Syllabus of
M. Sc. (Mathematics) Part II
University Courses

Semester III **Semester IV**

Compulsory Courses

MT-701. **Functional Analysis** MT-801. **Field Theory**

MT-702. **Ring Theory** MT-802. **Combinatorics**

Optional Courses (Any one out of three)

MT-703. **Mechanics** MT-803. **Differential Manifolds**

MT-704. **Measure and Integration** MT-804. **Algebraic Topology**

MT-705. **Graph Theory** MT-805. **Lattice Theory**

Departmental Courses (Any two)

MT 706 **Topics in Analysis-I**
MT 707 **Topics in Topology-I**
MT 708 **Topics in Algebra-I**
MT 709 **Topics in Discrete Mathematics-I**
MT 710 **Topics in Applied Mathematics-I**
MT 711 **Topics in Geometry-I**
MT 712 **Operations Research-I**
MT 713 **Cryptography**
MT 714 **Representation Theory of Groups**
MT 715 **Topics in Mechanics-I**
MT 716 **Topics in Number Theory-I**
MT 717 **Topics in Numerical Analysis-I**

MT 806 **Topics in Analysis-II**
MT 807 **Topics in Topology-II**
MT 808 **Topics in Algebra-II**
MT 809 **Topics in Discrete Mathematics-II**
MT 810 **Topics in Applied Mathematics-II**
MT 811 **Topics in Geometry--II**
MT 812 **Operational Research-II**
MT 813 **Coding Theory**
MT 814 **Representation Theory of Groups**
MT 815 **Topics in Mechanics-II**
MT 816 **Topics in Number Theory-II**
MT 817 **Topics in Numerical Analysis-II**

M.A. / M.Sc. (Mathematics) Syllabus

(For the Colleges Affiliated under Pune University)

PART- I			
Semester- I		**Semester- II**	
Course Code	**Course Title**	**Course Code**	**Course Title**
MT-501	Real Analysis	MT-601	General Topology
MT-502	Advanced Calculus	MT-602	Differential Geometry
MT-503	Linear Algebra	MT-603	Groups and Rings
MT-504	Number Theory	MT-604	Complex Analysis
MT-505	Ordinary Differential Equations	MT-605	Partial Differential Equations OR Object oriented Programming using C++

PART-II			
Semester-III		**Semester- IV**	
University Courses		**University Courses**	
(A) Compulsory Courses		**(A) Compulsory Courses**	
MT-701	Functional Analysis	MT-801	Field Theory
MT-702	Ring Theory	MT-802	Combinatorics
(B) Optional Courses (Any one)		**(B) Optional Courses (Any one)**	
MT-703	Mechanics	MT-803	Differential Manifolds
MT-704	Measure and Integration	MT-804	Algebraic Topology
MT-705	Graph Theory	MT-805	Lattice Theory
Departmental Courses (Any two)		**Departmental Courses (Any two)**	
MT-706	Topics in Analysis-I	MT-806	Topics in Analysis-II
MT-707	Topics in Topology-I	MT-807	Topics in Topology-II
MT-708	Topics in Algebra-I	MT-808	Topics in Algebra-II
MT-709	Topics in Discrete Mathematics-I	MT-809	Topics in Discrete Mathematics-II
MT-710	Topics in Applied Mathematics-I	MT-810	Topics in Applied Mathematics-II
MT-711	Topics in Geometry-I	MT-811	Topics in Geometry-II
MT-712	Operations Research-I	MT-812	Operations Research-II
MT-713	Cryptography	MT-813	Coding Theory
MT-714	Representation Theory of Groups	MT-814	Representation Theory of Groups
MT-715	Topics in Mechanics-I	MT-815	Topics in Mechanics-II
MT-716	Topics in Number Theory-I	MT-816	Topics in Number Theory-II
MT-717	Topics in Numerical Analysis-I	MT-817	Topics in Numerical Analysis-II

Paper MT-701

FUNCTIONAL ANALYSIS

1. Banach Spaces

The definition and some examples.
Continuous linear transformations.
The Hahn-Banach theorem.
The natural imbedding of N in N**.
The open mapping theorem.
The conjugate of an operator.

