and Search : State Space representation of problems. Problem solving as search. Constraints. Definition and examples of Production Systems. Heuristic search techniques. Two person games.
3. Knowledge Representation Issues: Procedural Knowledge Representation vs. Declarative Knowledge + Reasoning. Facts, General Assertions, Met knowledge. The Frame Problem.
4. Using First-Order Logic for Knowledge Representation : Propositional Logic: Semantics and Deduction. First Order Logic: Semantics and Deduction. Unification. Resolution-based theorem proving. Using theorem proving to answer questions about the truth of sentences or to identify individuals that satisfy complex constraints. Logic Programming.
5. Common Sense Reasoning : No monotonic reasoning and modal logics for nonmonotonic reasoning. How to deal with Agents and their Beliefs.
6. Weak Slot-and-Filler Structures: Semantic Nets and Frames. Scripts for representing prototypical combinations of events and actions.
7. Rule-Based Systems: Pattern-matching algorithms. The problem of Control in Rule Based Systems. The Rete Algorithm.
8. Planning: Representing plans. Partial order planning. Planning applications.
9. Statistical Reasoning: Use of Certainty Factors in Rule-Based Systems. Associating probabilities to assertions in first-order logic. Bayesian Networks. Fuzzy Logic.
10. Learning: Learning to classify concepts using features of their instances. Learning a concept [Induction] from examples. Explanation-Based Learning. Version Spaces. Neural Nets with back propagation.

Reference Book :

Artificial Intelligence: A Modern Approach : Prentice-Hall, 1995

## MT 12 - SYMMETRIES

1. Symmetry of plane figures of motions of the plan, finite groups of motions, discrete groups of motion, symmetry, cosets, counting formula, permutation representations, finite subgroups of the generators and relations
2. Operation of a group on itself, class equation of the isocahedral groups operations on subsets groups of order 12, free group generators and relations.
3. Bilinear forms, symmetric forms, orthogonality, geometry associated to a positive form, Hermitian forms, spectral theorem, conics and quadrics, normal operators, skew symmetric forms.

Reference Book :

Artin : Algebra (Prentice-Hall) Chapters

5, 6 (sections 1, 2, 3 ), 7.



## MT 13 - WAVELET ANALYSIS

1. Fourier Transform : Fourier transform on  $L^1(\mathbb{R})$  and  $L^2(\mathbb{R})$  and basic properties and examples
2. Windowed Fourier Transform : Motivation and definition of Windowed Fourier Transform and examples, Time frequency localization, the reconstruction formula.
3. Continuous Wavelet Transform : Motivation and Definition of the wavelet transform and examples, Basic properties, The reconstruction formula, Frequency localization, Orthonormal Wavelets.
4. Multiresolution Analysis : Definition of MRA and examples, Properties of scaling functions and orthonormal wavelets bases, Construction of orthonormal wavelets.

### Reference Book :

1. Bachman G, L. Narici & E. Beckenstein: Fourier and Wavelet Analysis, Springer-Verlage (2000)
2. Chui C. K. : An Introduction to Wavelets, Academic Press (1992)

## MT 14 - COMBINATORICS

1. Counting Methods for selections arrangements: Basic counting principles, simple arrangements and selections, arrangements and selection with repetition, distributions, binomial, generating permutations and combinations and programming projects.
2. Generating function : Generating function models, calculating of generating functions, partitions exponential generating functions, a summation method.
3. Recurrence Relations : Recurrence relation model, divide and conquer relations, solution of inhomogeneous recurrence relation, solution with generating functions.
4. Inclusion-exclusion: Counting with Venn diagrams inclusion formula, restricted positions and rook polynomials.
5. Ramsey Theory: Ramsey theorem, applications to geometrical problems.

### Reference Book :

1. Alan Tucker, Applied Combinatorics (third edition), John Wiley & sons , New York (1995)
2. V. Krishnamurthy, Combinatorial, Theory and Applications, East West Press, New Delhi (1989) Scientific, (1996)

## MT 15 - PARTIAL DIFFERENTIAL EQUATIONS

1. First order PDE, Linear Equations of first order, Charpit's method, Jacobi's method, Quasi-linear equations, Non-linear first order PDE.
2. Second ordered PDE: Genesis, Classification, One dimensional Wave equation, Laplace equation, Boundary Value Problems, Maximum and Minimum Principles, Cauchy Problem,
3. Heat Conduction Problem, Duhamel's Principle

Reference Book :

An Elementary Course in Partial Differential Equations by T. Amaranath (Narosa) Chapters 1-2.

