UNIVERSITY OF PUNE

S.Y.B.Sc. MATHEMATICS Question Bank

Practicals Based on Paper I

Semester-I: Calculus of Several Variables

Practical No. 1

Limit, Continuity and Partial Derivatives

1. Test the following function for existence of simultaneous limit and iterated limits at the origin where,
   $f(x,y) = \dfrac{x-y}{x+y},\ (x,y) \neq (0,0).$ [4]

2. Evaluate $\displaystyle\lim_{(x,y)\to(0,1)} \frac{x+y-1}{\sqrt{x}-\sqrt{1-y}}$, if it exits. [4]

3. Discuss the continuity of f at $(0,0)$ and $(1,1)$ where
   $$\begin{aligned} f(x,y) &= xy, \quad |x| \geq |y| \\ &= -xy, \ |x| < |y|. \end{aligned}$$ [4]

4. Using definition find $f_x(0,0)$ and $f_y(0,0)$ where,
   $$\begin{aligned} f(x,y) &= 2xy\frac{x^2-y^2}{x^2+y^2},\ (x,y) \neq (0,0) \\ &= 0, \qquad\qquad (x,y) = (0,0). \end{aligned}$$ [4]

5. If $V = f(x,y),\ x = r\cos\theta,\ y = r\sin\theta$, then prove that
   $$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{1}{r}\frac{\partial V}{\partial r}.$$ [8]

Practical No. 2
Differentiability 1

1. If f and g are twice differentiable functions and $z = f(y + ax) + g(y - ax)$, show that $z_{xx} = a^2 z_{yy}$. [4]

2. Let $f(x, y) = (x^2 + y^2)\tan^{-1}\frac{y}{x}$, $x \neq 0$ and $f(0, y) = \frac{\pi y^2}{2}$, show that $f_{yx}(0, 0) = 1$ while $f_{xy}(0, 0)$ does not exist. [8]

3. By using the definition, show that $f(x, y) = \sqrt{|xy|}$ is not differentiable at $(0, 0)$. [8]

4. If $u = (1 - 2xy + y^2)^{-1/2}$, show that, $\frac{\partial}{\partial x}[(1 - x^2)\frac{\partial u}{\partial x}] + \frac{\partial}{\partial y}[y^2\frac{\partial u}{\partial y}] = 0$. [4]

Practical No. 3
Differentiability 2

1. Given z is a function of u and v, where $u = x^2 - y^2 - 2xy$, $v = y$, find $(x+y)\dfrac{\partial z}{\partial x} + (x-y)\dfrac{\partial z}{\partial y}$. [4]

2. Let $u = \sin^{-1}(x^2+y^2)^{\frac{1}{5}}$. Using Euler's theorem show that,
$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{2}{5}\tan u$ and
$x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \dfrac{2}{25}\tan u(2\tan^2 u - 3)$. [8]

3. Using differentials find approximate value of $\sqrt{\dfrac{4.1}{25.01}}$ [4]

4. Prove that $\sin x \sin y = xy - \frac{1}{6}[(x^3+3xy^2)\cos\theta x \sin\theta y + (y^3+3x^2y)\sin\theta x \cos\theta y]$, for some $\theta \in (0,1)$. [8]

5. Let $f(x,y) = x^2\sin\dfrac{1}{x} + y^2\sin\dfrac{1}{y}$, when $xy \neq 0$
$f(x,0) = x^2\sin\dfrac{1}{x}$, when $x \neq 0$
$f(0,y) = y^2\sin\dfrac{1}{y}$, when $y \neq 0$
$f(0,0) = 0$. Show that

   (a) f_x and f_y are not continuous at $(0,0)$.

   (b) f is differentiable at $(0,0)$. [4]

Practical No. 4
Extreme Values

1. Locate the stationary points of the following functions :

   (a) $f(x, y) = \sin x + \sin y + \sin(x + y)$

   (b) $f(x, y) = x^3 + y^2 + x^2y - x^2 - y^2$. [4]

2. A rectangular box open at the top is to have a volume of 32 cu.m. What must be the dimensions so that the total surface area is mininmum ? [8]

3. Obtain the shortest distance of the point $(1, 2, -3)$ from the plane $2x - 3y + 6z = 20$, using Lagrange's method of undetermined coefficients. [8]

4. Given the following critical points of the function $3x^2y - 3x^2 - 3y^2 + y^3 + 2$, examine for extreme values $(0, 0)$, $(0, 2)$, $(1, 1)$, $(-1, 1)$. [4]

Practical No. 5
Multiple Integrals 1

1. Evaluate $\displaystyle\int\int\int_V \frac{1}{(x+y+z+1)^3}\ dx\ dy\ dz$, where V is the region bounded by the planes $x=0,\ y=0,\ z=0$ and $x+y+z=1$. [4]

2. Evaluate $\displaystyle\int_0^2\int_0^x\int_0^{x+y} e^{x+y+z}dx\ dy\ dz.$ [4]

3. Change the order of integration,
$\displaystyle\int_0^{2a}\int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f\ dy\ dx.$ [4]

4. Change the order of the integration and hence evaluate $\displaystyle\int\int y\ dx\ dy$ over the region bounded by the line $y=x$ and the parabola $y=4x-x^2$. [8]

5. By double integration, find the area of the region bounded by the curves $y=x^2-9,\ y=9-x^2$. [4]

Practical No. 6
Multiple Integrals 2

1. Evaluate $\int\int_R (x+y)^3\, dx\, dy$ where R is bounded by $x+y = 1,\ x+y = 4,\ x-2y = 1,\ x-2y = -2$ using the substitution $x + y = u,\ x - 2y = v.$ [8]

2. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integration. [4]

3. Evaluate $\int\int x^2y^2\, dx\, dy$ over the domain $\{(x, y) : x \geq 0, y \geq 0, x^2 + y^2 \leq 1\}$. [4]

4. Evaluate $\int\int\int_R (x^2 + y^2)\, dx\, dy\, dz$ where R is the region bounded by $x^2 + y^2 = 2z$ and $z = 2$ using cylindrical polar co-ordinates. [8]

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Practicals Based on Paper II(A)

Semester-I: Differential Equations

Practical No. 1
Homogeneous Differential Equations

1. (a) Find the order and the degree of the differential equation:
$$\frac{[1+(y')^2]^{3/2}}{yy''+1+(y')^2} = 1.$$
(b) Determine whether the following function is homogeneous. If homogeneous, state its degree.
$$f(x,y) = \frac{(x^2+y^2)^{1/2}}{(x^2-y^2)^{7/2}}.$$ [4]

