## Proposed Revised Syllabi for the first two semesters of M.A./M.SC. Statistics

The M. A. / M. Sc. Program in Statistics shall consist of 100 credits. Credits of a course are specified against the title of the course. A course with T in brackets indicates that it is a theory course whereas a course with P in brackets indicates that it is a practical course. Some of the practical courses are linked with a theory course and in such a case, both the courses will have the same number with T and P, indicating a theory and a practical course respectively.

A student can enroll for a practical course if

- (1) if (s)he has enrolled for the corresponding theory course(as indicated) in the same term
- or
- (2) if (s)he has passed the corresponding theory course in an earlier term or
- (3) if terms for the corresponding theory course have been granted in an earlier term.

Some courses have pre-requisites as specified. Unless a student passes the pre-requisites of a course, she/he will not be allowed to enroll in the said course. Prerequisites of a course are indicated in curly brackets against the course.

The Head of the Department, if necessary, may change the pre-requisites of a course on recommendations of the Departmental Committee.

# Semester Courses in Statistics at M. A. /M. Sc.

## Core courses

- ST 1 Mathematical Analysis (5)
- ST 2(T) Linear Algebra (3)
- ST 2(P) Linear Algebra (2)
- ST 3(T) Probability Distributions (5)
- ST 3(P) Probability Distributions (1)
- ST 4 Calculus (3)
- ST 5 (T) Numerical Analysis and Programming (2)
- ST 5 (P) Numerical Analysis and programming (3)
- ST 6 Probability theory (5) {ST 1}
- ST 7(T) Regression Analysis (4) {ST 2(T), ST 3(T)}
- ST 7(P) Regression Analysis (2)
- ST 8 Parametric Inference (5) {ST 3(T)}
- ST 9 (T) Multivariate Analysis (4) {ST 3(T), ST 4, ST2(T) }
- ST 9(P) Multivariate Analysis (2)
- ST 10 Stochastic Processes I (4)

## Detailed syllabi of the compulsory courses

#### **ST 1: Mathematical Analysis**

Countability, supremum and infimum of sets of real numbers. Limit points of a set – open sets, closed sets etc. (will be developed through general metric space and  $R^n$  will be considered as a special case), compactness, Bolzano-

Weierstrass theorem, Heine-Borel Theorem. Continuous functions, uniform continuity. absolute continuity.

Sequences and series of real numbers, limit superior, limit inferior and limit of a sequence. Cauchy sequences, convergence of series, tests for convergence of series, absolute convergence, Cauchy products.

Riemann and Riemann – Stieltjes integrals, integration by parts, mean value theorem.

Uniform convergence of sequences and series. Term by term differentiation and integration, applications to power series. Improper Riemann – Stieltjes integrals: Improper integrals of first and second kind for one variable. Uniform convergence of improper integrals, differentiation under the sign of integral – Leibnitz rule.

#### **Books Recommended**

1. Apostol, T. M. (1975) Mathematical Analysis : A Modern Approach to Advanced Calculus. (Addison - Wesley)

2. Bartle, R. G. (1976) Elements of Real Analysis (Wiley)

3. Rudin, W. (1985) Principles of Mathematical Analysis (McGraw - Hill)

#### ST 2(T): Linear Algebra

Matrix algebra, special types of matrices, determinants and their simple properties. Orthogonal and idempotent matrices: Linear dependence, basis of a vector space, an orthogonal basis, Gram-Schmidt orthogonalization, projection theorem, linear transformation, rank of matrix, linear equations, solution space and null space.

Characteristics roots of real matrices, right and left characteristic vectors, independence of characteristic vectors corresponding to distinct characteristic roots, algebraic and geometric multiplicities; Generalized inverse, Spectral decomposition theorem.

Definiteness of a real quadratic form, reduction of quadratic forms, simultaneous reduction of two quadratic forms, maxima and minima of ratios of two quadratic forms.

