UNIVERSITY OF PUNE
DEPARTMENT OF MATHEMATICS
SYLLABUS
M.Sc.

A two years duration course with total 100 credit points

FIRST YEAR

SEMESTER I
(All are compulsory and each course is of 5 credit points)

MT 101 Linear Algebra
MT 102 Topology
MT 103 Measure and Integration
MT 104 Algebra
MT 105 Numerical Analysis.

Total credits:25 points

SEMESTER II
(All are compulsory and each course is of 5 credit points)

MT 201 Functional Analysis
MT 202 Complex Analysis
MT 203 Field Theory
MT 204 Advanced Calculus
MT 205 Differential Equations

Total credits: 25 points

SECOND YEAR

(In each of semester III and semester IV, any five of the following courses which are running in the department should be opted. Each course is of 5 credit points.)

MT 01. Operations Research
MT 02. Integral Equations and Transforms
MT 03. Number Theory I
MT 04. Coding Theory
MT 05. Graph Theory I
MT 06. Lattice Theory I
MT 07. Computational Geometry
MT 08. Cryptography

MT 09. Financial Mathematics
MT 10. Modeling and Simulation
MT 11. Artificial Intelligence
MT 12. Symmetries
MT 13. Wavelets
MT 14. Combinatorics
MT 15. Partial Differential Equations
MT 16. Fuzzy Logic
MT 17. Statistics and Probability
MT 18. Fluid Dynamics
MT 19. Banach Algebra
MT 20. Boundary Value Problems
MT 21. Baer * Rings
MT 22. Matroid Theory I
MT 23. Sperner Theory
MT 24. Differential Equation and Dynamical System
MT 25. Mechanics
MT 26. Complex Analysis II
MT 27. Representation Theory of Groups
MT 28. Fourier Analysis on Finite Groups
MT 29. Differential Geometry
MT 30. Non-Linear Dynamical System
MT 31. Topics in Lie Groups.
MT 32. Algebraic Topology
MT 33. Advanced Calculus
MT 34. Projective Geometry
MT 35. Algebraic Geometry
MT 36. Algebraic Number Theory
MT 37. Algebraic Curves
MT 38. Commutative Algebra
MT 39. Advanced Lattice Theory II
MT 40. Graph Theory II
MT 41. Matroid Theory II
MT 42. Group Theory II
MT 43. Ring Theory
MT 44. Topics in Non Commutative Rings.

SEMESTER I

MIM 101 : Linear Algebra

1. **Prerequisites:** Vector Spaces: Definition and Examples, Subspaces, Bases and Dimensions, Linear Transformations, Quotient Spaces, Direct Sum, The matrix of Linear Transformation, Duality.

2. **Canonical Forms:** Eigenvalues and Eigenvectors, The minimal Polynomial, Diagonalisability, Triangular sable Operators, Jordan Forms, The Rational Forms.

3. **Inner Product Spaces:** Inner Product Spaces, Orthogonally, The Adjoint of Linear Transformation, Unitary operators, Self Adjoint and Normal Operators.

4. **Bilinear Forms:** Definition and Examples, The matrix of a Bilinear Form, Orthogonality, Classification of Bilinear Forms.

5. **Modules:** Definition and Examples, Further notions and Results.

6. **Free Modules:** Linear Independence, Bases of Free Modules, Matrices and Homeomorphisms.

Prescribed Books:

- **Luthar and Passi**, Modules (Narosa Publishing House).
- **Vivek Sahai and Vikas Bist,** Linear Algebra (Narosa Publishing House).

MT 102 : Topology

1. **Prerequisites:** Cartesian Products, Finite Sets, Countable and Uncountable Sets, Infinite Sets and Axiom of Choice, Well Ordered Sets.

2. **Topological Spaces :** Basis for a topology, Order topology, Subspace Topology, Product topology, closed sets and limit points, Continuous functions, Metric Topology

3. **Connected and Compact Spaces:** Connected spaces, Connected Subspaces of Real Line, Components and Local Connectedness, Compact spaces, Compact Subspaces of the Real Line, Limit point compactness, Local Compactness.

4. **Countablity and Separation Axioms:** Countability Axioms, Separation axioms Normal Spaces, Urysohn's Lemma(without proof), Titetz Extension Theorem (Without Proof), Metrization Theorem (without proof), Tychonoff's Theorem.

Prescribed Book:

- **J.R. Munkres**, Topology : A First Course. Second Edition.
  (Ch.1 : Sec 5,6,7,9,10; Ch.2 : Sec 12 to 21; Ch.3 : Sec 23 to 29; Ch.4 : Sec 30 to 35; Ch.5 : Sec 37).

MT 103 Measure and Integration

1. **Prerequisites:** Cardinal Numbers and Countability, Properties of Open Sets, Cantor Like Sets.