2. Hilbert Spaces

The definition and some simple properties.
Orthogonal complements.
Orthonormal sets.
The conjugate space H*
The adjoint of an operator.
Self-adjoint operators.
Normal and unitary operators.
Projections.

3. Finite-Dimensional spectral Theory

Matrices.
Determinants and the spectrum of an operator.
The spectral theorem.
A survey of the situation.

Text Book :

G. F. Simmons , Introduction to Topology and Modern Analysis, McGraw Hill.
Chapters : 9, 10, 11.

Reference Books :

1. B. V. Limaye, Functional Analysis, Wiley Eastern Ltd.
2. Bachman and Narici, Functional Analysis.
3. W. Rudin, Functional Analysis, Tata Mc Graw Hill Edition.

Paper MT -702

RING THEORY

1. Preliminaries: Rings- Definition, Examples, Ring Homomorphism, Ideals, Ring of Fractions.

2. Euclidean Domains, P.I.D.'s, U.F.D.'s.

3. Polynomial Rings: Definition, properties, Polynomial Rings over Fields, Polynomial Rings that are U.F.D.'s, Irreducibility Criteria.

4. Basic Definitions and Examples of Modules, Quotient Modules and Module Homomorphisms, Generation of Modules, Direct Sums and Free Modules.

Text Book:
Dummit and Foote, Abstract Algebra, second edition (Wiley India).

Sections: 7.1 - 7.5, 8.1 - 8.3, 9.1 - 9.5, 10.1 -10.3

Reference Books:

1. C. Musili, Rings and Modules, 2nd Revised Edition, Narosa Publishing House.

2. Luther and Passi, Algebra II, Narosa Publishing House.

3. Jain and Bhattacharya, Basic Abstract Algebra, Second Edition, Cambridge University Press.

Paper MT -703

MECHANICS

Elementary principles: Mechanics of a particle, Mechanics of a system of particles, Constraints, D'Alembert's principle and Lagrange's equations. Simple applications of the Lagrangian formulation.

Variational principles and Lagrange's equations : Hamilton's principle, Some techniques of the calculus of variations, Derivation of Lagrange's equations from Hamilton's principle, Conservation theorem and symmetry properties.

The two-body central force problem : Reduction to the one-body equivalent problem, Equations of motion and first integrals. The virial theorem, The differential equation for the orbit, and integrable power-law potentials, The Kepler problem: Inverse square law of force, The motion in time in the Kepler problem.

The kinematics of rigid body motion : The independent co-ordinates of a rigit body, Orthogonal transformations, The Euler angles, Euler's theorem on the motion of a rigid body, Finite rotations, Infinitesimal rotations.

The Hamilton equations of motion : Legendre transformations and the Hamilton equations of motion, Cyclic co-ordinates and conservation theorems. Derivation of Hamilton's equations from a variational principle.

Canonical transformations : The equations of canonical transformation, Examples of canonical transformations, Poisson brackets.

Text Book :

Goldsten H., Classical Mechanics, Addition-Wesley, Second Edition, Narosa Publishing House, 2002.

Chapter 1 : 1-1, 1-2, 1-3, 1-4, 1-6, **Chapter 2 :** 2-1, 2-2, 2-3, 2-4, 2-6.
Chapter 3 : 3-1, 3-2, 3-4, 3-5, 3-7, 3-8. **Chapter 4:** 4-1, 4-2, 4-4, 4-6, 4-7, 4-8.
Chapter 8 : 8-1, 8-2, 8-5, **Chapter 9:** 9-1, 9-2, 9-4.

Reference Books :

1. Tiwari, R.N. and Thakur, B.S., Classical Mechanics, Prentice-Hall of India, New Delhi, 2007.

2. Gregory, R. Dougals, Classical Mechanics, Cambridge University Press, 2006.

3. Pars L. A., A treatise on analytical dynamics, London : Heinemann, 1965.

Paper MT -704

MEASURE AND INTEGRATION

Review : Lebesgue Measure and Lebesgue Integration.