# **Books Recommended**

1. Graybill, F.E.(1961) Introduction to Matrices with Applications in Statistics (Wadsworth Pub. Co.)

2. Hohn, F. E. (1973) Elements of Matrix Algebra (Macmillan)

3. Rao, C. R. (1995) Linear Statistical Inference and Its Applications. (Wiley Eastern)

4. Searle, S. R. (1982) Matrix Algebra Useful for Statistics. (John Wiley & Sons

# **Additional Books for Reference**

1. Bellman, R. (1970) Introduction to Matrix Analysis (Tata McGraw Hill)

# ST2 (P)

The following practicals are to be done by using the software R/SYSTAT/Matlab.

- I. Calculation of determinant (4 x 4)
- II. Calculation of eigen values, eigen vectors, g- inverse
- III. Solution of simultaneous equations
- IV. Quadratic forms

# ST 3(T): Probability Distributions

Random Experiment and its sample space, random variables, c.d.f., p.d.f., p.m.f., absolutely continuous and discrete distributions, mixtures of probability distributions. Some common distributions. Transformations, moments, m.g.f., p.g.f., quantiles and symmetry. Random vectors, joint distributions, joint m.g.f. mixed moments, variance covariance matrix. Hazard rate and cumulative hazard rate, lack of memory property. IFR, IFRA, DFR and DFRA classes of distribution.

Independence, sums of independent random variables, convolutions, conditional expectation and variances, regression function and best linear regression function, multiple and partial correlation coefficients.

Sampling distributions of statistics from univariate normal random samples, such as linear and quadratic forms. Fisher Cochran theorem. Non-central  $x^2$ , t and F distributions.

Order statistics: distribution of r-th order statistics, joint distribution of several order statistics and their functions. Probability Integral Transformation, Rank orders and their exact null distributions. One and two sample examples of rank statistics such as sign statistic, Wilcoxon signed rank statistics, Wilcoxon two sample statistics etc. Recurrence relations for the null distribution of the Wilcoxon two sample statistics.

# **Books Recommended**

- 1. Hogg, R. V. and Craig, T. T. (1978) Introduction to Mathematical Statistics (Fourth Edition) (Collier-McMillan)
- 2. Rohatgi, V. K. (1988) Introduction to Probability Theory and Mathematical Statistics (Wiley Eastern)
- 3. C. R. Rao (1995) Linear Statistical Inference and Its Applications (Wiley Eastern) Second Edition
- 4. H, Cramer (1946) Mathematical Methods of Statistics, (Prinecton).
- 5. J. D. Gibbons & S. Chakraborti (1992) Nonparametric statistical Inference (Third Edition) Marcel Dekker, New York
- 6. W.J.Conover. (1999) Practical Nonparametric Statistics, Wiley. 3rd Edition
- Barlow R. E. & Proschan F. (1975) Statistical Theory of Reliability & Life testing. Holt, Rinehart & Winston Inc.

# ST 3(P)

- . Using software packages: SYSTAT, R and MINITAB
  - 1. Plotting of density functions, distribution functions. and failure rates.
  - Wilcoxon test, Wilcoxon signed-rank test, Kruskal Wallis test., rank correlation (2 practicals)
  - 3. Model sampling from standard distributions and mixtures of distributions.

#### ST 4: Multi-dimensional calculus

Review of calculus : differentiability, mean value theorem and Taylor series expansions.

Functions of several variables: Continuity, directional derivatives, differential of functions of one variable, differentials of functions of several variables, the gradient vector, differentials of composite functions and the chain rule, the mean value theorem, a sufficient condition for the existence of the differential, partial derivatives of higher order and Taylor's formula.

Multiple Integrals and evaluation of multiple integrals by repeated integration. Mean value theorem for multiple integrals.

Applications of partial differentiation: Jacobians, the inverse function theorem, the implicit function theorem, extremum problems.

1. Apostol T.M. (1975) Mathematical Analysis : A modern approach to advanced calculus, (Addison-Wesley).

2. Bartle, R. G. (1976) Elements of Real Analysis (Wiley)

3. Kreyszig, E. (1975) Advanced Engineering Mathematics (Wiley Eastern)

4. Rudin, W. (1985) Principles of Analysis (McGraw-Hill)

5 Williamson R.E. and Tratter H.F. (1996) Multivariable Mathematics, 3<sup>rd</sup> Ed. (Prentice Hall)

## ST 5(T): Numerical analysis and programming

Solutions to Non linear equations: Bisection method, NR. Steepest descent, Quadrature interpolation, Jacobi and Gauss Seidel Methods.

