UNIVERSITY OF PUNE, PUNE 411007
BOARD OF STUDIES IN MATHEMATICS
S.Y.B.A. (MATHEMATICS)
SYLLABUS

MG-2: Differential Eqns. And Linear Algebra.

AMG-2: Calculus of Several Variables and Vector Calculus.

FMG-2: Operations Research

MS-1: Prob. Course based on the Papers MG 2 and AMG 2
(Same as Paper III of SYBSc Mathematics)

OR

Any one of :
a) Combinatorics and Computational Geometry.
b) Graphs and Lattices.

MS-2: Number Theory and Complex Variables.

Detailed Syllabus:

Paper MG-2 : Differential Equations and Linear Algebra

First Term :- Differential Equations

1. Differential Equations of first order and first degree: [20 lectures]

(1) Variables separable form.
(2) Homogeneous Differential Equations and Exact Differential Equations. Examples of Non- Homogeneous equations.
(3) Condition for exactness. (Necessary and sufficient condition)
(4) Integrating factor, Rules of finding integrating factors (Statements only).
(5) Linear Differential Equations, Bernoulli's equation.

2. Application of Differential Equations : [8 lectures]

(6) Orthogonal trajectories.
(7) Growth and decay.

3. Linear Differential Equations with constant coefficients: [20 lectures]

(8) The auxiliary equations.
(9) Distinct roots, repeated roots, Complex roots, particular solution.
(10) The operator $\frac{1}{f(D)}$ and its evaluation for the functions $x^m, e^{ax}, e^{ax}v, xv$ and the operator $1/(D^2+a^2)^r$

acting on $\sin ax$ and $\cos ax$ with proofs.

(11) Method of undetermined coefficients, Method of variation of parameters, Method of reduction of order.

Text Books:

(1) Rainville and Bedient, Elementary Differential Equations, Macmillan Publication .
(2) Daniel. A.Murray, Introductory Course in Differential Equations, Orient Longman.

Reference books:

(1) Shanti Narayan, Integral Calculus, S. Chand and Company.
(3) G.F. Simmons and S. Krantz, Differential Equations with Applications and Historical notes, Tata Mc-Graw Hill.

<u>Second Term</u> :- **<u>Linear Algebra</u>**

1. **Vector Spaces** : [14 lectures]
   (1) Definitions and Examples.
   (2) Vector Subspaces.
   (3) Linear Independence.
   (4) Basis and Dimensions of a Vector Space.
   (5) Row and Column Spaces of a matrix.
   Row rank and Column rank.
2. **Linear Transformations**: [12 lectures]
   (1) Linear Transformation, representation by a matrix.
   (2) Kernel and Image of a Linear Transformation.
   (3) Rank-Nullity theorem.
   (4) Linear Isomorphism.
   (5) L (V, W) is a vector space. Dimension of L(V,W) (Statement only)
3. **Inner Product spaces:** [16 lectures]
   (1) The Euclidean space and dot product.
   (2) General inner product spaces.
   (3) Orthogonality, Orthogonal projection onto a line, Orthogonal basis.
   (4) Gram-Schmidt Orthogonalization.
   (5) Orthogonal Transformation.
4. **Eigen values and Eigen vectors:** [6 lectures]
   (1) Rotation of axes of conics.
   (2) Eigenvalues and eigenvectors.

Text Books:
S. Kumaresan , Linear Algebra: A Geometric Approach, Prentice Hall of India, New Delhi, 1999.
Chapters: 2, 4, 5 (excluding Arts 4.4.10 -4.4.12, 5.3. 5.6, 5.7, 5.9), 7.1, 7.2.

Reference Books:

(1) M. Artin, Algebra, Prentice Hall of India , New Delhi, (1994).
(2) K. Hoffmann and R. Kunze Linear Algebra, Second Ed. Prentice Hall of India New Delhi, (1998).
(3) S. Lang, Introduction to Linear Algebra, Second Ed. Springer-Verlag, New Yark, (1986).
(4) A. Ramchandra Rao and P. Bhimasankaran, Linear Algebra, Tata McGraw Hill, New Delhi (1994).
(5) G. Schay, Introduction to Linear Algebra, Narosa, New Delhi, (1998).
(6) L. Smith, Linear Algebra, Springer –Verlag, New York, (1978).
(7) G. Strang, Linear Algebra and its Applications. Third Ed. Harcourt Brace Jovanovich, Orlando, (1988).
(8) T. Banchoff and J. Werner, Linear Algebra through Geometry. Springer-Verlag, New Yark, (1984).
(9) H. Anton and C. Rorres, Elementary Linear Algebra with Applications, Seventh Ed., Wiley, (1994).