1. Measure spaces, Measurable functions, Integration, General convergence theorems, Signed measure, The Radon-Nikodym Theorem, The L^P- spaces.

2. Outer measure and measurability, The extension theorem, The Lebgesuge-Stieltjes integral, Product measure, Inner measure, Extension by sets of measure zero, Caratheodory outer measure, Hausdorff measure.

Text Book :

H. L. Royden, Real Analysis (Pearson Education).

Chapters : 11, 12 (except 12.5)

Reference Books :

1. P.R. Halmos , Measure Theory, Reprint (Springer-Verlag, 1974).

2. W. Rudin , Real and Complex Analysis, 3rd Edition, (Mc-Graw Hill).

3. C. D. Aliprants, O. Burkinshaw, Principles of Real Analysis ,(Harcourt Asia Pvt. Ltd.).

4. G. de Barra, Measure Theory and Integration, Wiley Eastern Ltd , 1981.

Paper MT -705

GRAPH THEORY

1. Fundamental Concepts :

Graphs, Matrices and isomorphism decomposition, connection in Graphs, bipartite graphs, Eulerian circuits, vertex degrees, and Graphic sequences.

2. Trees and Distance :

Trees, Distance in trees and Graphs, Enumeration of trees Caycley's formula, Spanning trees in graphs, minimum spanning trees, Kruskal's algorithm, shortest paths, Dijkstra's Algorithm.

3. Matchings :

Maximum Matchings, Hall's matching condition, Min-Max Theorems, Maximum bipartite Matching, weighted bipartite matching.

4. Connectivity and Paths :

Connectivity, edge-connectivity, blocks, 2-connected graphs,

5. Vertex Colorings and Upper Bounds. – Definitions and Examples, Brooks' Theorem.

Text Book :

West D.B. Introduction to Graph Theory (Second edition), Prentice Hall of India, New Delhi (2009).

Chapters : 1, 2, 3.1, 3.2, 4.1,5.1

Reference Books :

1. J. Clark, D.A. Holton, A First Look at Graph Theory, Allied Publishers.

2. R. J. Wilson, Introduction to Graph Theory, (Fourth Edition), Pearson Education, Singapore (2003).

Paper MT -801

FIELD THEORY

1. Algebraic extensions of fields :

Irreducible polynomials and Eisenstein criterion.
Adjunction of roots.
Algebraic extensions.
Algebraically closed fields, Existence and Uniqueness of algebraic closure (without proof).

2. Normal and separable extensions :

Splitting fields.
Normal extensions.
Multiple roots.
Finite fields.
Separable extensions.

3. Galois Theory :

Automorphism groups and fixed fields.
Fundamental theorem of Galois theory.
Fundamental theorem of algebra.

4. Applications of Galois Theory to classical problems.

Polynomials solvable by radicals.
Ruler and compass constructions.

Text Book : P. Bhattacharya and S. Jain, Basic Abstract Algebra, Second Edition, Cambridge University Press.
Chapters : 15, 16, 17, 18.3, 18.5.

Reference Books :

1. John M. Howie, Fields and Galois Theory, Springer Undergraduate Mathematics Series.
2. Dummit and Foote, Abstract Algebra, 2nd Edition, Wiley Eastern Ltd.
3. M. Nagata, Theory of Field, Marcel Dekker.
4. O. Zariski and P. Sammuel, Commutative Algebra, Vol. 1, Van Nostrand.

Paper MT -802

COMBINATORICS

1. General Counting Methods :

Counting Principles, Arrangements and selections, Arrangements and selections with Repetitions, Distributions, Binomial Identities,

2. Generating Functions :

Generating Function Models, Calculating coefficient of generating functions, Partitions, Exponential Generating Functions, A Summation Method.

3. Recurrence Relations:

Recurrence Relation Models, Divide and conquer Relations, Solution of Linear Recurrence Relations, Solution of Inhomogeneous Recurrence Relations, Solutions with Generating Functions.

4. Inclusion-Exclusion :

Counting with venn diagrams, Inclusion-Exclusion Formula, Restricted Positions and Rook polynomials.

Text Book :

Alan Tucker **:** Applied Combinations Fourth Edition (John Wiley and Sons, Inc).