2. **Measure on Real Line :** Lebesgue Outer Measure, Measurable Sets, Regularity, Measurable Functions, Borel and Lebesgue Measurability.

3. **Integration of Functions on Real Variable :** Integration of Non Negative Functions, General Integral, Integration of Series, Riemann and Lebesgue Integral.

4. **Differentiation :** Functions of Bounded Variation, Lebesgue Differentiation Theorem, Differentiation Theorem, Differentiation and Integration.

5. **Inequalities and L^p spaces :** The Lp Spaces, The Convex Functions, Jensen's Inequalities, Inequalities of Holder and Minkowski, Completion of L^p.

6. **Convergence :** Convergence in Measure, Almost Uniform Convergence, Convergence Diagrams, Counter Examples

Prescribed Book:

- **G. de Barra**, Measure Theory and Integration, New Age International Ltd, Publishers.
  (Sec 1.5 to 1.7., 2.1 to 2.5., 3.1 to 3.4., 4.1 to 4.5., 5.1 to 5.6., 6.1 to 6.5., 7.1 to 7.4.).
  Reference Book:
  - **H.L.Roydon,** Real Analysis (Third Ed.), Prentice Hall 1995.

MT 104 : Algebra

1. Prerequisites: Semigroups and groups, Homomorphisms, Subgroups and cosets. Rings, Examples of rings, types of rings, subrings and characteristic of a ring.
2. Cyclic groups, permutation groups, generators and relations.
3. Normal subgroups and quotient groups. Isomorphism theorems, automorphisms, conjugacy and G-sets.
4. Normal series, Solvable groups, Nilpotent groups.
5. Group Homomorphisms, First Isomorphism Theorem, Fundamental Theorem of Finite Abelian Groups.
6. Permutation Groups, Cyclic decomposition, Alternating group A_n, Simplicity of A_n.
7. Structure of groups, Direct products, Finitely Generated Abelian Groups, Invariants of a finite abelian group
8. Sylow Theorems, Groups of order p2, pq.
9. Ideals and homomorphisms, maximal and prime ideals, nilpotent and nil ideals, Zorn's lemma
10. . Unique Factorisation Domains, Principal Ideal Domains, Euclidean Domains. Polynomials over UFD.

Prescribed Book:

- **P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul,** Basic Abstract Algebra (Second Ed.), Cambridge Univ. Press (Indian Ed. 1995).

Reference Book:

- **Joseph A. Gallian,**Contemporary Abstract Algebra (Fourth Ed.), Narosa, 1999.
- **I. S. Luthar and I. B. S. Passi,** Algebra-Vol. 1: Groups, Narosa, New Delhi, 1996.

MT : 105 Numerical Analysis

1. **Iterative solutions of nonlinear equation:** bisection method. Fixed-point interation, Newton's method, secant method, acceleration of convergence, Newton's method for two non linear equations, polynomial equation methods.
2. **Polynomial interpolation:** interpolation polynomial, divided difference interpolation, Aitken's formula, finite difference formulas, Hermite's interpolation, double interpolation.
3. **Linear systems of Equations:** Gauss Elimination, Gauss-Jordan method, LU decomposition, iterative methods, and Gauss- Seidel iteration.
4. **Numerical Calculus :** Numerical differentiation, Errors in numerical differentiation, Numerical Integration, Trapezoidal rule, Simpson's 1/3 - rule, Simpson's 3/8 rule, error estimates for Trapezoidal rule and Simpson's rule.
5. **Numerical Solution of Ordinary differential Equations :** Solution by Taylor series, Picard Method of successive approximations, Euler's Method, Modified Eular Method, Runge- Kutta Methods, Predicator-Corrector Methods.
6. **Eigenvalue Problem :** Power method, Jacobi method, Householder method.
7. **Practicals with Scilab.**

Prescribed Book:

– **S. S. Sastry,** Introduction Methods of Numerical Analysis (4th Edition)(Prentice-Hall).

Refrence Book:

– **K .E. Atkinson,**: An Introduction to Numerical Analysis.

– **J. I. Buchaman and P. R. Turner,** Numerical Methods and Analysis..

SEMESTER II

MT 201 : Functional Analysis

1. Normed spaces, continuity of linear maps, Hahn - Banach theorems, Banach spaces.
2. Uniform bounded principle, Application - Divergence of Fourier Series of Continuous Functions, closed graph theorem, Open mapping theorem, bounded inverse theorem, spectrum of Bounded Operator.
3. Duals and transposes, duals of $L^P[a,b]$ and $C[a,b]$.
4. Inner product spaces, orthonormal sets, approximation and optimization, projections, Riesz representation theorem.
5. Bounded operators and adjoints on a Hibert space, normal, unitary and self adjoint operators.
6. Fourier Series and Integrals.