2. Solve: $xy^2dx + (y+1)e^x dy = 0$. [4]

3. Solve: $(x - y\ln y + y\ln x)\ dx + x(\ln y - \ln x)\ dy = 0$. [4]

4. Solve: $\dfrac{dy}{dx} + \dfrac{3x^2y}{1+x^3} = \dfrac{\tan^2 x}{1+x^3}$. [4]

5. Reduce the differential equation
$$(2x+y-3)\ dx = (2y+x+1)\ dy$$
to homogeneous form and find its solution. [8]

Practical No. 2
Exact Differential Equations

1. Solve $(xy+1)dx + x(x+4y-2)dy = 0$. [4]

2. Solve the equation $6y^2dx - x(2x^3+y)dy = 0$ by treating it as a Bernoulli's equation in the dependent variable x. [4]

3. Solve : $\mathrm{tanx}\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{y} = \mathrm{secx}$. [4]

4. Solve : $(x+a)y' = bx - ny; a, b, n$ are constants with $n \neq 0, n \neq -1$. [4]

5. Solve : $y(xy+2x^2y^2)dx + x(xy - x^2y^2)dy = 0$. [4]

Practical No. 3
Applications of Differential Equations

1. For the family $(x-a)^2+y^2=a^2$, find that member of the orthogonal trajectories which passes through $(1,2)$. [4]

2. Show that the family of curves $\dfrac{x^2}{c}+\dfrac{y^2}{c-\lambda}=1$ where c is a parameter, is self orthogonal. [4]

3. A bacterial population is known to have a logistic growth pattern with initial population 1000 and an equilibrium population of 10000. A count shows that at the end of 1hr there are 2000 bacteria present. Determine the population as a function of time. [4]

4. If half of a given quantity of radium decomposes in 1600 years, what percentage of the original amount will be left at the end of

   (a) 2400 years?

   (b) 8000 years? [8]

5. The decay rate of a certain substance is directly proportional to the amount present at that instant. Initially there are 27 gm. of the substance and 3 hours latter it is found that 8 gm. are left. Show that the amount left after one more hour is $\dfrac{16}{3}$ gm. [4]

Practical No. 4
Inverse Differential Operator

1. (a) Solve : $D^3(D^2+3D-2)y=0$.

   (b) Solve : $(4D^4-24D^3+35D^2+6D-9)y=0$. [4]

2. (a) Solve : $(D^3+2D^2+D)y=0$.

   (b) Find the particular solution of $(D^3+2D^2+D)y=e^{2x}$.

   (c) Find the particular solution of $(D^3+2D^2+D)y=x^2+x$. Hence find the general solution of $(D^3+2D^2+D)y=e^{2x}+x^2+x$. [8]

3. Solve : $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=x^2e^{3x}$. [4]

4. Solve : $(D^2+16)y=3\cos^2 2x+e^{2x}$. [4]

5. Solve : $(D^2+4)y=x\sin x$. [8]

Practical No. 5

Methods of Solving Second Order Differential Equations

1. (a) Solve : $(D^2 + D - 2)y = 0$.

   (b) Find the particular solution of $(D^2 + D - 2)y = 2x - 40\cos 2x$ by the method of undetermined coefficiants and hence write the general solution of $(D^2 + D - 2)y = 2x - 40\cos 2x$. [8]

2. (a) Solve : $(D^2 + 4D + 5)y = 0$.

   (b) Find the particular solution of $(D^2+4D+5)y = 10e^{-3x}$ by the method of undetermined coefficiants.

   (c) Find the particular solution of $(D^2+4D+5)y = 10e^{-3x}$, with initial conditions $y(0) = 4, y'(0) = 0$. [8]

3. (a) Solve : $(D^2 + 1)y = 0$.

   (b) Find the particular solution of $(D^2 + 1)y = \tan x$, by the method of variation of parameters and hence find the general solution of $(D^2 + 1)y = \tan x$. [8]

4. (a) Solve : $(D^2 - 3D + 2)y = 0$.

   (b) Find the particular solution of $(D^2 - 3D + 2)y = \frac{1}{1 + e^{-x}}$, by the method of variation of parameters and hence find the general solution of $(D^2 - 3D + 2)y = \frac{1}{1 + e^{-x}}$. [8]

5. (a) Solve : $y'' - 5y' + 6y = 0$.

   (b) Find the general solution of $y'' - 5y' + 6y = 2e^{x}$, by the method of reduction of order. [8]

Practical No. 6
Miscellaneous

1. Solve : $2y(x^2 - y + x)dx + (x^2 - 2y)dy = 0.$ [4]

2. According to data listed at www.census.gov, the words total population reached 6 billion persons in mid-1999, and was then increasing at the rate of about 212000 persons each day. Assuming that natural population growth at this rate continuous, answer the following questions.
   a) What is the annual growth rate,k?
   b) What will be the world population at the middle of 21 st century?
   c) How long will it take the world population to increase ten fold-there by reaching the sixty billion that some demographers believe to be the maximum for which the planet can provide adequate food supplies? [8]

3. a) Show that $y = 2x^2e^{2x}$ is a solution of the differential equation $D^2(D - 2)^2 = 16e^{2x}$. [2]
   b) Show that $y = x - 3cos4x$ is a solution of the differential equation $(D^2 + 2D + 1)y = 48e^{-x}cos4x$. [2]

4. Solve $(D^2 + D + 1)y = x$cosx. [4]

5. Show that the initial value problem $(D^2+1)y = 2$cosx,when $x = 0, y = 0$ and when $x = \pi, y = 0$,has infinitely many solutions. [8]

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Practicals Based on

Semester-I: Numerical Analysis P-II (B)

Practical No. 1
Title: Estimation of Errors and Solutions of Equations

1. (a) Round off the following numbers to two decimal places:
   48.21416, 2.375, 2.3642

   (b) Round off the following numbers to four significant figures:
   38.46235, 0.70029
   0.0022218, 19.235101 [4]

2. Find Absolute, Relative and Percentage errors of the following:
   An approximate value of π is given by 3.1428517 and its true value is 3.1415926. [4]

3. Using Sturm's theorem, find the number and position of the real roots of the equation $f(x) = x^3 - 3x^2 - 4x + 13 = 0$. [4]

4. Using Sturm's theorem, find the number and position of the real roots of the equation $f(x) = x^4 - x^3 - 4x^2 + 4x + 1 = 0$. [4]