Sections : 5.1 to 5.5, 6.1 to 6.5, 7.1 to 7.5, 8.1 to 8.3, , A4.

Reference Books :

1. V.K. Balkrishnan **:** Schaum's outline series. Theory and Problems of Combinations (Ms Graw Hill).
2. K.D. Joshi**:** Foundations of Discrete Mathematics (Wiley Eastern Limited).
3. Marshal Hall Jr.**:** Combinatorial Theory, Second Edition (Wiley Inter science Publications).

Paper MT -803

DIFFERENTIAL MANIFOLDS

Chapter 1 : Differential Manifolds

1. The volume of a Parallelopiped.
2. The volume of a Parametrized – Manifold.
3. Manifolds in $\mathbf{R}^n$.
4. The Boundary of a Manifold.
5. Integrating a Scalar Function over a Manifold.

Chapter 2 : Differential Forms

1. Multilinear Algebra.
2. Alternating Tensors.
3. The Wedge Product.
4. Tangent Vectors and Differential Forms.
5. The Differential Operator.
6. The Action of a Differentiable Map.

Chapter 3 : Stoke's Theorem

1. Integrating Forms over Parametrized-Manifolds.
2. Orientable Manifolds.
3. Integrating Forms over Oriented Manifolds.
4. The Generalized Stoke's Theorem.

Text Book :

James R. Munkres, Analysis on Manifolds, (Addision-Wesley Publishing Company).
Chapters : 5, 6(except Section 31), 7(except section 36,38)

Paper MT -804

ALGEBRAIC TOPOLOGY

Homotopy; Homotopy type and Retractions, Paths, Path connectedness, The Fundamental group (Homotopy group), Fundamental group of the circle, Covering spaces; Fibrations, simplexes and complexes, Simplicial Homotopy Theory.

Text Book:

1. B. K. Lahiri , A First course in Algebraic Topology (Second Edition), Narosa Publishing House, (2005).

Chapters : 3-11.

References:

1. M.A. Armstrong, Basic Topology, Springer Verlag (2004)(Chapters 5 and 8).
2. Munkres J. R., Topology, Prentice Hall (1975).

Paper MT -805

LATTICE THEORY

1. **Lattices :**

   (a) Equivalence of two definitions.
   (b) Homomorphisms
   (c) Sublattices, ideals and congruence relation.
   (d) Product of lattices
   (e) Polynomial identities in lattices.
   (f) Distributive and Modular lattices
   (g) Special elements in lattices.

2. **Characterization and Representation :**
   (a) Bickhoff's distributivity criterion.
   (b) Stone representation theorem.
   (c) Machbin Theorem.

3. **Modular and Semimodular lattices:**

   (a) Isomorphism Theorems.
   (b) Upper and lower covering conditions.
   (c) Semimodular lattices.
   (d) Jordan-Holder chain condition.

4. **Complete Lattice :**

   (a) Closure Operations
   (b) Embedding in complete lattices.
   (c) Conditional Completeness.
   (d) Fixpoint Theorem.

Text Books :

1. G. Gruatzer, General Lattice Theory, Academic Press, 1978.
   Chapters : 1, 2 (Section 1), 3 (section 1).

Reference Books :

1. G. Baskhoff , Lattice Theory, 3rd Edition, American Mathematical Society, 1940.
2. D. E. Rutherford, Introduction to Lattice Theory, Oliver and Boyd, London, 1965
3. G. Szasz, Introduction to Lattice Theory, Academic Press, New York, 1963.

Coding Theory

Error detection, correction and decoding

1.1. Communication channels
1.2. Maximum likelihood decoding
1.3. Hamming distance
1.4. Nearest neighbour/minimum distance decoding
1.5. Distance of a code
Exercises

Linear codes

2.1. Vector spaces over finite fields
2.2. Linear codes
2.3. Hamming weight
2.4. Bases for linear codes
2.5. Generator matrix and parity-check matrix
2.6. Equivalence of linear codes
2.7. Encoding with a linear code
2.8. Decoding of linear codes

Cyclic codes

3.1 Definitions
3.2 Generator polynomials
3.3 Generator and parity-check matrices
3.4 Decoding of cyclic codes
3.5 Burst-error-correcting codes
3.6 BCH codes

Text Book :

San Ling and Chaoping Xing., Coding Theory (A First Course)
Cambridge University Press.

