UNIVERSITY OF PUNE

S.Y.B.Sc. MATHEMATICS Question Bank

Practicals Based on Paper I

Semester-I: Calculus of Several Variables

Practical No. 1

Limit, Continuity and Partial Derivatives

1. Test the following function for existence of simultaneous limit and iterated limits at the origin where, $f(x,y) = \dfrac{x-y}{x+y}$, $x+y \neq 0$. [4]

2. Let $f(x,y) = \dfrac{x+y-1}{\sqrt{x}-\sqrt{1-y}}$. Determine the domain of f, draw it geometrically. Evaluate $\lim\limits_{(x,y)\to(0,1)} f(x,y)$, if it exists. [4]

3. Let $f(x,y) = \begin{cases} xy, & \text{if } |x| \geq |y| \\ -xy, & \text{if } |x| < |y| \end{cases}$. By drawing the domain of f, indicate how f is determined in various subsets of $\mathbb{R}^2$. Consider the function $f(x,1)$. Does the limit of $f(x,1)$ as $x \to 1$ exist? Discuss the continuity of f at $(1,1)$. [4]

4. Using definition find $f_x(0,0)$ and $f_y(0,0)$ where,
$$\begin{aligned} f(x,y) &= 2xy\frac{x^2-y^2}{x^2+y^2}, \quad (x,y) \neq (0,0) \\ &= 0, \qquad\qquad (x,y) = (0,0). \end{aligned}$$ [4]

5. Let $f(x,y) = (x^2+y^2)\tan^{-1}\dfrac{y}{x}$, for $x \neq 0$ and $f(0,y) = \dfrac{\pi y^2}{2}$. Show that $\dfrac{\partial^2 f}{\partial x \partial y}(0,0) = 1$ while $\dfrac{\partial^2 f}{\partial y \partial x}(0,0)$ does not exist. [8]

Practical No. 2
Differentiability 1

1. If f and g are twice differentiable functions and
$z = f(y+ax) + g(y-ax)$, show that $z_{xx} = a^2 z_{yy}$. [4]

2. If $V = f(x,y),\ x = r\cos\theta,\ y = r\sin\theta$, then prove that
$\dfrac{\partial^2 V}{\partial x^2} + \dfrac{\partial^2 V}{\partial y^2} = \dfrac{\partial^2 V}{\partial r^2} + \dfrac{1}{r^2}\dfrac{\partial^2 V}{\partial \theta^2} + \dfrac{1}{r}\dfrac{\partial V}{\partial r}$. [8]

3. By using the definition, show that $f(x,y) = \sqrt{|xy|}$ is not differentiable at $(0,0)$. [8]

4. If $u = (1-2xy+y^2)^{-1/2}$, show that,
$\dfrac{\partial}{\partial x}[(1-x^2)\dfrac{\partial u}{\partial x}] + \dfrac{\partial}{\partial y}[y^2\dfrac{\partial u}{\partial y}] = 0.$ [4]

Practical No. 3
Differentiability 2

1. Given z is a function of u and v, while $u = x^2 - y^2 - 2xy$, $v = y$, find $(x+y)\dfrac{\partial z}{\partial x} + (x-y)\dfrac{\partial z}{\partial y}$. [4]

2. Let $u = \sin^{-1}(x^2+y^2)^{\frac{1}{5}}$. Using Euler's theorem show that,
$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{2}{5}\tan u$ and
$x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \dfrac{2}{25}\tan u(2\tan^2 u - 3)$. [8]

3. Using differentials find approximate value of $\sqrt{\dfrac{4.1}{25.01}}$ [4]

4. Prove that $\sin x \sin y = xy - \frac{1}{6}[(x^3+3xy^2)\cos\theta x \sin\theta y + (y^3+3x^2y)\sin\theta x \cos\theta y]$, for some $\theta \in (0,1)$. [8]

5. Let $f(x,y) = x^2 \sin\dfrac{1}{x} + y^2 \sin\dfrac{1}{y}$, when $xy \neq 0$
$f(x,0) = x^2 \sin\dfrac{1}{x}$, when $x \neq 0$
$f(0,y) = y^2 \sin\dfrac{1}{y}$, when $y \neq 0$
$f(0,0) = 0$. Show that

   (a) f_x and f_y are not continuous at $(0,0)$.

   (b) f is differentiable at $(0,0)$. [4]

Practical No. 4
Extreme Values

1. Locate the stationary points of the following functions :

   (a) $f(x, y) = \sin x + \sin y + \sin(x + y)$

   (b) $f(x, y) = x^3 + y^2 + x^2y - x^2 - y^2$. [4]

2. A rectangular box open at the top is to have a volume of 32 cu.m. What must be the dimensions so that the total surface area is minimum ? [8]

3. Obtain the shortest distance of the point $(1, 2, -3)$ from the plane $2x - 3y + 6z = 20$, using Lagrange's method of undetermined coefficients. [8]

4. Given the following critical points of the function $3x^2y - 3x^2 - 3y^2 + y^3 + 2$, examine for extreme values $(0, 0)$, $(0, 2)$, $(1, 1)$, $(-1, 1)$. [4]

Practical No. 5
Multiple Integrals 1

1. Evaluate $\displaystyle\int\int\int_V \frac{1}{(x+y+z+1)^3}\,dx\ dy\ dz$, where V is the region bounded by the planes $x=0,\ y=0,\ z=0$ and $x+y+z=1$. [4]

2. Evaluate $\displaystyle\int_0^2\int_0^x\int_0^{x+y} e^{x+y+z}dx\ dy\ dz.$ [4]

3. Change the order of integration,
$\displaystyle\int_0^{2a}\int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f\ dy\ dx.$ [4]

4. Change the order of the integration and hence evaluate $\displaystyle\int\int y\ dx\ dy$ over the region bounded by the line $y=x$ and the parabola $y=4x-x^2$. [8]

5. By double integration, find the area of the region bounded by the curves $y=x^2-9,\ y=9-x^2$. [4]

Practical No. 6
Multiple Integrals 2

1. Evaluate $\int\int_R (x+y)^3 \, dx \, dy$ where R is bounded by $x+y=1,\ x+y=4,\ x-2y=1,\ x-2y=-2$ using the substitution $x+y=u,\ x-2y=v$. [8]