Simple Optimization method. Direct search, grid search, Hooke & Jeeve's method Interpolatory search, Gradient search.

Progamming in R.

Random number generation methods

i.	i.	Mixed congruential
ii.	ii.	Multiplicative congruential
iii.	iii.	Rejection
iv.	iv.	Distribution specific methods

Testing for randomness of a sequence. Runs test, digit frequency test, gap test, serial correlation

Methods to compute integrals: quadrature formula, Monte Carlo Methods.

Applications of Monte Carlo methods to compute expected values of

random variables..

# **Books Recommended:**

- 1. Thisted R. A.(1988) Elements of statistical computing, (Chapman and Hall)
- 2. Kennedy W. J. & Gentle J. E. (1980) Statistical computing (Marcel Dekker)
- Rajaraman V. (1993) Computer oriented numerical methods, 4<sup>th</sup> ed. (Prentice-Hall).

4. Krishnamurthy V. & Sen (1993) Numerical Agorithm Computation in Science and Engineering 2<sup>nd</sup> Ed. (Affiliated East West Press)

- 5. Ross, S. (2000) Introduction to Probability Models.(Academic Press)
- 6. Ripley B. D.(1987) Stochastic Simulation (Wiley)

# ST 5(P)

Elementary Statistics as listed below using software packages : SYSTAT, R and MINITAB

. Calculation of summary statistics (mean, median, maximum, minimum, s.d.)

Calculation of regression and correlation coefficients.

ANOVA for one-way and two way models

Analysis of 2 x 2 contingency table (tables of O, E,  $(O-E)^2/E$ ).

Computation of integrals by Riemann & RS sums.

Calculation of p-value for standard normal distribution (for given Z value)

Preparing frequency distribution of given data.

Calculation of double integrals.

SRSWOR, Calculation of order statistic Plotting of curves (standard & non standard) Limits of functions Box Plot Computing integrals by statistical methods Computing expectations of complicated functions Calculation of empirical power & level of significance.

Empirical confidence coefficient

Test for Randomness

## **Books Recommended:**

- 7. Thisted R. A.(1988) Elements of statistical computing, (Chapman and Hall)
- 8. Kennedy W. J. & Gentle J. E. (1980) Statistical computing (Marcel Dekker)
- Rajaraman V. (1993) Computer oriented numerical methods, 4<sup>th</sup> ed. (Prentice-Hall).
- Krishnamurthy V. & Sen (1993) Numerical Agorithm Computation in Science and Engineering 2<sup>nd</sup> Ed. (Affiliated East West Press)
- 11. Ross, S. (2000) Introduction to Probability Models.(Academic Press)
- 12. Ripley B. D.(1987) Stochastic Simulation (Wiley)
- Handbook of Statistics Vol 4 (The Art of Computer Generation of Random Variables): M. T. Boswell, S D Gore, G. P. Patil and C. Tallie.

# **ST 6: Elements of Probability Theory**

Algebra of sets, fields and sigma-fields, limits of sequences of subsets, sigma-field generated by a class of subsets, Borel fields.

Probability measure on a sigma-field, probability space, continuity of a probability measure, Real and vector-valued random variables, distribution functions (d.f.), discrete r.v.s, r.v.s of the continuous type, decomposition of a d.f., Independence of two events and n (> 2) events, sequence of independent events,

independent classes of events,  $\pi$ -system and  $\lambda$ -system of events, Dynkin's theorem, independence of r.v.s, Borel zero-one law.

Expectation of a real r.v. and of a complex-valued r.v. Linear properties of expectations, Characteristic functions, their simple properties, Parseval relation, uniqueness theorem.

Convergence of a sequence of r.v.s., convergence in distribution, convergence in probability, almost sure convergence and convergence in quadratic mean, their inter-relations, Khintchin's weak law of large numbers, Kolmogorov strong law of large numbers (without proof), monotone convergence theorem and dominated convergence theorem, continuity theorem for characteristic functions. Lindeberg's CLT and its particular cases, Cramer's theorem on composition of convergence in distribution and convergence in probability.