5. Using Regula-Falsi Method, solve $x^3 - 9x + 1 = 0$ for the roots lying between 2 and 4. [8]

Semester-I: Numerical Analysis P-II (B)

Practical No. 2

Title: Solution of Equations

1. Obtain Newton-Raphson formula to find $\sqrt[3]{c}$ and $\sqrt[4]{c}$ where $c \geq 0$ and hence find
   a) $\sqrt[3]{12}$ b) $\sqrt[4]{72}$ [4+4]

2. Using Newton-Raphson method, find the roots of the equations $x^3 + x^2 + 3x + 4 = 0$ [4]

3. Using Newton-Raphson method, find the real roots of following equations:
   (a) $x = e^{-x}$ (b) $x\text{sinx} + \text{cosx} = 0$ upto four decimals. [4+4]

4. Solve the following system of equations by Gauss-Seidel iteration method:
   $27x + 6y - z = 85$
   $6x + 15y + 2z = 72$
   $x + y + 54z = 110$ [8]

5. Solve the following system of equations by Gauss-Seidel iteration method:
   $2x - y + z = 5$
   $x + 3y - 2z = 7$
   $x + 2y + 3z = 10$ [4]

Semester-I: Numerical Analysis P-II (B)

Practical No. 3

Title: Fitting of Polynomials

1. The table below gives the temperature T (in ^0c) and length l (in mms)of a heated rod. If $l = a_0 + a_1T$, find the values of a_0 and a_1 using linear least squares.

T	40	50	60	70	80
l	600.5	600.6	600.8	600.9	601.0

[4]

2. The weights of a calf taken at weekly intervals are given below. Fit a straight line using the method of least squares and calculate the average rate of growth per week.

Age (x)	1	2	3	4	5
Weight (y)	52.5	58.7	65.0	70.2	75.4
Age (x)	6	7	8	9	10
Weight (y)	81.1	87.2	95.5	102.2	106.4

[8]

3. Determine the constants a, b and c by the least-squares method such that $y = a + bx + cx^2$, fits the following data:

x	1.0	1.5	2.0	3.0	3.5	4.0
y	1.1	1.2	1.5	2.8	3.3	4.1

[8]

4. Find the function of the type $y = ax^b$ to the following data:

x	2	4	7	10	20	40	60	80
y	43	25	18	13	8	5	3	2

[8]

5. Find the best values of c and d if the curve $y = ce^{dx}$ is fitted to the data:

x	0	0.5	1.0	1.5	2.0	2.5
y	0.10	0.45	2.15	9.15	40.35	180.75

[8]

Semester-I: Numerical Analysis P-II (B)

Practical No. 4

Title: Interpolation

1. Represent the function $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$ and its successive differences in factorial notation. [4]

2. (a) Evaluate $\Delta^2(cos2x)$

   (b) Prove that
   $u_0+u_1+u_2+\ldots+u_n = {}^{n+1}C_1u_0+ {}^{n+1}c_2\Delta u_0+ {}^{n+1}c_3\Delta^2u_0+ \ldots+ \Delta^n u_0$ [8]

3. Given that
   $\log 310 = 2.4913617$, $\log 320 = 2.5051500$,
   $\log 330 = 2.5185139$, $\log 340 = 2.5314781$,
   $\log 350 = 2.5440680$, $\log 360 = 2.5563025$
   Find the value of $\log 337.5$ [4]

4. Find the form of the function for following data:

x :	0	1	2	3	4
$f(x)$:	3	6	11	18	27

[4]

5. Use Lagranges interpolation formula to express the following function as sums of partial fractions.
   $$f(x) = \frac{x^2 + 6x + 1}{(x-1)(x+1)(x-4)(x-6)}$$ [8]

Semester-I: Numerical Analysis P-II (B)

Practical No. 5

Title: Numerical Integration

1. Given the set of tabulated points $(1, -3)$, $(3, 9)$, $(4, 30)$ and $(6, 132)$, obtain the values of y when $x = 5$ using Newton's divided-difference formula. [4]

2. Compute the value of $\log 2$ from the formula $\log 2 = \int_1^2 \frac{1}{x}\, dx$ by using Trapezoidal rule taking 10 subintervals. [4]

3. The velocities of a car (running on a straight road) at the intervals of 2 minutes are given below:

Time in min.	0	2	4	6	8	10	12
Velocity in km/hr.	0	22	30	27	18	7	0

   Apply Simpson's $1/3^{rd}$ rule to find the distance covered by the car. [4]

4. Using Simpson's $3/8^{th}$ rule, evaluate $I = \int_0^1 \frac{1}{1+x}\, dx$ with $h = 1/6$ and compare the result. [4]

5. Evaluate $\int_4^{5.2} \log_e x \; dx$ by using Simpson's $1/3^{rd}$ and $3/8^{th}$ rule using six equal subintervals. [8]

Semester-I: Numerical Analysis P-II (B)

Practical No. 6

Title: Numerical Solutions of First Order Ordinary Differential Equations

1. Using Euler's method, solve the differential equation $\frac{dy}{dx} = x^2 + y^2$ with initial condition $y(0) = 0$ by taking interval $h = 0.1$ and compute $y(0.5)$. [4]

2. Solve by Euler's Method the equation $\frac{dy}{dx} = xy$ with $y(0) = 1$ and find $y(0.4)$ by taking $h = 0.1$ [4]

3. By using Euler's Modified Method, solve $\frac{dy}{dx} = \log(x + y)$ with initial condition $y(0) = 1$, find $y(0.2)$ and $y(0.5)$ [4+4]

4. By using Euler's Modified Method, solve $\frac{dy}{dx} = x + y$ with initial condition that $y(0) = 1$, find $y(0.05)$ and $y(0.1)$ [8]

5. Use Runge-Kutta Method to approximate y, when $x = 0.1$ and $x = 0.2$ given that $x = 0$ when $y = 1$ and $\frac{dy}{dx} = x + y$. [8]

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S.Y.B.Sc. MATHEMATICS Question Bank

Practicals Based on Paper I

Linear Algebra

Practical No. 1

Subspace and Linear Dependence

1. Let $V = \{(x, y) \in \mathbb{R}^2 | x, y > 0\}$. For $u = (x_1, y_1)$ and $v = (x_2, y_2) \in \mathbb{R}^2, k \in \mathbb{R}$ define $+$ and $\cdot$ operations as $u + v = (x_1x_2, y_1y_2)$ and $k \cdot u = (x_1^k, y_1^k)$. Show that V is real vector space w.r.t. these operations. **[8]**