2. Find the volume of the sphere $x^2+y^2+z^2=a^2$ using triple integration. [4]

3. Evaluate $\int\int x^2y^2 \, dx \, dy$ over the domain $\{(x,y) : x \geq 0, y \geq 0, x^2+y^2 \leq 1\}$. [4]

4. Evaluate $\int\int\int_R (x^2+y^2) \, dx \, dy \, dz$ where R is the region bounded by $x^2+y^2=2z$ and $z=2$ using cylindrical polar co-ordinates. [8]

Semester-I: Differential Equations

Practical No. 1

Homogeneous Differential Equations

1. (a) Find the order and the degree of the differential equation:

$$\frac{[1+(y')^2]^{3/2}}{yy''+1+(y')^2} = 1.$$

   (b) Determine whether the following function is homogeneous? If homogeneous, state it's degree.

$$f(x,y) = \frac{(x^2+y^2)^{1/2}}{(x^2-y^2)^{7/2}}.$$ [4]

2. Solve: $xy^2dx + (y+1)e^x dy = 0.$ [4]

3. Solve: $(x - y\ln y + y\ln x)\ dx + x(\ln y - \ln x)\ dy = 0.$ [4]

4. Solve: $\dfrac{dy}{dx} + \dfrac{3x^2y}{1+x^3} = \dfrac{\tan^2 x}{1+x^3}.$ [4]

5. Reduce the differential equation $(2x+y-3)\ dx = (2y+x+1)\ dy$ to homogeneous form and find it's solution. [8]

Practical No. 2

Exact Differential Equations

1. Solve $(xy+1)dx + x(x+4y-2)dy = 0.$ [4]

2. Solve the equation $6y^2dx - x(2x^3+y)dy = 0$ by treating it as a Bernoulli's equation in the dependent variable x. [4]

3. Solve : $\tan x \dfrac{dy}{dx} + y = \sec x.$ [4]

4. Solve : $(x+a)y' = bx - ny; a, b, n$ are constants with $n \neq 0, n \neq -1.$[4]

5. Solve : $y(xy+2x^2y^2)dx + x(xy - x^2y^2)dy = 0.$ [4]

Practical No. 3
Applications of Differential Equations

1. For the family $(x-a)^2 + y^2 = a^2$, find that member of the orthogonal trajectories which passes through $(1, 2)$. [4]

2. Show that the family of curves $\frac{x^2}{c} + \frac{y^2}{c-\lambda} = 1$ where c is a parameter, is self orthogonal. [4]

3. The population of a certain town is known to increase at a rate proportional to itself. After 2 years, the population doubled, and after one more year the population was 10,000. What was the original population? [4]

4. If half of a given quantity of radium decomposes in 1600 years, what percentage of the original amount will be left at the end of
   (a) 2400 years?
   (b) 8000 years? [8]

5. The decay rate of a certain substance is directly proportional to the amount present at that instant. Initially there are 27 gm. of the substance and 3 hours latter it is found that 8 gm. are left. Show that the amount left after one more hour is $\frac{16}{3}$ gm. [4]

Practical No. 4
Inverse Differential Operator

1. (a) Solve : $D^3(D^2+3D-2)y = 0$.
   (b) Solve : $(4D^4 - 24D^3 + 35D^2 + 6D - 9)y = 0$. [4]

2. (a) Solve : $(D^3 + 2D^2 + D)y = 0$.
   (b) Find the particular solution of $(D^3 + 2D^2 + D)y = e^{2x}$.
   (c) Find the particular solution of $(D^3 + 2D^2 + D)y = x^2 + x$. Hence find the general solution of $(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x$. [8]

3. Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 e^{3x}$. [4]

4. Solve : $(D^2 + 16)y = 3\cos^2 2x + e^{2x}$. [4]

5. Solve : $(D^2 + 4)y = x \sin x$. [8]

Practical No. 5

Methods of Solving Second Order Differential Equations

1. (a) Solve : $(D^2 + D - 2)y = 0$.

   (b) Find the particular solution of $(D^2 + D - 2)y = 2x - 40\cos 2x$ by the method of undetermined coefficients and hence write the general solution of $(D^2 + D - 2)y = 2x - 40\cos 2x$. [8]

2. (a) Solve : $(D^2 + 4D + 5)y = 0$.

   (b) Find the particular solution of $(D^2 + 4D + 5)y = 10e^{-3x}$ by the method of undetermined coefficients.

   (c) Find the particular solution of $(D^2 + 4D + 5)y = 10e^{-3x}$, with initial conditions $y(0) = 4, y'(0) = 0$. [8]

3. (a) Solve : $(D^2 + 1)y = 0$.

   (b) Find the particular solution of $(D^2 + 1)y = \tan x$, by the method of variation of parameters and hence find the general solution of $(D^2 + 1)y = \tan x$. [8]

4. (a) Solve : $(D^2 - 3D + 2)y = 0$.

   (b) Find the particular solution of $(D^2 - 3D + 2)y = \frac{1}{1+e^{-x}}$, by the method of variation of parameters and hence find the general solution of $(D^2 - 3D + 2)y = \frac{1}{1+e^{-x}}$. [8]

5. (a) Solve : $y'' - 5y' + 6y = 0$.