#### **Books Recommended**

- 1. Bhat, B. R. (1985) Modern Probability Theory (Wiley Eastern)
- 2. Billingsley, P. (1986) Probability and Measure (John Wiley and Sons)

#### Additional Books for Reference

1. Feller, W. (1969) Introduction to Probability and its Applications vol.II (Wiley Eastern Ltd.)

- 2. Loeve, M. (1978) Probability Theory (4th Edn) (Springer Verlag)
- 3. Gnedenko, B.V. (1988) Probability Theory (Mir.Pub.)

#### ST 7(T): Regression Analysis

Simple regression with one independent variable(X), assumptions, estimation of parameters, standard error of estimator, testing of hypothesis about  $\beta$ , standard error of prediction. Testing of hypotheses about parallelism, equality of intercepts, congruence. Extrapolation, optimal choice of X.

Diagnostic checks and correction: graphical techniques, tests for normality, uncorrelatedness, homoscedasticity, lack of fit, modifications like polynomial regression, transformations on Y or X, WLS. Inverse regression X(Y).

Multiple regression: Standard Gauss Markov Setup. Least square(LS) estimation, Error and estimation spaces. Variance- Covariance of LS estimators. Estimation of error variance, case with correlated observations. LS estimation with restriction on parameters. Simultaneous estimation of linear parametric functions. Test of Hypotheses for one and more than one linear parametric functions. Confidence intervals and regions. ANOVA.

Non Linear regression: Linearization transforms, their use & limitations, examination of non linearity, initial estimates, iterative procedures for NLS, grid search, Newton- Raphson, steepest descent, Marquardt's methods.

Logistic Regression: Logit transform, ML estimation. Tests of hypotheses, Wald test, LR test, score test. Test for overall regression.

Multiple logistic regression, forward, backward method. Interpretation of parameters relation with categorical data analysis.

Generalized Linear model: link functions such as Poisson, binomial, inverse binomial, inverse Gaussian, gamma.

#### **Books Recommended:**

- Draper, N. R. & Smith, H(1998) Applied Regression Analysis, 3<sup>rd</sup> Ed. (John Wiley).
- McCullagh, P & Nelder, J. A. (1989) Generalized Linear Models (Chapman & Hall).
- 3. Ratkowsky, D.A. (1983) Nonlinear Regression Modelling (Marcel Dekker).
- Hosmer, D.W. & Lemeshow, S. (1989) Applied Logistic Regression (John Wiley).
- 5. Seber, G.E.F. and Wild, C.J. (1989) Nonlinear Regression (Wiley)
- Neter, J., Wasserman, W., Kutner, M.H. (1985) Applied Linear Statistical Models. (Richard D. Irwin)
- Montgomery, Douglas C.; Peck, Elizabeth A.; Vining, G. Geoffrey: (2003) Introduction to Linear Regression Analysis. (Wiley)

## ST 7(P)

Simple regression, regression diagnostics. (2 practicals ) Multiple regression, forward method, backward method (2 practicals) Non-linear regression – two or three non-standard models (2 practicals)

Logistic regression (2 practicals)

Following practicals are to be done with the help of a software package.

1. best subset selection, (2) GLM, (3) Linearization transforms.

## ST 8: Parametric Inference

Sufficiency, completeness, Uniformly minimum variance unbiased estimators, C-R inequalities, exponential class of densities and its properties, some special classes of distributions admitting complete sufficient statistics, extensions of these results to multi-parameter situation.

Test function, Neyman- Pearson lemma for test functions. Uniformly most powerful tests for one sided alternative for one parameter exponential class of densities and extension to the distributions having monotone likelihood ratio property.

Confidence Intervals, shortest expected length confidence intervals, relations with testing of hypotheses, uniformly most accurate confidence intervals.

Bayesian estimation, prior distributions, posterior distribution, loss function, principle of minimum expected posterior loss, quadratic and other common loss functions, conjugate prior distributions. Common examples. Bayesian HPD confidence intervals.

#### **Books for Reference**

- 1. Kale, B.K. (1999) A First Course on Parametric Inference (Narosa).
- Casella G. and Berger Robert L. (2002) Statistical Inference. 2<sup>nd</sup> Edition, Duxbury Advanced series.