Prescribed Book :

– **B.V. Limaye,** Functional Analysis (Second Edition) - New Age International Limited.
(Ch. 1: ; Ch. 2: Sec 5 to 8; Ch. 3: Sec 9 to 12; Ch. 4: Sec 13, 14; Ch. 6: Sec 21 to 24; Ch. 7: Sec 25, 26).

MT 202 : Complex Analysis

1. **Pre-requisites:**
   (a) **Topological and Analytical Preliminaries:** Point sets in the plane, sequences, compactness, stereographic projection, continuity.
   (b) **Elementary Functions:** Exponential functions, mapping properties, logarithmic function, complex exponents.
2. **Analytic Functions:** Cauchy-Riemann Equations, analyticity, harmonic functions.
3. **Power Series:** Sequences, uniform convergence, Maclaurin and Taylor series, operations on power series.
4. **Complex Integration and Cauchy's Theorem:** Curves, parameterizations, line integral, Cauchy's Theorem.
5. **Applications of Cauchy's Theorem:** Cauchy's integral formula, Cauchy's inequality and applications, maximum modulus theorem.
6. **Laurent Series and Residue Theorem:** Laurent series, classification of singularities, evaluation of real integrals, argument principle.
7. **Bilinear Transformations and Mappings:** Basic mappings, linear fractional transformations, other mappings.

Prescribed Book: S. Ponnuswamy and Herb Silverman, *Complex Variables with Applications*, Birkhäuser.

Reference Book: J. B. Convey, *Functions of one complex variables*, Narosa Publishing House.

MT 203 : Field Theory

1. **Prerequisites:** Definitions and basic properties Rings and fields, Ideals and homomorphisms, Characteristic of fields, Euclidean domains, Unique factorization, Polynomials.
2. **Field Extensions:** The degree of an extension, Extensions and polynomials, Polynomials and extensions.
3. **Applications to Geometry:** Ruler and compasses construction, An algebraic approach.
4. **Splitting Fields.**
5. **Finite Fields.**
6. **The Galois Group:** Monomorphisms between fields, Automorphisms, Groups and subfields, Normal extensions, Separable extensions, The Galois correspondence, The fundamental theorem, An example.
7. **Equations and Groups:** Solution by radicals of quadratics, cubics and quartics. Cyclotomic polynomials, cyclic extensions.
8. **Groups and Equations:** Insoluble quintics, General polynomials.
9. **Prescribed Book:**
   - J. M. Howie, Fields and Galois Theory, Springer Undergraduate Mathematics Series, 2006.
     (Chapters 1 to 8 and Chapter 10).

   Reference Books:
   - **M. Artin,** Algebra, Prentice-Hall, Englewood Cliffs, N.J., 1991.
   - **P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul,** Basic Abstract Algebra, Second Ed., Cambridge University Press, Cambridge, 1995.

MT 204 : Advanced Calculus

1. Compact and Connected Subsets of $\mathbb{R}^n$.
2. Differentiation : Derivative, Continuously Differentiable functions, Chain rule, Inverse function theorem, Implicit function theorem.
3. Integration: integral over a rectangle, Existence of the Integral, evaluation of the integral, integral over a bounded set and rectifiable sets, improper integrals
4. Change of Variable Theorem (Proof of one variable) and Statement of n-variables (with Illustrations)
5. Line Integrals with Applications

Prescribed Book:

- **J.R. Munkres**, Analysis on Manifolds.
  (Sections 4 to 15 and Section 17).

Reference Book:

- **T.M. Apostol**, Calculus (Volume II).
  (Chapter 10 : Sections 10.1 to 10.9).

MT 205 : Differential Equations

1. **Prerequisites:** Linear equations of the first order.
2. **Linear equations with constant coefficients :** Second order homogeneous equations, Initial value problems, Linear dependence and independence, Nonhomogeneous equations of n-th order, Algebra of constant coefficients.
3. **Linear equations with variable coefficients :** Initial value problems, Solutions of the homogeneous equation, Wronskian and linear independence, Reduction of order, Nonhomogeneous equations, Legendre equation.
4. **Linear Equations with regular singular points :** Euler equation, Second order equation with regular singular points, Exceptional cases, Bessel equation.
5. **Existence and uniqueness of solutions to first order equations:** Separation of variables, exact equations, Method of successive approximations, Lipschitz condition, Approximation to and uniqueness of solutions.
6. **Existence and uniqueness of solutions to systems and n-th order equations:** Complex n-dimensional space, Systems as vector equations, Existence and uniqueness of solutions to systems, Existence, Uniqueness for linear systems and equations of order n.

Prescribed Book:

- **E. A. Coddington,** An Introduction to Ordinary Differential Equatins (Prentice- Hall).

Reference Book:

- **G. F. Simmons and S. G. Krantz,** Differential Equatins (Tata McGraw-Hill).