   (b) Find the general solution of $y'' - 5y' + 6y = 2e^x$, by the method of reduction of order. [8]

Practical No. 6
Miscellaneous

1. Solve : $2y(x^2 - y + x)dx + (x^2 - 2y)dy = 0$. [4]

2. (a) The rate at which an ice-ball melts is proportional to the amount of ice at that instant. If the half the quantity of ice melts in 20 minutes, show that after one hour the amount of the ice left will be $\frac{1}{8}$th of the original. [4]

   (b) Show that the family $y^2 = 4a(x + a)$ is self orthogonal. [4]

3. a) Show that $y = 2x^2e^{2x}$ is a solution of the differential equation $D^2(D-2)^2 = 16e^{2x}$. [2]
   b) Show that $y = -3e^{-x}\cos 4x$ is a solution of the differential equation $(D^2 + 2D + 1)y = 48e^{-x}cos4x$. [2]

4. Solve $(D^2 + D + 1)y = x\cos x$. [4]

5. Show that the initial value problem $(D^2+1)y = 2\cos x$,when $x = 0, y = 0$ and when $x = \pi, y = 0$,has infinitely many solutions. [8]

UNIVERSITY OF PUNE
S.Y.B.Sc. Mathematics Question Bank
Practicals based on Paper II(B)
Semester-II: Discrete Mathematics

Practical No. 1
Mathematical Induction

1. Prove by mathematical induction that if $A_1, A_2, \ldots, A_n$ and B are any $n+1$ sets, then
   $(\cup_{i=1}^{n} A_i) \cap B = \cup_{i=1}^{n}(A_i \cap B)$, $\forall n \in \mathbb{N}$. [8]

2. Prove that $(3+\sqrt{5})^n + (3-\sqrt{5})^n$ is divisible by 2^n, $\forall n \in \mathbb{N}$. [8]

3. Prove that $n^3 + 2n$ is divisible by 3, $\forall n \in \mathbb{N}$. [4]

4. Prove that any amount of postage greater than or equal to 2 rupees can be built using only 2 rupees and 3 rupees stamps. [4]

5. Prove that $1 + 2^n < 3^n$, for $n \geq 2$. [4]

Practical No. 2
Permutations and Combinations

1. Let $A = \{a, b, c, d\}$.
   a) List the subsets and the number of subsets of each possible size for the set A.
   b) List the permutations of elements of A taken two at a time. [4]

2. In how many ways can six men and six women be seated in a row if
   a) any person may sit next to any other on the 12 chairs.
   b) men and women must occupy alternate seats on the 12 chairs.
   c) any person may sit next to any other if 15 chairs are placed in a row. [4]

3. If 8 fair coins are tossed and the results are recorded, how many
   a) sequences are possible?
   b) sequences contain exactly 3 tails?
   c) sequences contain exactly k tails where $k \leq 8$? [4]

4. In how many ways can Harry, Ron and Hermione share 15 pastries of different flavours if
   a) each takes 5 pastries?
   b) if Hermione takes 7 pastries and the other two take 4 each?
   c) if one of them takes 7 pastries and the remaining two

each take 4?

d) if one of them takes 7 pastries and the remaining two each take 4, where all pastries are identical? [8]

5. From a standard deck of 52 cards, how many 5-card hands
   a) can be drawn?
   b) consists only of spades?
   c) consists of cards from a single suit?
   d) have 2 clubs and 3 hearts?
   e) have 2 cards of one suit and 3 cards of a different suit?
   f) contain 2 aces and 3 kings? [8]

Practical No. 3
Pigeon Hole Principle and Recurrence Relations

1. Show that if seven integers from 1 to 12 are chosen, then two of them will add up to 13. [4]

2. Show that if any eight positive integers are chosen, two of them will have the same remainder when divided by 7. [4]

3. Solve the recurrence relation
$a_n = 5a_{n-1} - 6a_{n-2}; \;\; a_0 = 1, a_1 = 6.$ [4]

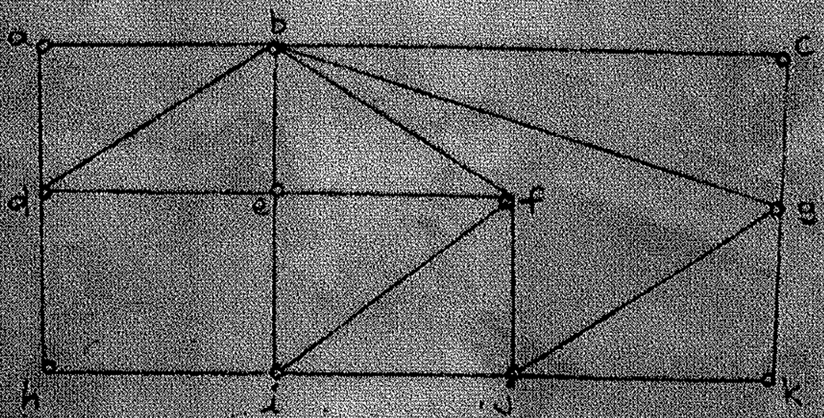
4. Solve the recurrence relation
$a_n - 3a_{n-2} + 2a_{n-3} = 0; \;\; a_0 = 0, a_1 = 8, a_2 = -2.$ [8]

5. Solve the recurrence relation
$a_n = a_{n-1} + a_{n-2}; \;\; a_0 = 1, a_1 = 1.$ [8]

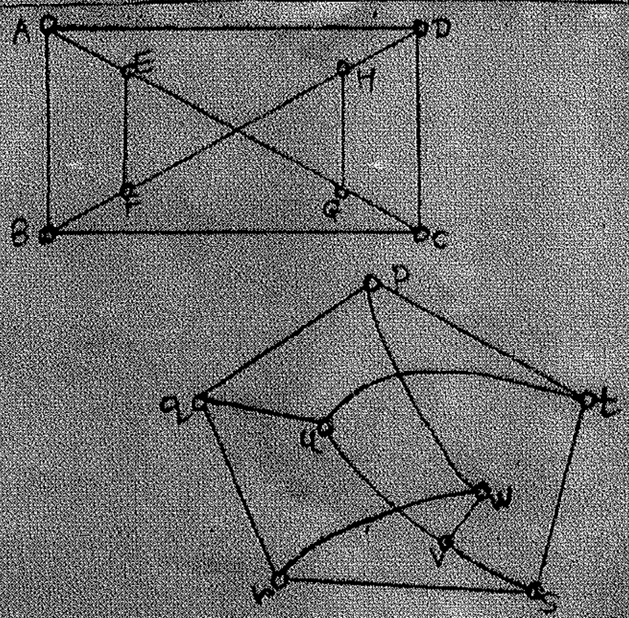
Practical No. 4
Topics in Graph Theory-I

1. For each of the following sequence, determine if there exists a graph whose degree sequence is the one specified. In each case, either draw a graph, or explain why no graph exists.
   a) 5, 4, 3, 2, 1 b) 5, 5, 4, 3, 2, 1 c) 4, 4, 3, 3, 2. [8]