## MT 16 - FUZZY LOGIC

1. Fuzzy Sets and Operations on Them
2. Fuzzy Relations
3. Fuzzy Rules
4. Approximate Reasoning
5. Fuzzy Logic
6. Fuzzy Systems (e.g., Fuzzy Logic Control)
7. Fuzzy Logic in Pattern Recognition
8. Fuzzy Decision Making
9. Fuzzy Logic Applications

Reference Book :

George J. Klir, Bo Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, Prentice Hall PTR, 1995.

Hao Ying, *Fuzzy Control and Modeling: Analytical Foundations and Applications*, IEEE Press, 2000. (Do not need to buy it. A handout will be distributed).

## MT 17 - STATISTICS AND PROBABILITY

1. Introduction to Discrete Probability : Intuitive concepts: probability of an event as a measure between 0 and 1; random variable; probability distribution; frequency interpretation of probability; random numbers; coins, dice, and other games; simulations; odds; historical development of probability; random walks.
2. Formal concepts: sample space, outcomes, and events; random variable; discrete distribution functions and axioms of probability; unions, intersections, and complements; properties of probabilities, principle of inclusion and exclusion; tree diagrams; uniform distributions over finite sets, symmetry; infinite sample spaces with discrete probabilities.
3. Introduction to Continuous Probability: The intuitive problems with probabilities over space (line, plane,  $\mathbb{R}^n$  in general). Monte Carlo simulations, Buffon's needle. Formal concepts: density function for a continuous random variable; integration; cumulative distribution functions; derivatives; exponential density function;
4. Conditional Probability : Intuitive concept of conditional probability; formal definition of conditional probability; Bayes' formula for inverting conditional probabilities; independent events; joint distribution functions; independent random variables; independent trials. Conditional density functions for continuous distributions; the beta distribution
5. Distributions and Densities : Uniform continuous distributions; geometric distribution; Poisson distribution; exponential and gamma distributions; introduction to queuing theory; normal (Gaussian) distribution; Chi-squared distribution
6. Expected Value and Variance : Expected value for discrete random variables, expectation; linearity of expectation; expectation of independent random variables; conditional expectation; variance and standard deviation; variance of various distributions. Expectation and variance for continuous random variables.
7. Sums of Random Variables : Analysis of sums of independent random variables with identical distributions, that is, independent trials.
8. Law of Large Numbers : Chebychev inequality, law of averages, law of large numbers.
9. The Central Limit Theorem : The central limit theorem for Bernoulli trials, binomial distributions again, the normal distribution, the general central limit theorem.

Reference Book :

Charles M. Grinstead and J. Laurie Snell's textbook *Introduction to Probability*, published by the American Mathematical Society, 1997

## MT 18 - FLUID DYNAMICS

1. Physical Properties of fluids. Concept of fluids, Continuum Hypothesis, density, specific weight, specific volume.
2. Kinematics of Fluids : Eulerian and Lagrangian methods of description of fluids, Equivalence of Eulerian and Lagrangian method, General motion of fluid element, integrability and compatibility conditions, strain rate tensor, stream line, path line, streak lines, stream function, vortex lines, circulation.
3. Stresses in Fluids : Stress tensor, symmetry of stress tensor, transformation of stress components from one co-ordinate system to another, principle axes and principle values of stress tensor.
4. Conservation Laws : Equation of conservation of mass, equation of conservation of momentum, Navier Stokes equation, equation of moments of momentum, Equation of energy, Basic equations in different co-ordinate systems, boundary conditions.
5. Irrotational and Rotational Flows : Bernoulli's equation, Bernoulli's equation for irrotational flows, Two dimensional irrotational incompressible flows, Blasius theorem, Circle theorem, sources and sinks, sources sinks and doublets in two dimensional flows, methods of images.

### Reference Book :

1. An introduction to fluid dynamics, R.K. Rathy, Oxford and IBH Publishing Co. 1976.
2. Theoretical Hydrodynamics, L. N. Milne Thomson, Macmillan and Co. Ltd.
3. Textbook of fluid dynamics, F. Chorlton, CBS Publishers, Delhi.
4. Fluid Mechanics, L. D. Landau and E.N. Lipschitz, Pergamon Press, London, 1985.

## MT 19 - BANACH ALGEBRA

1. Relatively compact sets, compactly continuous operators, finite dimensional operators, transformation that is bounded but not completely continuous, a type of transformation that is always completely continuous, further properties of completely continuous transformations.
2. Spectra and the resolvent set, Approximate proper values, Banach Algebra With identity, compactness of the spectrum, the resolvent operator, Spectral radius and spectral mapping theorem for polynomials, the Gelfand Theory.
3. Sesquilinear functions: Spectral results for normal and completely continuous operators, numerical range
4. The Fredholm alternative theory, the spectral theorem for bounded, normal Finite dimensional operators.
5. Commutative Banach Algebras, ideals and homomorphisms.