2. Check whether $W = \{(x, y, z) | x - y + z = 0\}$ is a subspace of vector space $\mathbb{R}^3$. Give a geometrical interpretaion of W. **[4]**

3. Let $S = \{e_1, e_2, e_1 + e_2\}$ where $e_1 = (1, 0, 0), e_2 = (0, 1, 0)$. Find $L(S)$, linear span of S. Give a geometrical interpretaion of $L(S)$. **[4]**

4. Check whether the set $S = \{(-1, 2, 3), (2, 5, 7), (3, 7, 10)\}$ is linearly dependent in $\mathbb{R}^3$. **[4]**

5. For which values of λ do the following vectors $v_1 = (\lambda, -1/2, -1/2), v_2 = (-1/2, \lambda, -1/2)$, $v_3 = (-1/2, -1/2, \lambda)$ are linearly dependent in $\mathbb{R}^3$? **[4]**

Practical No. 2
Basis and Dimension

1. Let $v_1 = (1,2,1), v_2 = (2,9,0), v_3 = (3,3,4)$. Show that the set $S = \{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$. Find the co-ordinates of the vector $v = (5,-1,9)$ w.r.t. S. **[4]**

2. Show that the set $S = \{1, t+1, t^2+1\}$ is a basis for P_2. Express $p(t) = t^2 + t + 1$ as a linear combination of vectors in S. **[4]**

3. Find a basis and dimension of the linear subspaces of $\mathbb{R}^n$ given by $\{(x_1, x_2, \ldots, x_n) : x_1 + x_2 + ... + x_n = 0\}$. **[4]**

4. In P_n show that each of $W_1 = \{f(0) = 0\}, W_2 = \{f(1) = 0\}, W_3 = \{f(a) = 0\}, W_4 = \{f(0) = f(1) = 0\}$ is a subspace of P_n. Find their dimensions. **[8]**

5. Find a basis for the null space, row space and column space of $A = \begin{pmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{pmatrix}$ **[8]**

Practical No. 3
Linear Transformations

1. Check which of the following are linear transformations :

   (a) $T : R^3 \to R^2$ is defined as $T(x, y, z) = (x, yz)$

   (b) $T : R^3 \to R^3$ is defined as
   $T(x, y, z) = (x + 2y, y - 3z, x + z)$

   (c) $T : R^2 \to R^2$ is defined as $T(x, y) = (x + y, |y|)$

   (d) $T : R^2 \to R^3$ is defined as $T(x, y) = (x, y, y + 1)$ **[8]**

2. Find the range and kernel of the linear transformation $T : R^3 \to R^3$ defined as $T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + z \\ x + y + 2z \\ 2x + y + 3z \end{pmatrix}$. Also find rank T and nullity T. **[8]**

3. Let $S = u_1 = (-1, 0, 1), u_2 = (0, 1, -2), u_3 = (1, -1, 1)$ be a basis for $\mathbb{R}^3$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be linear transformation for which $T(u_i) = e_i, i = 1, 2, 3$ where $\{e_1, e_2, e_3\}$ is a standard basis for $\mathbb{R}^3$. Find formula for $T(x, y, z)$ and use it to compute $T(2, 1, -3)$. **[4]**

4. Let $T : R^3 \to R^3$ be linear map with $Te_1 = e_2, Te_2 = e_3, Te_3 = 0$ Then show that $T \neq 0, T^2 \neq 0, T^3 = 0$. **[4]**

5. Let $T : P_1 \to R^2$ be function defined by the formula
   $T(p(x)) = (p(0), p(1))$.
   (a) Find $T(1 - 2x)$ (b)Show that T is linear isomorphism (c)Find $T^{-1}(2, 3)$. **[8]**

Practical No. 4
Inner product spaces

1. For any $x, y \in \mathbb{R}^2$, where $x = (x_1, x_2), y = (y_1, y_2)$, show that $< x, y >= y_1(x_1 + 2x_2) + y_2(2x_1 + 5x_2)$ defines an inner product on $\mathbb{R}^2$. **[4]**

2. Compute the angle between

   (a) $v = e_1, w = e_1 + e_2$ in $\mathbb{R}^2$ where $e_1 = (1, 0)$, $e_2 = (0, 1)$.

   (b) $v = (x, y)$ and $w = (-y, x), x \neq 0, y \neq 0$ in $\mathbb{R}^2$. **[4]**

3. In an inner product space, show that $\|x + y\| = \|x\| + \|y\|$ if and only if one is non negative multiple of the other. **[4]**

4. Let P_n be the space of all polynomials of degree $\leq n$. What is dimension of P_n? Define $< p, q >= \int_{-1}^{1} p(x)q(x)dx$. What is the length of $p(x) = x$ in P_2. Apply the Gram Schmidt process to the basis $\{1, x, x^2\}$ with respect to the above inner product. **[8]**

5. Apply Gram Schmidt process to obtain an orthonormal basis from $\{(1, 0, 1), (1, -1, 0), (1, 1, 1)\}$. **[8]**

Practical No. 5
Eigen Values and Eigen Vectors

1. Find the eigenvalues and eigenvectors of the matrix
$$A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{pmatrix}.$$ [**8**]

2. Verify Caley Hamilton theorem for a matrix
$$A = \begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}.$$ [**4**]

3. Let $T : R^3 \to R^3$ be a linear transformation given by $T(x, y, z) = (x + y + z, 2y + z, 2y + 3z)$. Find eigenvalues of T and eigenspace of each eigenvalue. [**8**]

4. Find the matrix P (if it exists) which diagonalises
$$A = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & -1 \\ 2 & 1 & 3 \end{pmatrix}.$$ [**4**]

5. Find the matrix P (if it exists) which diagonalises
$$A = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix}.$$ [**4**]

Practical No. 6
Miscellaneous

1. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T(x, y, z) = (x + y, y + z, z + x).$$

   Find a similar formula for T^{-1}. **[4]**

2. Let $\{x, y, z\}$ be linearly independent set of vector space V. Let $u = x, v = x + y, w = x + y + z$. Prove that $\{u, v, w\}$ is linearly independent set. **[4]**

3. Find a basis for the following subspaces of $\mathbb{R}^3$.

   (a) $\{(x, y, z) : z = x + y\}$

   (b) $\{(x, y, z) : x = y\}$ **[4]**

4. Apply Gram Schmidt process to $\{x_1 = (1, -2, 2),$ $x_2 = (-1, 0, 1),\ x_3 = (5, -3, -7)\}$ in $\mathbb{R}^3$ with the dot product. **[8]**