2. Use Fleury's Algorithm to find an Eulerian circuit in the following graph: [8]



3. Determine if the following graphs are isomorphic: [4]



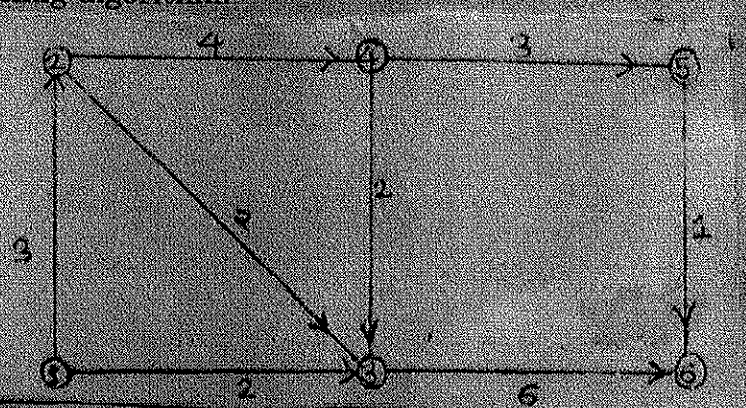
4. Construct an example of a graph which is Eulerian, but not Hamiltorian. Justify your answer. [4]

5. For a complete graph on $n-$ vertices, $K_n, n \geq 3$,
   a) how many edges must a Hamiltonian circuit have?
   b) how many different Hamiltonian circuits, beginning at a fixed vertex, are there? [4]

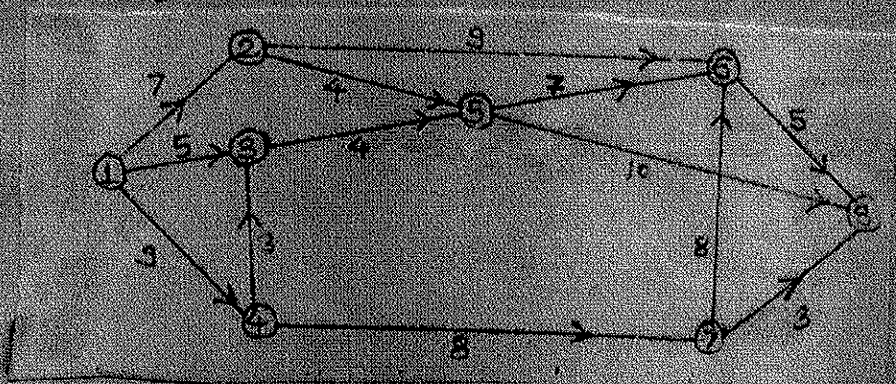
Practical No. 5
Topics in Graph Theory-II

1. List all possible disticnt Hamitonian circuits of a complete graph K_4. [8]

2. Find a maximum flow in the following network by using the labeling algorithm: [8]



3. Give example of two cuts and their capacities for the following network: [4]

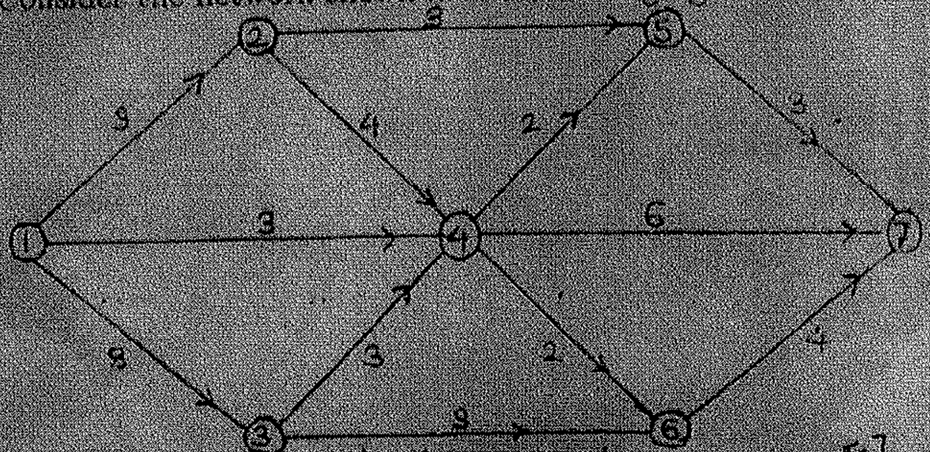


4. Consider the matrix M_R for a relation from A to B given below:

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}.$$

Find a maximal matching for A, B and R. [4]

5. Consider the network shown in the following figure:

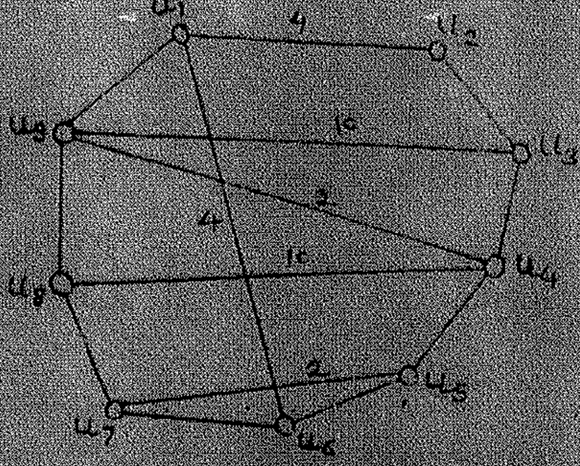


Construct two flows in the above network. [4]