### Reference Book:

1. Walter Rudin: Functional Analysis, Tata McGraw Hill Publishing co. New Delhi.

## MT 20 - BOUNDARY VALUE PROBLEMS

1. Definition of boundary Value Problems, the heat equation, wave equation, Laplace's equation, the Fourier method, Linear Operators, Principle of Superposition, series solutions, uniform convergence (Weierstrass M-test), separation of variables, non homogeneous conditions, Sturm-Liouville problems, formal solutions, the vibrating string.
2. Orthogonal sets of functions, Generalized Fourier series, Best approximation in the mean, Convergence in the mean, the orthonormal trigonometric functions, other types of orthogonality.
3. Sturm-Liouville Problem and applications, orthogonality and uniqueness of eigenfunctions, method of solutions, surface heat transfer other boundary value problems.
4. Bessel function  $J_n$ , recurrence relation, the zero of  $J_0(X)$  and related functions, Fourier-Bessel series, Temperatures in a long cylinder.
5. Legendre polynomials, orthogonality of Legendre polynomials, Legendre series, Dirichlet Problem in spherical regions.

### Reference Book

R.V. Churchill and J. Brown.: Fourier Series and Boundary Value Problems (4<sup>th</sup> edition)(Publisher: McGraw-Hill Book Company)



## MT 21 - BAER\* RINGS

1. Rings with involution
2. Poset of projections
3. Proper involutions and C\*-algebras
4. Rickart \*-rings and Bear \*-ring
5. Weakly Rickart \*-rings and unitification
6. Central cover
7. Additivity of projections
8. Comparability axioms and parallelogram law
9. Finite and abelian projections
10. Structure Theorem

Reference Book:

Bear \*-rings, S.K. Berberian, Springer

## MT 22 - MATROID THEORY

### 1. Basic definitions and examples

Independent sets and circuits, bases, rank, closure, geometric representations of matroids of small rank, transversal matroids, the lattice of flats, the greedy algorithm.

### 2. Duality

The definition and basic properties, duals of representable matroids, duals of graphic matroids, duals of transversal matroids.

### 3. Minors

Contraction, Minors of certain matroids, flats and the sum theorem

### 4. Connectivity

Connectivity, for graphs and matroids, properties of matroid connectivity, more properties of connectivity.

Reference Book :

James G. Oxley, Matroid Theory Science Publications, Oxford (1992)(Chapter 1 to 4)

## MT 23 - SPERNER THEORY

1. Introduction and Sperner's Theorem: A Simple intersection result, Sperner's theorem, Theorem of Bollobaas.
2. Normalized Matching and rank numbers: Sperner's proof, system of distinct representatives, L Y M inequalities, and normalized matching property. Rank numbers, some examples.
3. Symmetric Chain: Symmetric chain decompositions, Dilworth's theorem, symmetric chains of sets, Application to Nested chains, posets with symmetric chain decompositions.
4. Rank numbers of multisets. Unimodality and log connectivity, the normalized matching property. The largest size of a rank number.

### Reference Book

Ian Anderson : Combinatorics of Finite Sets. (Oxford Science Publications)

Konrad Engel: Sperner Theory (Cambridge University Press)

## MT 24 - DIFFERENTIAL EQUATIONS AND DYNAMICAL

### SYSTEMS

- 1) Liner Systems: Uncoupled Liner Systems, Diagonalization, Exponential of operators  
Fundamental theorem for liner systems, liner systems in  $\mathbb{R}$ , Complex eigenvalues, multiple eigenvalues, Jordan Canonical Forms, stability theory Nonhomogeneous Liner systems.
- 2) Nonlinear Systems: Local Theory, Fundamental existence theorem dependence on initial conditions and parameters, the maximal interval of existence, Flow defined by a differential equation. Linearization, stable manifold theorem, Hartman- Grobman theorem, Stability and Lipunov functions, Saddles, Nodes, Foci and centers, Nonhyperbolic critical points in  $\mathbb{R}_n$ , Gradient and Hamiltonian system.

#### Reference Book:

- (1) L. Perko- Differential Equations and Dynamical systems (1991) Springer-verlag
- (2) Hirsch and Smale – Differential Equations, Dynamical Systems, and Liner Algebra - Academic Press, New York, (1974)

## MT 25 - MECHANICS

1. Sec 1.1-1.6 Survey of Elementary Principles.
2. Sec. 2.1-2.7 Variational Principles & Lagrange`s Equation
3. Sec.3.1-3.7 Central Force problem
4. Sec. 4.1-4.10 Kinematics of rigid body motion
5. Sec. 8.1-8.2 Hamilton Equations of motion
6. Sec.9.1-9.9 Canonical Transformations

### Reference Book :

Classical Mechanics by Goldstein, Poole and Safko (Third Edition) 2002, Person Education Inc. Supplementary Reading (1) Rana & Joag Classical Mechanics (Tata McGraw Hill)