Paper AMG-2 : Calculus of Several Variables & Vector Calculus

First Term :- **Calculus of Several Variables**

1. **Limits and Continuity** : [6 lectures]
   (1) Functions of two and three variables.
   (2) Notions of limits and continuity.
   (3) Examples.
2. **Partial Derivatives** : [4 lectures]
   (1) Definition and examples.
   (2) Chain Rules.
3. **Differentiability** : [14 lectures]
   (1) Differential and differentiability and necessary and sufficient conditions for differentiability.
   (2) Higher ordered partial derivatives.
   (3) Schwartz's theorem, Young's theorem with proof.
   (4) Euler's theorem for homogeneous functions.
   (5) Mean Value theorem, Taylor's theorem for functions of two variables.
4. **Extreme Values** : [8 lectures]
   (1) Extreme values of functions of two variables.
   (2) Necessary conditions for extreme values.
   (3) Sufficient conditions for extreme values.
   (4) Lagrange's method of undetermined coefficients.
5. **Multiple Integrals** : [16 lectures]
   (1) Double integrals, evaluation of double integrals.
   (2) Change of order of integration for two variables.
   (3) Double integration in Polar co-ordinates.
   (4) Triple integrals.
   (5) Evaluation of triple integrals.
   (6) Jacobians, Change of variables.(Results without proofs)
   (7) Applications to Area and Volumes.

Text book:

Shanti Narayan and P.K. Mittal, A Course of Mathematical Analysis (12^{th} Edition, 1979), S. Chand and Co..

(Art. 12.1 to 12.3, 12.4, 12.5, 13.1, 13.3 to 13.9, 16.6 to 16.8,16..11, 18.5, 18.8).

References:

(1) M.R. Spiegel, Advanced Calculus: Schaum Series.
(2) D.V. Widder, Advanced Calculus (IInd Edition), Prentice Hall of India, New Delhi, (1944).

T.M. Apostol, Calculus Vol. II (IInd Edition), John Willey, New York, (1967).

<u>Second Term</u> :- <u>Vector Calculus</u>

1. Vector functions of one variable: [10 lectures]

1) Limit and continuity.
2) Derivatives.
3) Derivability in relation to algebraic operations: constant vector functions.
4) Limits, continuity and partial derivatives of vector function of two and three variables.
5) Total differentials

2. Curves in three dimensional spaces: [6 lectures]

1) Curves in three dimensional spaces.
2) Tangent vector.
3) Normal plane and osculating plane.
4) Normal plane at a point and fundamental planes.
5) Orthonormal triad of unit vectors

3. Differential operators: [18 lectures]

1) The operator del, scalar and vector fields.
   Gradient of a scalar point function, properties and its geometrical interpretation.
2) Directional derivatives of a scalar point function.
3) Divergence and curl of a vector point function and its properties.
4) Physical interpretation of Divergence and Curl , Solenoidal and Irrotational vector field.

4. Vector Integration : [14 lectures]

1) Line Integral.
2) Surface Integral.
3) Volume Integral.
4) Green's theorem with proof.
5) Gauss's Divergence Theorem(statement only).
6) Stokes's Theorem(Statement only), Examples on sphere, cube, cylinder.

Text book:

1) Shanti Narayan, R.K. Mittal, A Text-book of Vector Calculus, S.Chand and Company,(2005).
Articles:1.1 to 1.13, 2.1 to 2.5, 6.1 to 6.17, 7.1 to 7.11.

Reference books:

(1) M.R. Spiegel, Advanced Calculus : Schaum Series.
(2) D.V. Widder, Advanced Calculus (IInd Edition), Prentice Hall of India , New Delhi,(1944).
(3) John M. H. Olmsted, Advanced Calculus, Eurasia Publishing House, New Delhi(1970)
(4) T.M. Apostol , Calculus Vol. II (IInd Edition), John Wiley, New York, (1967).