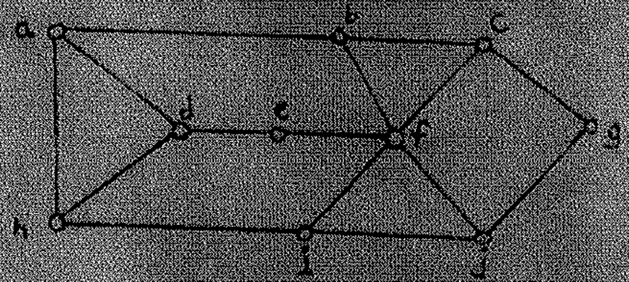
Practical No. 6
Trees

1. Draw all possible nonisomorphic trees on 4 vertices. [4]

2. Apply Kruscal's Algorithm to find the shortest spanning tree of the following weighted graph. [8]

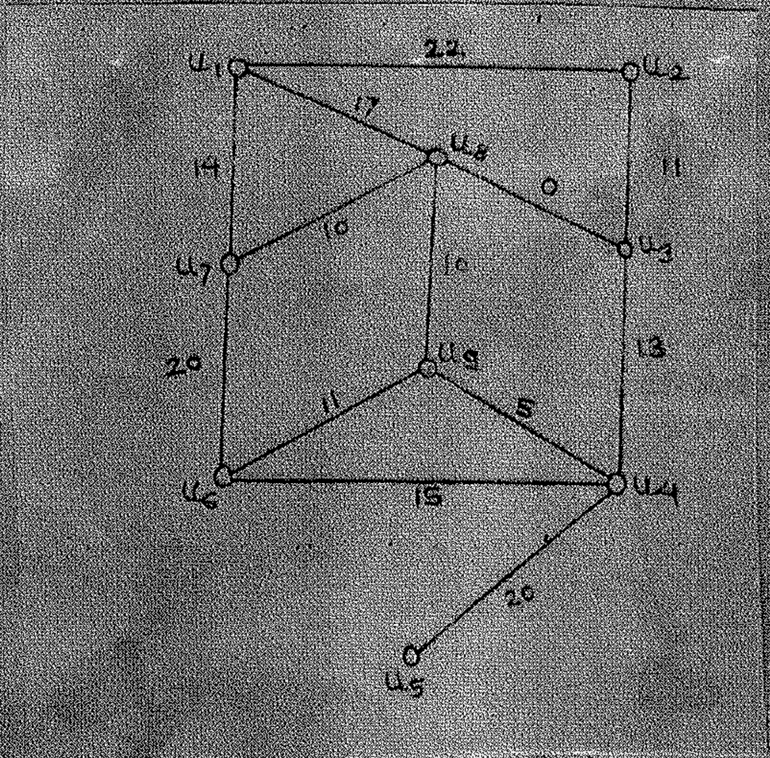


3. Determine whether the following graph is Hamiltonian. If yes, find the Hamiltonian circuit. [4]



4. Give an example of complete bipartite graph that has a complete matching. [4]

5. Consider the following weighted graph. Obtain any three spanning trees and their weights. [4]



Semester-II: Linear Algebra

Practical No. 1
Subspace and Linear Dependence

1. Let $V = \{(x, y) \in \mathbb{R}^2 | x, y > 0\}$. For $u = (x_1, y_1), v = (x_2, y_2) \in V$, $k \in \mathbb{R}$ define $+$ and $\cdot$ operations as $u + v = (x_1x_2, y_1y_2)$ and $k \cdot u = (x_1^k, y_1^k)$. Show that V is real vector space w.r.t. these operations. **[8]**

2. Check whether $W = \{(x, y, z) | x - y + z = 0\}$ is a subspace of vector space $\mathbb{R}^3$. Give a geometrical interpretation of W. **[4]**

3. Let $S = \{e_1, e_2, e_1 + e_2\}$ where $e_1 = (1, 0, 0), e_2 = (0, 1, 0)$. Find $L(S)$, linear span of S. Give a geometrical interpretation of $L(S)$. **[4]**

4. Check whether the set $S = \{(-1, 2, 3), (2, 5, 7), (3, 7, 10)\}$ is a linearly dependent set in $\mathbb{R}^3$. **[4]**

5. For which values of λ do the following vectors $v_1 = (\lambda, -1/2, -1/2)$, $v_2 = (-1/2, \lambda, -1/2)$,$v_3 = (-1/2, -1/2, \lambda)$ are linearly dependent in $\mathbb{R}^3$? **[4]**

Practical No. 2
Basis and Dimension

1. Let $v_1 = (1,2,1), v_2 = (2,9,0), v_3 = (3,3,4)$. Show that the set $S = \{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$. Find the coordinate vector of the vector $v = (5,-1,9)$ w.r.t. S. [4]

2. Show that the set $S = \{1, t+1, t^2+1\}$ is a basis for P_2. Express $p(t) = t^2 + t + 1$ as a linear combination of vectors in S. [4]

3. Find a basis and the dimension of the linear subspace of $\mathbb{R}^n$ given by $\{(x_1, x_2, \ldots, x_n) : x_1 + x_2 + \ldots + x_n = 0\}$. [4]

4. Find basis and dimension of the solution space of the following system of equations:

$$\begin{aligned} x + 2y - z + 3w &= 0 \\ 2x - y + z + w &= 0 \\ 3x + y + 4w &= 0 \end{aligned}$$

5. Find a basis for the null space, row space and column space of

$$A = \begin{pmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{pmatrix}$$ [8]

Practical No. 3

Linear Transformations

1. Check which of the following are linear transformations :

   (a) $T : \mathbb{R}^3 \to \mathbb{R}^2$ is defined as $T(x, y, z) = (x, yz)$

   (b) $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined as $T(x, y, z) = (x + 2y, y - 3z, x + z)$

   (c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined as $T(x, y) = (x + y, |y|)$

   (d) $T : \mathbb{R}^2 \to \mathbb{R}^3$ is defined as $T(x, y) = (x, y, y + 1)$ [8]

2. Find the range and kernel of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined as $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + z \\ x + y + 2z \\ 2x + y + 3z \end{pmatrix}$. Also find rank and nullity of T.
   [8]

3. Let $S = u_1 = (-1, 0, 1), u_2 = (0, 1, -1), u_3 = (1, -1, 1)$ be a basis for $\mathbb{R}^3$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be linear transformation for which $T(u_i) = e_i, i = 1, 2, 3$ where $\{e_1, e_2, e_3\}$ is standard basis for $\mathbb{R}^3$. Find formula for $T(x, y, z)$ and use it to compute $T(2, 1, -3)$. [4]

4. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be linear map with $Te_1 = e_2, Te_2 = e_3, Te_3 = 0$ Then show that $T \neq 0, T^2 \neq 0, T^3 = 0$. [4]

5. Let $T : P_1 \to \mathbb{R}^2$ be function defined by the formula $T(p(x)) = (p(0), p(1))$.

   (a) Show that T is linear isomorphism.