5. If $A = \begin{pmatrix} -2 & 2 & 3 \\ -2 & 3 & 2 \\ -4 & 2 & 5 \end{pmatrix}$, then find eigevalues and basis of eigenspaces of A. **[8]**

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S.Y.B.Sc. MATHEMATICS Question Bank

Practicals Based on Paper II (a)

Semester-II: Vector Calculus

Practical No. 1

Vector Functions of One Variable

1. If $\bar{f}(x) = \frac{\tan 3x}{x}\bar{i} + \frac{\log(1+x)}{x}\bar{j} + \frac{2^x - 1}{x}\bar{k}, x \neq 0$ find $\bar{f}(0)$ so that $\bar{f}$ is continuous at 0. **[4]**

2. If $\bar{f}(x) = \frac{\sin^{-1} x - \sin^{-1} a}{x - a}\bar{i} + \frac{e^x - e^a}{x - a}\bar{j} + \frac{x \sin a - a \sin x}{x - a}\bar{k}, x \neq a$ find $\lim_{x \to a} \bar{f}(x)$. **[4]**

3. $\hat{\mathbf{r}}$ is a unit vector in the direction of $\bar{r}$ then prove that

$$\hat{r} \times \frac{d\bar{r}}{dt} = \frac{1}{r^2}\bar{r} \times \frac{d\bar{r}}{dt}.$$

[4]

4. Show that $\bar{r} = \bar{a}e^{kt} + \bar{b}e^{lt}$ is a solution of the linear differential equation $\frac{d^2\,\bar{r}}{dt^2} + p\frac{d\bar{r}}{dt} + q\bar{r} = 0$, where k and l are distinct roots of the equation $m^2 + pm + q = 0$ and $\bar{a}$ and $\bar{b}$ are constant vectors. **[4]**

5. If $\bar{u}, \bar{v}, \bar{w}$ are derivable functions of t such that $\frac{d\bar{u}}{dt} = \bar{w} \times \bar{u}$ and $\frac{d\bar{v}}{dt} = \bar{w} \times \bar{v}$, show that $\frac{d}{dt}(\bar{u} \times \bar{v}) = \bar{w} \times (\bar{u} \times \bar{v})$. **[8]**

Practical No. 2
Curves in Three Dimensional Space

1. Consider the right circular helix $\bar{r} = a\cos t\bar{i} + a\sin t\bar{j} + bt\bar{k}$, where $b = \cot\alpha$ and $0 < \alpha < \pi/2$. Find $\bar{t}, \bar{n}, \bar{b}$. [**4**]

2. Find $\bar{t}, \bar{n}, \bar{b}$ and the equation of the osculating plane at any point of the curve $x = a\cos 2t, y = a\sin 2t, z = 2a\sin t$. [**8**]

3. A particle moves along the curve $\bar{r} = 2t^2\bar{i} + (t^2 - 4t)\bar{j} + (3t - 5)\bar{k}$. Find its velocity and acceleration at $t = 1$ in the direction of the vector $\bar{n} = \bar{i} - 3\bar{j} + 2\bar{k}$.

4. If $\bar{a}, \bar{b}, \bar{c}$ are pairwise perpendicular unit vectors and are derivable functions of t, show that $\dfrac{d\bar{a}}{dt} = \pm\left(\dfrac{d\bar{b}}{dt} \times \bar{c} + \bar{b} \times \dfrac{d\bar{c}}{dt}\right)$. [**4**]

5. If $\bar{r} = \bar{a}\cos\omega t + \bar{b}\sin\omega t$, and $\bar{a}, \mathrm{b}, \omega$ are constants, show that (i) $\bar{r} \times \dot{\mathbf{r}}$ is a constant function and (ii) $\ddot{\mathbf{r}} = -\omega^2\bar{r}$. [**4**]

Practical No. 3
Differential Operators-I

1. Find the equations of tangent plane and normal line to the surface $x^3 - xy^2 + yz^2 - z^3 = 0$ at the point $(1, 1, 1)$. [**4**]

2. Find the directional derivative of the function $f(x, y, z) = xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t, y = t^2, z = t^3$ at $t = 1$. [**4**]

3. If $\bar{\mathrm{r}} = \mathrm{x}\bar{\mathrm{i}} + \mathrm{y}\bar{\mathrm{j}} + \mathrm{z}\bar{\mathrm{k}}$ and $\bar{\mathrm{m}}, \bar{\mathrm{n}}$ are constant vectors then prove that $\nabla \cdot [(\bar{\mathrm{m}} \cdot \bar{\mathrm{r}})\bar{\mathrm{n}}] = \bar{\mathrm{m}} \cdot \bar{\mathrm{n}}$. [**4**]

4. If $\bar{\mathrm{f}} = (\mathrm{y}^2 \cos \mathrm{x} + \mathrm{z}^3)\bar{\mathrm{i}} + (2\mathrm{y} \sin \mathrm{x} - 4)\bar{\mathrm{j}} + (3\mathrm{xz}^2 + 2)\bar{\mathrm{k}}$ then show that $\bar{\mathrm{f}}$ is conservative. Find scalar potential ϕ such that $\bar{\mathrm{f}} = \nabla\phi$. [**8**]

5. If $\nabla\phi = \dfrac{\bar{\mathrm{r}}}{r^5}$ and $\phi(1) = 0$ then show that $\phi(r) = \dfrac{1}{3}(1 - \dfrac{1}{r^3})$. [**4**]

Practical No. 4
Differential Operators-II

1. Show that $(\bar{\mathrm{q}} \cdot \nabla)\bar{\mathrm{q}} = \frac{1}{2}\nabla \mathrm{q}^2 - \bar{\mathrm{q}} \times (\nabla \times \bar{\mathrm{q}})$. [**4**]

2. If $\bar{f} = (xyz)^p(x^q\bar{\mathrm{i}} + \mathrm{y}^\mathrm{q}\bar{\mathrm{j}} + \mathrm{z}^\mathrm{q}\bar{\mathrm{k}})$ is irrotational then prove that $p = 0$ or $q = -1$. [**4**]

3. Find the constant a such that at any point of intersection of the two surfaces

$$(x-a)^2 + y^2 + z^2 = 3 \qquad \text{and} \qquad x^2 + (y-1)^2 + z^2 = 1$$

their tangent planes will be perpendicular to each other. [**8**]

4. If $\bar{\mathrm{f}} = (3\mathrm{x}^2\mathrm{y} - \mathrm{z})\bar{\mathrm{i}} + (\mathrm{xz}^3 + \mathrm{y}^4)\bar{\mathrm{j}} - 2\mathrm{x}^3\mathrm{z}^2\bar{\mathrm{k}}$, find $\nabla(\nabla \cdot \bar{\mathrm{f}})$ at the point $(2, -1, 0)$. [**4**].