Paper FMG-2 : Operations Research

First Term :-

1. Linear programming :
   Statement of L.P.P., Formulation of L.P.P., Definition of slack variable, Surplus variable and artificial variable, L.P.P. in standard form and canonical form, Definition of a solution, Feasible solution, Basic Feasible solution (Degenerate and non-degenerate), Optimal solution, Basic and non-basic variables, Solution of a L.P.P. by graphical method and by simplex method. Criteria for unbounded solution, no solution, multiple solutions. Big M-method, Examples, Duality, Dual simplex method [24 lectures]
2. Transportation problem:
   Statement of balanced and unbalanced transportation problem (T.P.), Methods of finding initial basic feasible solution (I.B.F.S.)
   (I) North West Corner Method
   (II) Matrix - Minima / Least Cost Method
   (III) Vogel's Approximation Method (VAM)
   Optimum solution of a T.P., Multiple optimum solutions, Degeneracy and method of resolving degeneracy.
   [18 lectures]
3. Assignment Problem : Statement of m x n assignment problem, Solution of assignment problem. [6 lectures]

Second Term :-

1. Sequencing : Statement of a sequencing problem of 2 machines and *n* jobs. 3-machines and *n* jobs (reducible to 2 machines and *n* jobs).
   Calculation of total time elapsed, Idle time of machine. Simple numerical problem. [6 lectures]
2. Replacement Problem : Replacement of depreciable asset- discrete case when money value is not considered and when money value is considered. [8 lectures]
3. Theory of Games : Definition of two person zero-sum game. Saddle point. Value of game , Maximum and minmax strategy, mixed strategies, Method of solving a 2×2 game. Use of dominee property. Graphical method (For $m \times 2$ and $2 \times n$ games) , Games as L.P.P. [15 lectures]
4. CPM- Definition of (a) event (b) activity (c) Critical activity (d) Project duration Construction of network. Definitions of (a) node (b) earliest event – time (c) least event time (d) Critical path float. Total float, Free float , Independent float [9 lectures]
5. Pert - Pessimistic time estimate : optimistic time estimate, most likely time estimate, Calculations of S.D. of project. [10 lecture]

Text Book :

Hamdy Taha, Operations Research; an Introductory Approach, Tata McGraw Hill, 1981

Reference Books :

1. Gupta and Hira, Operations Research
2. S.D. Sharma, Operations Research
3. L.S. Srinath, PERT and CPM
4. J. K. Sharma, Operations Research, Macmillan India Ltd., 1997

Paper MS-1 : Problem Course

Prob. Course based on the Papers MG - 2 and AMG -2 : Same as the syllabus outlined for Paper III of SYBSc Mathematics Practical Paper

OR

Paper MS-1 : Combinatorics and Computational Geometry

First Term :- **Combinatorics**

1. General Counting Methods

1.1 Two Basic counting Principles : Addition Principle and Multiplication Principle

1.2 Simple Arrangements and selections

1.3 Arrangements and selections with repetitions :

$P(n, r_1, r_2, \ldots, r_m) = n! / (r_1!\ r_2!\ \ldots\ r_m!)$

1.4 Distributions

Number of distributions of r distinct objects into n distinct boxes is n^r

Number of distributions of r identical objects into n distinct boxes is

$C(n+r-1, r)$ = The number of nonnegative integer solutions to

$X_1 + X_2 + \ldots + X_n = r$

1.5 Binomial Coefficients : Binomial Identities (omit generalized binomial coefficient and generalized binomial theorem), Multinomial Theorem (Ex.40 Section 5.5) [20 lectures]

2. Inclusion – Exclusion Principle

2.1 Counting with Venn diagrams

2.2 Inclusion – Exclusion formula, Derangements, Simple Examples. [10 lectures]

3. Pigeonhole Principle

3.1 Pigeonhole Principle [10 lectures]

4. Recurrence Relations

4.1 Recurrence relation models.

4.2 Solution of Linear Homogeneous recurrence relations (Methods without proof) [8 lectures]

Text book :

Alan Tucker, Applied Combinatorics, John Wiley and Sons, 2nd Edition, 1984

Sections : 5.1,5.2,5.3,5.4,5.5,7.1,7.4,8.1,8.2 and Appendix 4

Reference Books :

1. Balkrishanan, Theory and problems of combinatorics including concepts of Graph Theory, Schaum's Outline Series, McGraw Hill, New York, 1995
2. Richard A. Brualdi, Introductory Combinatorics, North Holland, New York, 1977

Second Term :- Computational Geometry

1. Two dimensional transformations. [16 lectures]

   Introduction

   (a) Representation of points
   (b) Transformations and matrices.
   (c) Transformations of points
   (d) Transformations of straight lines.
   (e) Midpoint transformations
   (f) Transformations of parallel lines.
   (g) Transformations of intersecting lines.
   (h) Transformations : rotation, reflection, scaling.
   (i) Combined Transformations.
   (j) Transformations of a unit square.
   (k) Solid body Transformations.
   (l) Translations and homogeneous coordinates.
   (m) Rotation about an arbitrary point.
   (n) Reflection through an arbitrary line.
   (o) Projection – a geometric interpretation of homogeneous coordinates
   (p) Overall scaling
   (q) Point at infinity
   (r) Transformations conventions