   (b) Find $T(1 - 2x)$. Find $T^{-1}(2, 3)$. [8]

Practical No. 4
Inner product spaces

1. For any $x, y \in \mathbb{R}^2$, where $x = (x_1, x_2), y = (y_1, y_2)$, show that $< x, y >= y_1(x_1 + 2x_2) + y_2(2x_1 + 5x_2)$ defines an inner product on $\mathbb{R}^2$. [4]

2. Compute the angle between

   (a) $v = e_1, w = e_1 + e_2$ in $\mathbb{R}^2$ where $e_1 = (1, 0)$, $e_2 = (0, 1)$.

   (b) $v = (x, y)$ and $w = (-y, x), x \neq 0, y \neq 0$ in $\mathbb{R}^2$. [4]

3. In an inner product space, show that $\|x + y\| = \|x\| + \|y\|$ if and only if one is non negative multiple of the other. [4]

4. Let P_n be the space of all polynomials of degree $\leq n$. What is dimension of P_n? Define $< p, q >= \int_{-1}^{1} p(x)q(x)dx$. What is the length of $p(x) = x$ in P_2. Apply the Gram Schmidt process to the basis $\{1, x, x^2\}$ w.r.t above inner product. [8]

5. Apply Gram Schmidt process to obtain an orthonormal basis from $\{(1, 0, 1), (1, -1, 0), (1, 1, 1)\}$. [8]

Practical No. 5
Eigenvalues and Eigenvectors

1. Find the eigenvalues and eigenvectors of the matrix
$A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{pmatrix}.$ [8]

2. Verify Cayley Hamilton theorem for a matrix
$A = \begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}.$ [4]

3. Let $T : R^3 \to R^3$ be linear transformation given by
$T(x, y, z) = (x + y + z, 2y + z, 2y + 3z)$. Find eigenvalues of T and eigenspace of each eigenvalue. [8]

4. Find eigenvalues and eigenvectors of A, where $A = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & -1 \\ 2 & 1 & 3 \end{pmatrix}.$
Are the eigenvectors of A linearly independent? [8]

Practical No. 6
Miscellaneous

1. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (x + y, y + z, z + x)$. Find a similar formula for T^{-1}. **[4]**

2. Let $\{x, y, z\}$ be linearly independent set of vector space V. Let $u = x$, $v = x + y, w = x + y + z$. Prove that $\{u, v, w\}$ is linearly independent set. **[4]**

3. Find a basis for the following subspaces of $\mathbb{R}^3$.

   (a) $\{(x, y, z) : z = x + y\}$

   (b) $\{(x, y, z) : x = y\}$ **[4]**

4. In P_n, each of

$$\begin{aligned} W_1 &= \{f \in P_n | f(0) = 0\}, \\ W_2 &= \{f \in P_n | f(1) = 0\}, \\ W_3 &= \{f \in P_n | f(0) = f(1) = 0\} \end{aligned}$$

   is a subspace of P_n. Find their dimensions. **[8]**

5. Apply Gram Schmidt process to $x_1 = (1, -2, 2), x_2 = (-1, 0, 1),\ x_3 = (5, -3, -7)$ in $\mathbb{R}^3$ with the dot product. **[8]**

UNIVERSITY OF PUNE

S.Y.B.Sc. MATHEMATICS Question Bank

Practicals Based on

Semester-I: Numerical Analysis P-II (B)

Practical No. 1
Title: Errors and Solutions of Equations

1. (a) Round off the following numbers to two decimal places:

   48.21416, 2.375, 2.3642

   (b) Round off the following numbers to four significant figures:

   38.46235, 0.70029

   0.0022218, 19.235101 [4]

2. Find Absolute, Relative and Percentage errors of the following:
   An approximate value of π is given by 3.1428517 and its true value is 3.1415926. [4]

3. Using Sturm's theorem, find the number and position of the real roots of the equation $f(x) = x^3 - 3x^2 - 4x + 13 = 0$. [4]

4. Using Sturm's theorem, find the number and position of the real roots of the equation $f(x) = x^4 - x^3 - 4x^2 + 4x + 1 = 0$. [4]

5. Using Regula-Falsi Method, find a root of the equation $x^3 - 9x + 1 = 0$ lying between 2 and 4 correct to 4 decimal places. [8]

Semester-I: Numerical Analysis P-II (B)

Practical No. 2

Title: Solution of Equations

1. Obtain Newton-Raphson formula to find $\sqrt[3]{c}$ and $\sqrt[4]{c}$ where $c \geq 0$ and hence find a) $\sqrt[3]{12}$ b) $\sqrt[4]{72}$ [4+4]

2. Using Newton-Raphson method, find a root of the equation $x^3+x^2+3x+4=0$ correct to 2 decimal places which lies between -2 and -1. [4]

3. Using Newton-Raphson method, find the real roots of following equations: (a) $x = e^{-x}$ (b) $x \sin x + \cos x = 0$ upto four decimals. [4+4]

4. Solve the following system of equations by Gauss-Seidel iteration method:

   $27x + 6y - z = 85$

   $6x + 15y + 2z = 72$

   $x + y + 54z = 110$ [8]

5. Solve the following system of equations by Gauss-Seidel iteration method:

   $2x - y + z = 5$

   $x + 3y - 2z = 7$

   $x + 2y + 3z = 10$ [4]

Practical No. 3

Title: Fitting of Polynomials

1. The table below gives the temperature T (in ^0c) and length l (in mms)of a heated rod. If $l = a_0 + a_1T$, find the values of a_0 and a_1 using linear least squares.

T	40	50	60	70	80
l	600.5	600.6	600.8	600.9	601.0

[4]

2. The weights of a calf taken at weekly intervals are given below. Fit a straight line using the method of least squares and calculate the average rate of growth per week.