5. Let $f(r)$ be a differentiable function of r. Prove that

$$\nabla \cdot (\frac{f(r)}{r}\bar{\mathrm{r}}) = \frac{1}{\mathrm{r}^2}\frac{\mathrm{d}}{\mathrm{dr}}(\mathrm{r}^2\mathrm{f}(\mathrm{r})).$$

[**8**]

Practical No. 5
Vector Integration-I

1. Evaluate line integral $\int_C (xy\bar{\text{i}} + (\text{x}^2 + \text{y}^2)\bar{\text{j}}) \cdot d\bar{\text{r}}$ where C is the x-axis from $x = 2$ to $x = 4$ and the line $x = 4$ from $y = 0$ to $y = 12$. [**4**]

2. Verify Green's theorem in the plane for the line integral

$$\oint_C (2x - y^3)dx - xydy$$

   where C is the boundary of the region enclosed by the circles $x^2+y^2 = 1$ and $x^2 + y^2 = 9$. [**8**]

3. Verify Green's theorem in the plane for the line integral

$$\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$$

   where C is the boundary of the region enclosed by the parabolas $y = \sqrt{x}$ and $y = x^2$. [**8**]

4. Find the work done by the force $\bar{\text{F}} = 3x^2\bar{\text{i}} + (2xz - y)\bar{\text{j}} + z\bar{\text{k}}$ in moving a particle

   (i) along the line segment C_1 from $O(0, 0, 0)$ to $C(2, 1, 3)$,

   (ii) along the curve $C_2 :\ \bar{\text{r}} = \bar{\text{f}}(t) = 2t^2\bar{\text{i}} + \text{t}\bar{\text{j}}(4\text{t}^2 - \text{t})\bar{\text{k}}$, $t \in [0, 1]$,

   (iii) along the curve C_3 given by $x^2 = 4y,\ 3x^2 = 8z,\quad x \in [0, 2]$. [**8**]

5. If $\bar{\text{f}} = 4\text{xz}\bar{\text{i}} - \text{y}^2\bar{\text{j}} + \text{yz}\bar{\text{k}}$, then evaluate $\iiint\limits_V \nabla \cdot \bar{\text{f}}\text{dV}$, where V is the volume bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. [**4**]

Practical No. 6
Vector Integration-II

1. Evaluate surface integral $\iint\limits_S (yzi + zxj + xyk).ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. **[8]**

2. Apply Stokes' theorem to prove that line integral $\int_C ydx + zdy + xdz = -2\sqrt{2}\pi a^2$ where C is the curve given by $x^2+y^2+z^2-2ax-2ay = 0, x+y = 2a$ and begins at the point $(2a, 0, 0)$ and goes at first below the Z-plane. **[4]**

3. Using Gauss's Divergence theorem evaluate surface integral
$$\iint\limits_S x^3dydz + x^2ydzdx + x^2zdxdy$$
over the closed surface S bounded by the planes $z = 0$, $z = b$ and the cylinder $x^2 + y^2 = a^2$. **[4]**

4. Let $\bar{\mathrm{F}} = (x + y)\bar{\mathrm{i}} + (2x - z)\bar{\mathrm{j}} + (y + z)\bar{\mathrm{k}}$. Verify Stokes' theorem for the function $\bar{\mathrm{F}}$ over the part of the plane $3x+2y+z = 6$ in the first octant. **[8]**

5. Using divergence theorem, show that
$$\iint\limits_S (ax\bar{\mathrm{i}} + \mathrm{by}\bar{\mathrm{j}} + \mathrm{cz}\bar{\mathrm{k}}) \cdot \mathrm{ds} = \frac{4}{3}\pi(\mathrm{a} + \mathrm{b} + \mathrm{c}),$$
where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. **[4]**

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S.Y.B.Sc. MATHEMATICS Question Bank

Practicals Based on Paper II(B)

Semester-II: Discrete Mathematics

Practical No. 1

1. Prove by mathematical induction that if $A_1, A_2, ..., A_n$ and B are any n+1 sets, then $(\bigcup_{i=1}^{n} A_i) \bigcap B = \bigcup_{i=1}^{n} (A_i \bigcap B)$, $\forall n \in \mathbf{N}$. [8

2. Prove that $(3+\sqrt{5})^n + (3-\sqrt{5})^n$ is divisible by 2^n, for all n $\in \mathbf{N}$. [8

3. Prove that $n^3 + 2n$ is divisible by 3, for all n $\in \mathbf{N}$. [4

4. Prove that for any positive integer n, the number $2^n + (-1)^{n+1}$ is divisible by 3. [4

5. Prove that $1 + 2^n < 3^n$ for $n \geq 2$. [8

Practical No. 2

1. Show that if seven integers from 1 to 12 are chosen, then two of them will add up to 13. [4

2. Show that if any eight positive integers are chosen, two of them will have the same remainder when divided by 7. [2

3. Show that the minimum number of socks to be chosen from amongst 15 pairs of socks to assure at least one matching pair is 16. [2

4. Solve the recurrence relation

$$a_n - 3a_{n-2} + 2a_{n-3} = 0, a_0 = 0, a_1 = 8, a_2 = -2.$$

[8

5. Solve the recurrence relation

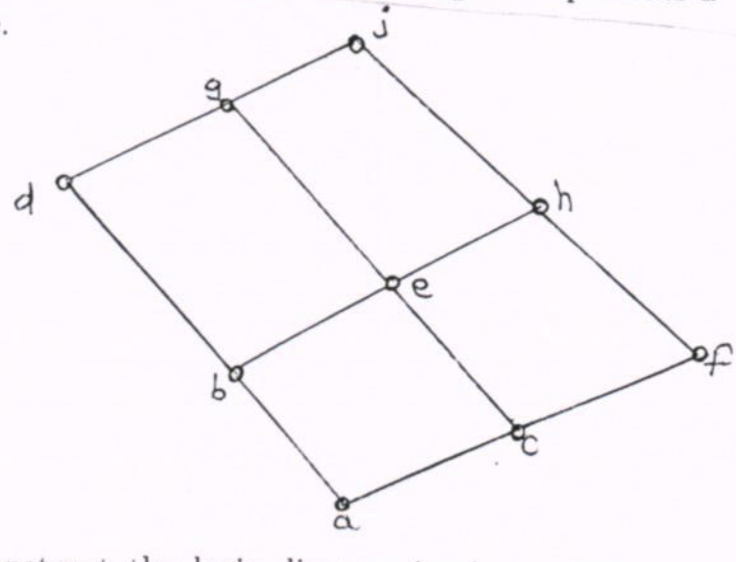
$$a_n = a_{n-1} + a_{n-2}, a_0 = 1, a_1 = 1.$$