2. Three – Dimensional Transformations. [16 lectures]

   (a) Introduction
   (b) Three-Dimensional Scaling, shearing, rotation , reflection, translation

(c) Multiple Transformations
(d) Rotation about – an axis parallel to a coordinate axis, an arbitrary axis in space.
(e) Reflection through an arbitrary plane
(f) Affine and perspective geometry
(g) Orthographic Transformations.
(h) Axonometric Projections
(i) Oblique Transformations
(j) Single point perspective transformations
(k) Vanishing points

3. Plane Curves [10 lectures]
(l) Introduction
(m) Curve representation
(n) Non-parametric curves
(o) Parametric curves.
(p) Parametric representation of circle.
(q) Parametric representation of an ellipse.
(r) Parametric representation of parabola.
(s) Parametric representation of hyperbola.

4. Space Curves [6 lectures]
(t) Beizer curves- introduction, definition, properties (without proofs), curve fitting (up to $n=3$), equation of the curve in matrix form (up to $n = 3$).
(u) B – spline curves -introduction, definition, properties (without proof)

Text Books :
1. David Rogers and J. Alan Adams, Mathematical elements for computer graphics, McGraw Hill International Edition
2. M.E. Mortenson, Computer Graphics Handbook, Geometry and Mathematics, Industrial Press Inc.

OR

Paper MS-1 : Graph Theory and Lattice Theory

First Term :- **Graph Theory**

1. Introduction :
   Graphs, Finite graph, infinite graph, null graph, incidence and degree. [2 lectures]
2. Paths and Circuits :
   Isomorphism, Subgraphs, Walk, Path, Circuit, Connected and disconnected graphs, Components, self complementary graph, Union, intersection, ring sum of graphs, Euler graph, Fleury's Algorithm, Hamiltonian graph,
   If u and v are any two non – adjacent vertices of a graph G with $d(u)+d(v) \geq n$, then G is Hamiltonian iff $G + uv$ is Hamiltonian, Chinese postman problem, Traveling salesman problem. [8 lectures]
3. Trees and fundamental circuits :
   Trees, Equivalent characterizations, Distance and center in a tree, Rooted and binary trees, Spanning trees, Fundamental circuits, Finding all spanning trees, If e is cyclic edge of G then $\tau(G)= \tau(G-e)+ \tau(G/e)$ [10 lectures]
4. Cut- Sets and Cut-Vertices :
   Cut –Sets, Fundamental circuits, Fundamental cut-sets, Vertex connectivity, edge Connectivity, Separability. [8 lectures]
5. Vector spaces of a graph :
   Vectors and Vector space associated with a graph, Basis vectors of a graph, Circuit subspaces, Cut-set subspaces. [7 lectures]
6. Matrix representation :
   Incidence matrix, Adjacency matrix, Powers of adjacency matrix and connectedness of a graph. [5 lectures]
7. Directed graphs :
   Types of digraphs, Connectedness in a digraph, Euler digraph, Arborescence and Polish notation. [8 lectures]

Text Book :

Narsing Deo, Graph Theory, Prentice Hall of India Pvt. Ltd, 1987

Reference Books :

1. Clark and Holton , A first look at Graph Theory, Allied Publishers Limited, 1991
2. K.R. Parthasarathy, Basic Graph Theory, Tata McGraw Hill Publ.Co. Ltd.

<u>Second Term</u> :- <u>Lattice Theory</u>

1. Lattices:

   Poset, diagrammatic representation, maximal/ minimal elements of a subset of a poset , Infimum and supremum of a subset of a poset, least and greatest elements of a subset of a poset. Isomorphism, duality principle, dual poset, Lattice as a poset and as an algebra, Equivalence of two definitions of a lattice, distributive and modular inequalities in a lattice, sublattice, semilattice, complete lattice. [12 lectures]
2. Ideals and homomorphism:

   Ideals, union and intersection of ideals, dual ideal, prime ideal, Principal ideal, prime dual ideal, principal dual ideal, complements and relative complements
   Jordan – Dedekind Condition,
   The lattices n_5 and m_5
   Atoms, dual atoms, join/meet homomorphism ,
   Quotient lattice
3. Distributive Lattice :

   Distributive lattice, examples, Necessary and sufficient condition for a lattice to be distributive lattice, homomorphic image of a distributive lattice, Birkhoff's characterization of a distributive lattice. [10 lectures]
4. Boolean Algebra :