Age (x)	1	2	3	4	5
Weight (y)	52.5	58.7	65.0	70.2	75.4
Age (x)	6	7	8	9	10
Weight (y)	81.1	87.2	95.5	102.2	106.4

[8]

3. Determine the constants a, b and c by the least-squares method such that $y = a + bx + cx^2$, fits the following data:

x	1.0	1.5	2.0	3.0	3.5	4.0
y	1.1	1.2	1.5	2.8	3.3	4.1

[8]

4. Find the function of the type $y = ax^b$ to the following data:

x	2	4	7	10	20	40	60	80
y	43	25	18	13	8	5	3	2

[8]

5. Find the best values of c and d if the curve $y = ce^{dx}$ is fitted to the data:

x	0	0.5	1.0	1.5	2.0	2.5
y	0.10	0.45	2.15	9.15	40.35	180.75

[8]

Semester-I: Numerical Analysis P-II (B)

Practical No. 4

Title: Interpolation

1. Represent the function $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$ and it's successive differences in factorial notation. [4]

2. (a) Evaluate $\Delta^2(\cos 2x)$ [8]

   (b) Prove that

   $$u_0 + u_1 + \cdots + u_n = {}^{n+1}C_1 u_0 + {}^{n+1}c_2 \Delta u_0 + {}^{n+1}c_3 \Delta^2 u_0 + \cdots + \Delta^n u_0$$

3. Given that

   $\log 310 = 2.4913617$, $\log 320 = 2.5051500$, $\log 330 = 2.5185139$,

   $\log 340 = 2.5314781$, $\log 350 = 2.5440680$, $\log 360 = 2.5563025$

   Find the value of $\log 337.5$ [4]

4. Find the missing term in the following table

x	0	1	2	3	4
y	1	3	9	?	81

   Explain why the result differ from 27. [4]

5. Use Lagranges interpolation formula to express the following functions as sums of partial fractions.

   $$f(x) = \frac{x^2 + 6x + 1}{(x-1)(x+1)(x-4)(x-6)}$$ [8]

Semester-I: Numerical Analysis P-II (B)

Practical No. 5

Title: Numerical Integration

1. Given the set of tabulated points $(1, -3)$, $(3, 9)$, $(4, 30)$ and $(6, 132)$, obtain the values of y when $x = 5$ using Newton's divided-difference formula. [4]

2. Compute the value of $\log 2$ from the formula $\log 2 = \int_1^2 \frac{1}{x}\, dx$ by using Trapezoidal rule taking 10 subintervals. [4]

3. The velocities of a car (running on a straight road) at the intervals of 2 minutes are given below:

Time in min.	0	2	4	6	8	10	12
Velocity in km/hr.	0	22	30	27	18	7	0

   Apply Simpson's $1/3^{rd}$ rule to find the distance covered by the car. [4]

4. Using Simpson's $3/8^{th}$ rule, evaluate $I = \int_0^1 \frac{1}{1+x}\, dx$ with $h = 1/6$ and compare the result. [4]

5. Evaluate $\int_4^{5.2} \log_e x\, dx$ by using Simpson's $\frac{1}{3}^{\text{rd}}$ and $\frac{3}{8}^{\text{th}}$ rule using six equal subintervals. [8]

Semester-I: Numerical Analysis P-II (B)

Practical No. 6

Title: Numerical Solutions of First Order Ordinary Differential Equations

1. Using Euler's method, solve the differential equation $\frac{dy}{dx} = x^2 + y^2$ with initial condition $y(0) = 0$ by taking interval $h = 0.1$ and compute $y(0.5)$. [4]

2. Solve by Euler's Method, the equation $\frac{dy}{dx} = xy$ with $y(0) = 1$ and find $y(0.4)$ by taking $h = 0.1$ [4]

3. By using Euler's Modified Method, given that $\frac{dy}{dx} = \log(x + y)$ with initial condition $y(0) = 1$, find $y(0.2)$ and $y(0.5)$ [4+4]

4. By using Euler's Modified Method, given that $\frac{dy}{dx} = x + y$ with initial condition that $y(0) = 1$, find $y(0.05)$ and $y(0.1)$ [8]

5. Use Runge-Kutta Method to approximate y, when $x = 0.1$ and $x = 0.2$ given that $x = 0$ when $y = 1$ and $\frac{dy}{dx} = x + y$. [8]

Practical No. 1
Vector functions of One Variable

1. If $\mathbf{f}(x) = \dfrac{\tan 3x}{x}\mathbf{i} + \dfrac{\log(1+x)}{x}\mathbf{j} + \dfrac{2^x - 1}{x}\mathbf{k}, x \neq 0$ find $\mathbf{f}(0)$ so that $\mathbf{f}$ is continuous at 0. [4]

2. If $\mathbf{f}(x) = \dfrac{\sin^{-1} x - \sin^{-1} a}{x-a}\mathbf{i} + \dfrac{e^x - e^a}{x-a}\mathbf{j} + \dfrac{x \sin a - a \sin x}{x-a}\mathbf{k}, x \neq a$ find $\lim_{x\to a} \mathbf{f}(x)$. [4]

3. If $\mathbf{r} = \mathbf{a}\cos\omega t + \mathbf{b}\sin\omega t$, and $\mathbf{a}, \mathbf{b}, \omega$ are constants, show that (i) $\mathbf{r} \times \dot{\mathbf{r}}$ is a constant function and (ii) $\ddot{\mathbf{r}} = -\omega^2 \mathbf{r}$. [4]

4. Show that $\mathbf{r} = \mathbf{a}e^{kt} + \mathbf{b}e^{lt}$ is a solution of the linear differential equation $\frac{d^2\mathbf{r}}{dt^2} + p\frac{d\mathbf{r}}{dt} + q\mathbf{r} = 0$, where k and l are distinct roots of the equation $m^2 + pm + q = 0$ and $\mathbf{a}$ and $\mathbf{b}$ are constant vectors. [4]

5. If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are derivable functions of t such that $\dfrac{d\mathbf{u}}{dt} = \mathbf{w} \times \mathbf{u}$ and $\dfrac{d\mathbf{v}}{dt} = \mathbf{w} \times \mathbf{v}$, show that $\dfrac{d}{dt}(\mathbf{u} \times \mathbf{v}) = \mathbf{w} \times (\mathbf{u} \times \mathbf{v})$. [8]

Practical No. 2
Curves in three dimensional space

1. Consider the right circular helix $\mathbf{r} = a\cos t\mathbf{i} + a\sin t\mathbf{j} + bt\mathbf{k}$, where $b = \cot\alpha$ and $0 < \alpha < \pi/2$. Find $\mathbf{t}, \mathbf{n}, \mathbf{b}$, and κ, τ. [8]

2. Find the length of the given curve:

   (i) $\mathbf{r} = 3t\mathbf{i} + 4t\mathbf{j} + 5\log\sec t\mathbf{k}, t \in [0, \pi/3]$,

   (ii) $\mathbf{r} = e^t\cos 2t\mathbf{i} + e^t\sin 2t\mathbf{j} + e^t\mathbf{k}, t \in [1, 4]$.