[8

Practical No. 3
Order Relations and Structures

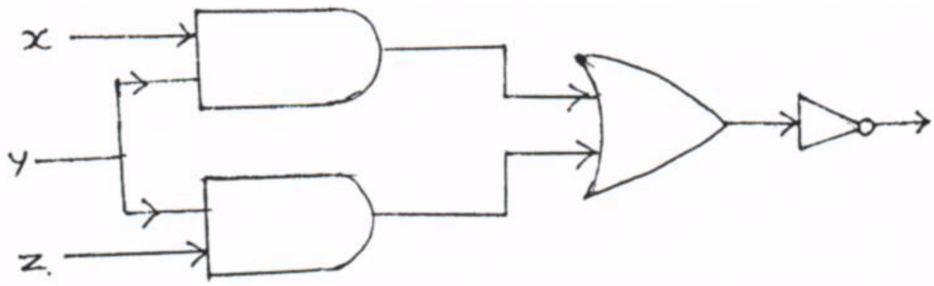
1. Draw the Hasse diagram for each of the following poset:
(i) D_{24} with respect to divisibility, where D_n denotes the set of all positive divisors of n.
(ii)$A = \{a, b, c, d, e\}$ with respect to the relation
$R = \{(a,a), (b,b), (c,c), (a,c), (c,d),$
$(c,e), (a,d), (d,d), (a,e),$
$(b,c), (b,d), (b,e), (e,e)\}$ [4

2. Determine if the following Hasse diagram represents a lattice. [4



3. (a) Construct the logic diagram implementing the function;
$f(x, y, z) = (x \vee (y' \wedge z)) \vee (x \wedge (y \wedge 1))$

(b) Give the Boolean function described by the logic diagram given below: [8



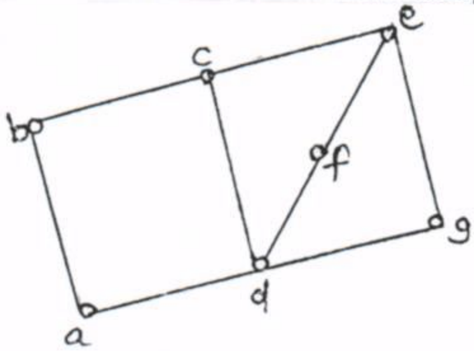
4. Use the Karnaugh map method to find Boolean expression for the function 'f' given below:

x	y	z	w	$f(x, y, z, w)$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	0	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

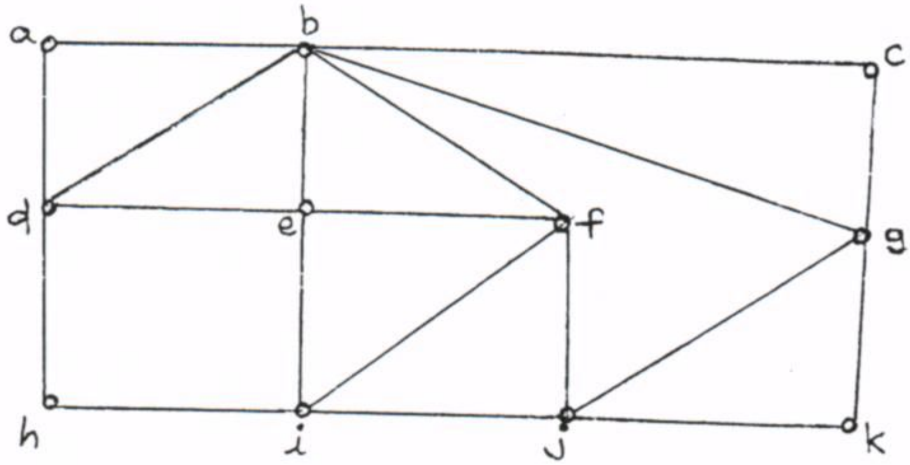
[8

Practical No. 4

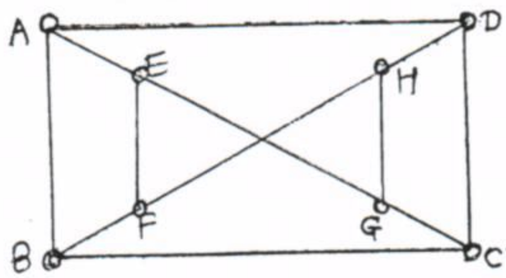
1. Determine whether the following lattice is distributive. Determine the elements which have complements. [8