   Boolean lattice, Axiomatic Boolean Algebra, Properties of a Boolean algebra, Algebra of sets, Propositions, electric circuits form a Boolean algebra. Finite Boolean Algebra is isomorphic to Boolean algebra of all subsets of some finite set, ideals in a Boolean algebra.
   CNF and DNF
   Switching circuits and their simplification. [14 lectures]

Text Book :

V.K. Khanna, Lattices and Boolean Algebras,
Vikas Publ. House Pvt. Ltd, New Delhi

Reference Books :

1. Discrete Mathematics, Schaum's Outline Series
2. Birkhoff and Bartee, Modern Applied Algebra, McGraw Hill, 1970

Paper MS-2 : Number Theory & Complex Variables

First Term :- **Number Theory**

1. Divisibility :

Revision (Fundamental Theorem of arithmetic, g.c.d. of numbers expressed as their linear combination, Euclidean algorithm). There are infinitely many prime numbers. Fermat Primes, Mersenne Primes, There are arbitrary large gaps in the sequence of Primes. There are infinitely many Primes of the form $4n+3, 6n+1, 4n+1$. The statement of Dirichlet's theorem on Primes in arithmetic Progression.

[10 lectures]

2. Congruences :

Revision complete and reduced residue systems, Euler's theorem and Fermat's theorem as its corollary. Wilson's theorem, solution of congruences, Chinese remainder theorem, Prime modulus, Primitive roots and power residues.

(See Article Number) [20 lectures]

3. Some functions of number theory :

Greatest integer function, Arithmetic functions. The Mobius Inversion Formula.

[10 lectures]

4. Some Diophantine Equations :

The equation $ax + by = c$, Pythagorean Triangles. [8 lectures]

Text Book:

I. Niven, H. Zuckerman, H.L. Montgomery, An Introduction to the Theory of Numbers, 5th Edition, John Wiley & Sons, 1991

Ch. 1, Art. 1.1 to 1.3 (except Theorem 1.19); Ch. 2, Art. 2.1 to 2.3(except Theorem 2.20), Art. 2.7, Art. 2.8(Theorems 2.39, 2.40 and 2.41 statements only); Ch., 4, Art. 4.1 to 4.3; Ch. 5, Art. 5.1, Art. 5.3

Reference Books :

1. David Burton, Elementary Number Theory, 3rd Edition, WCB Publications, 1989
2. Neal Koblitz, A Course in Number Theory and Cryptography, 2nd Edition, Springer, 2000

Second Term : (Complex Variables)

1. Functions of complex variables :
   - 1.1 Definition and examples.
   - 1.2 Limit. Theorems on limits.
   - 1.3 Continuity.
   - 1.4 Derivative. Differentiable functions. Algebra of differentiable functions. Chain rule (without proof).
   - 1.5 Cauchy-Riemann equations. Sufficient conditions. C. R. equations in polar form. Formula for f1(Z0).
   - 1.6 Definition of analytic function. The difference between analytic and differentiable function.
   - 1.7 Harmonic functions. Given harmonic function to find corresponding analysis function. 14 Lectures
2. Elementary functions :
   - 2.1 Definition of exponential function and It's properties.
   - 2.2 Trigonometric functions, their properties.
   - 2.3 Hyperbolic functions, their properties.
   - 2.4 Logz and branches of logz. 8 Lectures

3. Intergrals :
   - 3.1 Contour, simple arc. Line integral. Proof of the result
   - 3.2 Statement of Cauchy-Goursat theorem. Definition of simply and multiply connected regions. Antiderivatives and independence of path.
   - 3.3 Cauchy Integral formula. Derivatives of analytical functions.
   - 3.4 Tayler series and Laurant series (statements only). Examples Zeros of analytic function. 14 Lectures

4. Residues and poles :
   - 4.1 Definition and examples of residue of a function.
   - 4.2 Residue Theorem. Principal part of the function.
   - 4.3 Poles and calculations of residues at poles.
   - 4.4 Evaluation of improper real integrals. Improper intergrals involving trigonometric functions. Definite intergrals of trigonometric functions. (Examples involving simple poles only).

12 Lectures

Prescribed Books

Complex Variables and Applications – R. V. Churchill, J. W. Brown (Fourth Edition) International Students Edition), Chapter 2 : Sections 9 to 20, Chapter 3 : Sections 21 to 25, Chapter 4 : Sections 29 to 33, 36 definitions only), Chapter 5 : Sections 44 to 46, 53, Chapters 6 : Sections 54 to 57, 59, 60, 61.