Numerical Analysis

0. Preliminaries :

Convergence,Floating Point Number Systems, Floating Point Arithmetic.

1. Root finding methods :

Fixed Point Interaction Schemes, Newton's Method, Secant Method, Accelerating Convergence.

2. System of Equations :

Formation of Systems of Equations, Gaussian Elimination, Pivoting Strategies, Errors Estimates and Condition Number, LU decomposition, Direct Factorization, Iterative Techniques for Linear Systems, Nonlinear Systems of Equations.

3. Eigenvalues and Eigenvectors :

The Power Method,The Inverse Power Method, Reduction to Symmetric Tridiagonal form, Eigenvalues of Symmetric Tridiagonal Matrices.

4. Differentiation and Integration:

Numerical differentiation, using Lagrange's Interpolating polynomial, Numerical Integration, Newton-cotes Quadrature, Composite Newton-Cotes Quadrature.

5. Initial Value Problems of Ordinary Differential Equations :

Euler's Method, Runge-Kutta Methods, Multistep Methods, Convergence and Stability Analysis.

Text Book :

Brian Bradie, A Friendly Introduction to Numerical Analysis, Pearson Prentice Hall 2007.

Articals from Text Book : 1.2 – 1.4, 2.3 – 2.6, 3.1, 3.2, 3.4 -3.6, 3.8, 3.10, 4.1, 4.2, 4.4, 4.5, 6.1, 6.2,6.4-6.6, 7.2-7.6.

Websites : www.pcs.cnu.edu/~bbradie/textbookanswers.html

Prepared by : Prof. S.D. Deshmane.

Paper – MT 802

Title of Paper – Probability / Mathematical Statistics

Unit I : **Introduction :**

Combinatorial Methods, Binomial Coefficients.

Unit II : **Probability :**

Sample spaces, Event, Probability of an event, Rules of probability, Conditional probability, Independent Events, Baye's theorem.

Unit III : **Probability Distributions and Probability Densities :**

Random variables, Probability Distributions, Continuous Random Variables, Probability Density Functions, Multivariate Distributions, Marginal Distributions, Conditional Distributions.

Unit IV : **Mathematical Expectation :**

The expected value of a random variable, Moments, Chebyshev's theorem, M.G.F, Product moments, Moments of linear combinations of random variables,conditional expectations.

Unit V : **Special Probability Distributions :**

Discrete Uniform Distribution, Bernoulli, Binomial, Negative Binomial, Geometric, Hypergeometric Distribution, Poisson Distribution.

Unit VI : **Special Probability Densities :**

Uniform Distribution, Gamma, Exponential and Chi-square Distribution, Beta Distribution, Normal Distribution, Normal Approximation to the Binomial Distribution, Bivariate Normal Distribution.

Unit VII : **Function of Random Variables:**

Distribution function, Transformation Technique, Moment Generating Function Technique.

Unit VIII : Sampling Distributions:

Distribution of the mean, finite populations, chi-square distribution, t-Distribution, F- Distribution, Order Statistics.

Unit IX : Tests of Hypothesis :

Tests Concerning Means, Differences Between Means, Variances.

Unit X : Regression and Correlation :

Linear Regression, The method of least squares, Normal Regression Analysis, Normal Correlation Analysis, Multiple Linear Regression.

Text Book:

I. Miller and M. Miller, John E. Freund's **:** Mathematical Statistics with Applications. (Seventh Edition) (Pearson Education)

CHAPTER: 1 to 8, 13, 14.

Reference Books:

(1) Hogg, Mokean, Craig **:** Introduction to Mathematical Statistics, (Pearson Education).

(2) S. Arora and B. Lal **:** New Mathematical Statistics (Satya Prakashan).

R.E. Walpole and R.H. Myers **:** Probability and Statistics for Engineers and Scientists. (Second Edition) (Collier Macmillan).