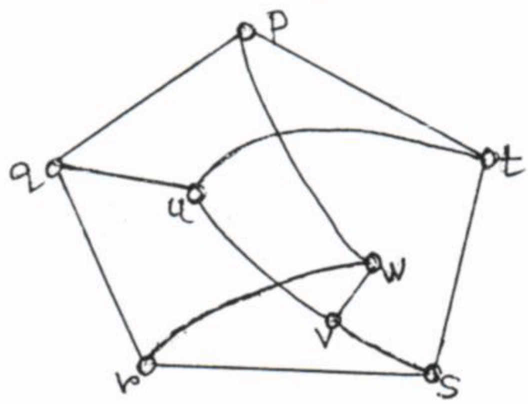


2. Use Fleury's Algorithm to find an Eulerian circuit in the following graph: [8



3. Determine whether the following graphs are isomorphic: [4



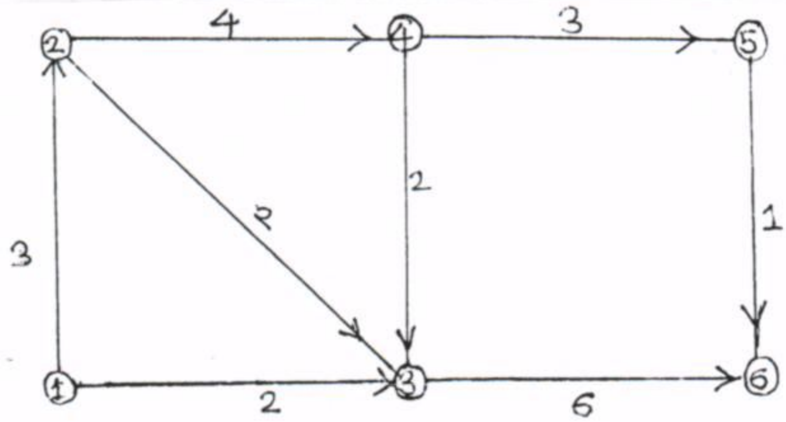


4. Construct an example of a graph which is Eulerian, but not Hamiltonian. Justify your answer. [4

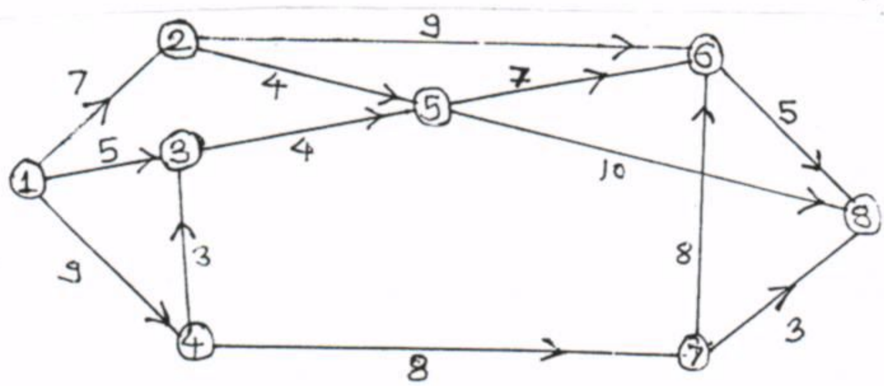
Practical No. 5

1. List all possible distinct Hamitonian circuits of a complete graph K_4. [8

2. Find a maximum flow in the following network by using the labeling algorithm: [8



3. Give example of two cuts and their capacities for the following network: [4

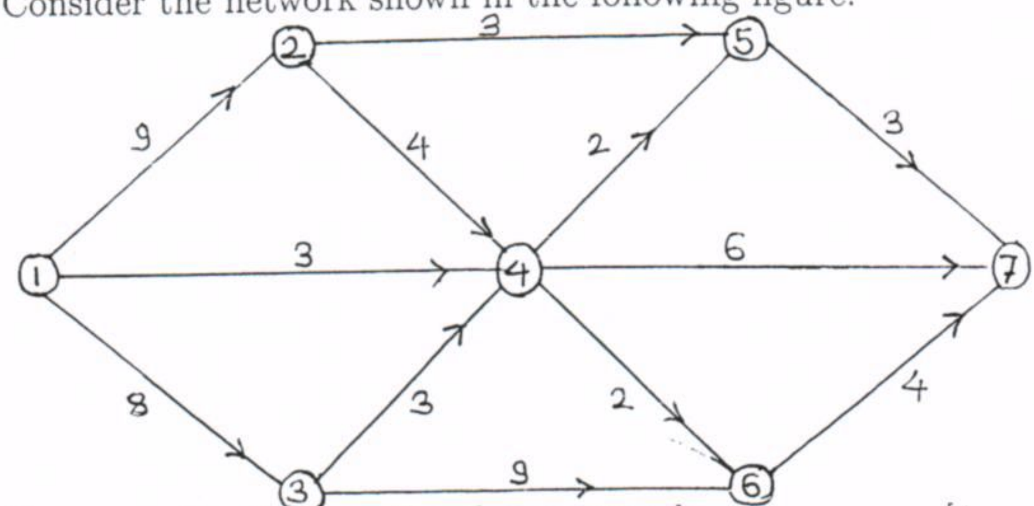


4. Consider the matrix M_R for a relation from A to B given below:

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}.$$

Find a maximal matching for A, B and R. [4

5. Consider the network shown in the following figure:

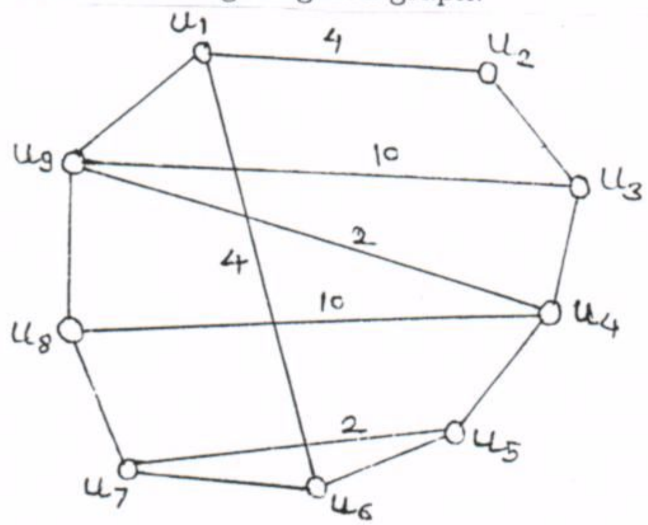


Construct two flows in the above network. [4

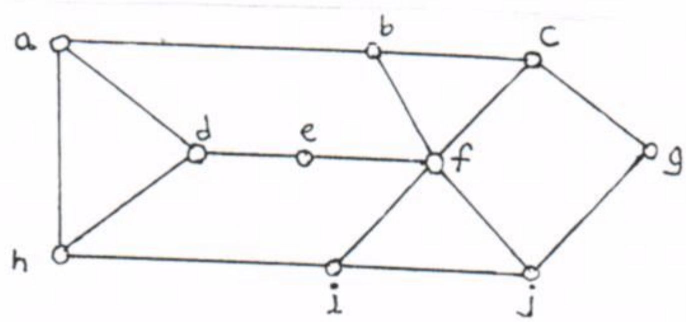
Practical No. 6
Trees

1. Draw all possible nonisomorphic trees on 4 vertices. [4

2. Apply Kruscal's Algorithm to find the shortest spanning tree of the following weighted graph: [8



3. Determine whether the following graph is Hamiltonian. If yes, find the Hamiltonian circuit. [4



4. Give an example of complete bipartite graph that has a perfect matching. [4

5. Consider the following weighted graph. Obtain any three spanning trees and their weights. [4