3. A particle moves along the curve $\mathbf{r} = 2t^2\mathbf{i} + (t^2 - 4t)\mathbf{j} + (3t - 5)\mathbf{k}$. Find its velocity and acceleration at $t = 1$ in the direction of the vector $\mathbf{n} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$. [4]

4. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are pairwise perpendicular unit vectors and are derivable functions of t, show that $\frac{d\mathbf{a}}{dt} = \pm\left(\frac{d\mathbf{b}}{dt} \times \mathbf{c} + \mathbf{b} \times \frac{d\mathbf{c}}{dt}\right)$. [4]

5. $\hat{\mathbf{r}}$ is a unit vector in the direction of $\mathbf{r}$ then prove that

$$\hat{\mathbf{r}} \times \frac{d\mathbf{r}}{dt} = \frac{1}{r^2}\mathbf{r} \times \frac{d\mathbf{r}}{dt}.$$

[4]

Practical No. 3
Differential Operators-I

1. Find the equations of tangent plane and normal line to the surface $x^3 - xy^2 + yz^2 - z^3 = 0$ at the point $(1, 1, 1)$. [4]

2. Find the directional derivative of the function $f(x, y, z) = xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t, y = t^2, z = t^3$ at $t = 1$. [4]

3. If $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\mathbf{m}, \mathbf{n}$ are constant vectors then prove that $\nabla \cdot [(\mathbf{m} \cdot \mathbf{r})\mathbf{n}] = \mathbf{m} \cdot \mathbf{n}$. [4]

4. If $\mathbf{f} = (y^2 \cos x + z^3)\mathbf{i} + (2y \sin x - 4)\mathbf{j} + (3xz^2 + 2)\mathbf{k}$ then show that $\mathbf{f}$ is conservative. Find scalar potential ϕ such that $\mathbf{f} = \nabla\phi$. [8]

5. If $\nabla\phi = \frac{\mathbf{r}}{r^5}$ and $\phi(1) = 0$ then show that $\phi(r) = \frac{1}{3}(1 - \frac{1}{r^3})$. [4]

Practical No. 4
Differential Operators-II

1. If $\bar{f} = (xyz)^p(x^q\mathbf{i} + y^q\mathbf{j} + z^q\mathbf{k})$ is irrotational then prove that $p = 0$ or $q = -1$. [4]

2. Find the constant a such that at any point of intersection of the two surfaces $(x-a)^2 + y^2 + z^2 = 3$ and $x^2 + (y-1)^2 + z^2 = 1$ their tangent planes will be perpendicular to each other. [8]

3. If $\mathbf{f} = (3x^2y - z)\mathbf{i} + (xz^3 + y^4)\mathbf{j} - 2x^3z^2\mathbf{k}$, find $\nabla(\nabla \cdot \mathbf{f})$ at the point $(2, -1, 0)$. [4].

4. Let $f(r)$ be a differentiable function of r. Prove that [8].

$$\nabla \cdot \left(\frac{f(r)}{r}\mathbf{r}\right) = \frac{1}{r^2}\frac{d}{dr}(r^2 f(r)).$$

Practical No. 5
Vector Integration-I

1. Evaluate line integral $\int_C (xy\mathbf{i} + (x^2 + y^2)\mathbf{j}) \cdot d\mathbf{r}$ where C is the x-axis from $x = 2$ to $x = 4$ and the line $x = 4$ from $y = 0$ to $y = 12$. [4]

2. Use Green's theorem in the plane to evaluate the line integral

$$\oint_C (2x - y^3)dx - xydy$$

   where C is the boundary of the region enclosed by the circles $x^2+y^2 = 1$ and $x^2 + y^2 = 9$. [4]

3. Verify Green's theorem in the plane for the line integral

$$\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$$

   where C is the boundary of the region enclosed by the parabolas $y = \sqrt{x}$ and $y = x^2$. [8]

4. Find the work done by the force $\mathbf{F} = 3x^2\mathbf{i} + (2xz - y)\mathbf{j} + z\mathbf{k}$ in moving a particle

   (i) along the line segment C_1 from $O(0, 0, 0)$ to $C(2, 1, 3)$,

   (ii) along the curve C_2 : $\mathbf{r} = \mathbf{f}(t) = 2t^2\mathbf{i} + t\mathbf{j}(4t^2 - t)\mathbf{k}$, $t \in [0, 1]$,

   (iii) along the curve C_3 given by $x^2 = 4y$, $3x^2 = 8z$, $x \in [0, 2]$. [8]

5. If $\mathbf{f} = 4xz\mathbf{i} - y^2\mathbf{j} + yz\mathbf{k}$, then evaluate $\iiint_V \nabla \cdot \mathbf{f} dV$, where V is the volume bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. [4]

Practical No. 6
Vector Integration-II

1. Evaluate surface integral $\iint\limits_S (yzi + zxj + xyk).ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. [8]

2. Using Gauss's Divergence theorem evaluate surface integral

$$\iint\limits_S x^3 dydz + x^2 ydzdx + x^2 zdxdy$$

over the closed surface S bounded by the planes $z = 0$, $z = b$ and the cylinder $x^2 + y^2 = a^2$. [4]

3. Let $\mathbf{F} = (x + y)\mathbf{i} + (2x - z)\mathbf{j} + (y + z)\mathbf{k}$. Verify Stokes' theorem for the function $\mathbf{F}$ over the part of the plane $3x + 2y + z = 6$ in the first octant. [8]

4. Using divergence theorem, show that

$$\iint\limits_S (ax\mathbf{i} + by\mathbf{j} + cz\mathbf{k}) \cdot ds = \frac{4}{3}\pi(a + b + c),$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. [4]