- 3. Dudewicz, E. J. and Mishra, S.N.(1988) Modern Mathematical Statistics (John Wiley)
- 4. Roussas, G. G. (1973) First Course in Mathematical Statistics (Addison Wesley)
- 5. Silvey, S. D. (1975) Statistical Inference (Chapman and Hall)
- 6. Wilks, S. S. (1962) Mathematical Statistics (John Wiley)
- 7. .Lehmann, E. L. (1986) Testing of Statistical hypothesis (John Wiley)
- 8. .Lehmann, E. L. (1988) Theory of Point Estimation (John Wiley)
- Rohatgi, V. K. (1976) Introduction to theory of probability and Mathematical Statistics (John Wiley & Sons)

# ST 9(T): Multivariate Analysis

Multivariate Normal distribution, pdf and mgf, singular and nonsingular normal distributions, distribution of a linear form and a quadratic form of normal variables, marginal and conditional distributions.

Multiple regression and multiple and partial correlation coefficients, Definition and Relationships

MLE's of the parameters of multivariate normal distribution and their sampling distributions (including derivation of Wishart distribution) Properties of the Wishart Distribution.

Tests of hypothesis about the mean vector of a multinormal population, Hotelling's  $T^2$  -statistic and its distribution. Rao's U -statistic and its distribution, Applications of Hotelling's  $T^2$  -statistic.

Introduction to Principle Components, Canonical correlation coefficients and canonical variables.

**Cluster Analysis** 

Classification problem. Discriminant analysis, Mahalanobis D<sup>2</sup>-statistic. Methods and applications of MANOVA (without derivation of the distribution of Wilks'  $\lambda$ ).

## **Books Recommended :**

1. Anderson, T. W.(1984) Introduction to Multivariate Analysis (John Wiley)

2. Kshirsagar, A. M. (1983) Multivariate Analysis (Marcel Dekker)

3. Morrison, D.F.(1990) Multivariate Statistical Methods (McGraw Hill Co.)(3<sup>rd</sup> ed.)

4. Rao, C. R.(1995) Linear Statistical Inference and its Applications. (Wiley Eastern Ltd.)

5. Johnson R.A. & Wichern, D.W. (1988) Applied Multivariate Statistical Analysis (Prentice Hall Inc.)

# ST 9 (P)

- 1. 1. Model Sampling from multivariate normal distribution
- 2. 2. Applications of Hotelling's T<sup>2 (2 practicals)</sup>
- 3. 3. MANOVA
- 4. 4. Discriminant Analysis
- 5. 5. Principal Component Analysis
- 6. 6. Cannonical Correlations
- 7. 7. Cluster Analysis

# ST 10 Stochastic Processes I

Markov chains with stationary transition probabilities, properties of transition functions, classification of states, Stationary distribution of a Markov chain, existence and uniqueness, convergence to the stationary distribution. Methods based on Markov chains for simulation of random vectors. MCMC algorithm.

Branching processes, Gambler's ruin problem, Transient states.

Estimation of transition probabilities.

Numerical Illustrations and calculations of transition probabilities.

Introduction to Wiener Process and Brownian Motion.

## **Books Recommended**

- 1. Taylor, H. M. and Karlin, S. (1984) An Introduction to Stochastic Modelling. (Academic Press)
- 2. Medhi, J.(1982) Stochastic Processes (Wiley Eastern Ltd.)
- 3. Ross, S. (2000) Introduction to Probability Models (7<sup>th</sup> Ed.) Academic Press
- 4. Bhat B.R.(2000) Stochastic Models: Analysis and Applications (New Age Internationals)

## **Additional Books for Reference**

1. Feller, W. (1972)	An Introduction to Probability Theory and its Applications, Vol.1 3 <sup>rd</sup> rd. (Wiley Eastern Ltd.)
2. Karlin, S & Taylor, H. N	<ul> <li>I. (1969) A First Course in Stochastic Processes</li> <li>(Second.Edition) (Academic Press)</li> </ul>
3 Hoel P G Port S C &	Stone C. J. (1972) Introduction to Stochastic

3. Hoel, P. G. Port, S.C.& Stone, C. J. (1972) Introduction to Stochastic Processes (Houghton Mifflin)